How Single-Photon Switching Is Quenched with Multiple Λ–Level Atoms

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Single-photon nonlinearity, namely, the change in the response of the system as the result of the interaction with a single photon, is generally considered an inherent property of a single quantum emitter. Although the dependence on the number of emitters is well understood for the case of two-level systems, deterministic operations such as single-photon switching or photon-atom gates inherently require more complex level structures. Here, we theoretically consider single-photon switching in ensembles of emitters with a Λ-level scheme and show that the switching efficiency vanishes with the number of emitters. Interestingly, the mechanism behind this behavior is the quantum Zeno effect, manifested in a slowdown of the photon-controlled dynamics of the atomic ground states.

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Nonlinear behavior at the single-quantum level is a fundamental physical phenomenon that is also at the heart of quantum technology applications. In optics, the practical absence of photon-photon interactions at the single-photon level in a vacuum makes it necessary to rely on coupling with quantum emitters in order to attain single-photon nonlinearity. The fact that strong light-matter coupling can be collectively enhanced in ensembles [1–6], makes it tempting to try to enhance the nonlinearity by exploiting multiple quantum emitters. However, coupling to multiple emitters has been found to reduce the nonlinearity rather than enhance it in certain cases, such as ensembles of two-level atoms interacting with a single optical mode. There, the central mechanism behind the nonlinearity, the photon blockade, was shown to be suppressed with the number of atoms [7,8]. This is unfortunate since obtaining high collective cooperativity $C_1 N_{\rm at} \gg 1$ with $N_{\rm at} > 1$ is typically easier than attaining large single-atom cooperativity $C_1 \gg 1$, which typically requires ultrahigh-quality resonators with microscopic cross sections.

Here, we wish to explore the possibility of enhancing a different kind of nonlinear behaviour, one that can support single-photon switching and deterministic photon-atom gates. Such operations cannot be performed by two-level atoms since they do not have any memory mechanism, as the system returns to the same ground state regardless of how many photons were scattered. This makes this type of nonlinearity similar to $\chi^{(2)}$ or Kerr, which is insufficient for photon-atom logic gates (due to time-bandwidth conflict) [9]. Switching, namely, a permanent change in the system after interaction with a single photon, requires at least a three-level Λ configuration with two ground states which resolves the time-bandwidth conflict [10]. Since ensembles of such three-level atoms can "count" the number of photons they interacted with, it is interesting to explore if this type of switching nonlinearity also vanishes with the number of emitters, and if so—what is the underlying mechanism. We do so by considering an intriguing "riddle" that involves multiple atoms in an optical cavity. Specifically, we consider atoms with the two ground levels $|\pm\rangle$ and the excited state $|0\rangle$ in a single-sided cavity. The cavity equally enhances the two modes of the "legs" of the Λ system (here taken to be σ_+ and σ_- polarizations), as depicted in Fig. 1(a). This configuration can be used to implement a single-photon memory [11] and photon-atom gates such the entangling $\sqrt{\rm SWAP}$ gate [12] and the SWAP gate [13,14], which maps a single photon qubit into an atomic qubit and vice versa. We wish to explore how this nonlinearity changes with the number of atoms $N_{\rm at}$ while keeping the collective cooperativity constant: $C_1 N_{\rm at} = {\rm const} \gg 1$.

The riddle we consider is a three-step protocol, depicted in Fig. 2: (a) By shining multiple V-polarized photons, all

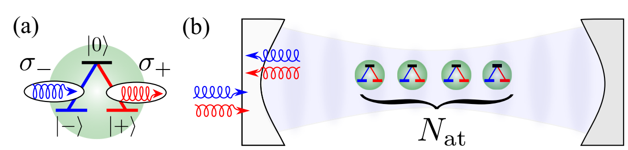

FIG. 1. (a) Illustration of the two atomic transitions selectively coupled to σ_+- and σ_- circularly polarized photons. (b) One-sided cavity (shown here with only the left mirror being partially transmissive) with an array of Λ atoms, coupled to right- and left-circularly polarized photons.

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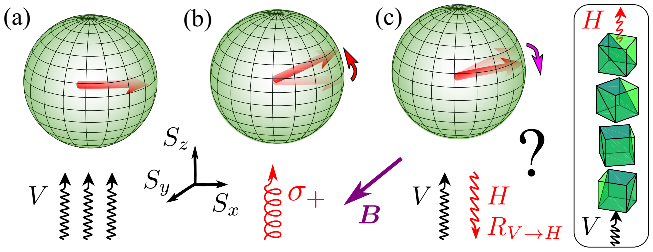

FIG. 2. Evolution of the collective spin of the Λ-atoms ensemble, driven by photons: (a) Spin is oriented along x by a strong V-polarized excitation. (b) One σ_+ photon slightly rotates the spin towards z. (c) The spin returns towards x after more V photons are sent. The question is how many of these photons are reflected with H polarization. External magnetic field $\boldsymbol{B}\|y$ can be also applied to rotate the spin. Inset in (c) illustrates an analogy with the quantum Zeno effect and nondemolition measurements: adiabatic photon polarization conversion by N gradually rotating beam splitters.

the atoms in the cavity are eventually pumped to the H superposition (dark) state:

$$|\psi_{\rm in}\rangle = \frac{|+_1\rangle + |-_1\rangle}{\sqrt{2}} \otimes \frac{|+_2\rangle + |-_2\rangle}{\sqrt{2}} \otimes \dots \frac{|+_N\rangle + |-_N\rangle}{\sqrt{2}}. \tag{1}$$

The fact that Eq. (1) is an equilibrium state will be rigorously proved below. It can be also understood intuitively by noticing that this state does not interact with V photons due to the parity selection rules. (b) A single σ_+ polarized photon is incident on the cavity. The V polarization component interacts with a bare cavity, while the H polarization interacts with a cavity strongly coupled to multiple atoms. The π phase shift associated with this condition flips the polarization of the photon to σ_- as it reflects from the cavity. (c) Clearly, the state of the atoms has now been perturbed, and cannot be assumed to remain the perfect dark state for V-polarized photons. We aim to verify this expectation by sending again many V-polarized photons, and asking, *how many of these photons will return as H, and not stay V as they would have, had we not sent the σ_+ photon in step (b)*?.

We believe that this problem is not trivial. It is not immediately obvious how to apply intuitions derived from the Kerr-type nonlinearity with two-level atoms [7,8] to the memory-type nonlinearity (see also [15]). One could expect, for example, that since the dark state of the ensemble was modified by reflecting one photon, it will also need to reflect one photon in total before going back to its original state. This would amount to a strong single-photon nonlinearity. Another expectation is that this reflection process will be faster when the ensemble has more atoms, as in the case of Dicke superradiance [23]. We will show that both these expectations are wrong. The average total number of reflected H photons $N^H_{\rm tot}$ is always below 0.5 (being the limit of a perfect SWAP gate with a single atom), and is suppressed by a factor of $1/N_{\rm at}$ for a large number of atoms $N_{\rm at} \gg 1$; moreover, the dynamics are slowed down and more V photons are required to reach this value. This leads to the intriguing question: "where did the angular momentum of the σ_+ photon (reflected as σ_-) go?"

Quantum Zeno effect—We can gain some intuition by considering another well-known scenario that involves repeated measurements: the quantum Zeno effect (QZE) [24,25]. In the QZE, frequent quantum nondemolition (QND) measurements are applied at an axis that is identical to the initial state of the system. If the measurements are frequent enough so that the system can barely evolve or be perturbed between them, and assuming (as is usually the case) that for slight perturbations, the overlap with the original state drops quadratically, then the system is most likely to continuously collapse to the original state. This effect can be used either to "freeze" the evolution (hence the name Zeno), or to adiabatically "drag" the state by gradually changing the axis of the QND measurement. As an example, consider a vertically polarized photon going through a series of N cubic polarizing beam splitters, which gradually rotate by $90°/N$ each until the last one is perpendicular to the first [inset in Fig. 2(c)]. The probability that each beamsplitter will reflect the photon is proportional to $1/N^2$, and the overall scattering probability goes down like $1/N$. The intriguing element here is that seemingly nothing is happening: the photon is never scattered, and yet its polarization is rotated. Here as well, we may ask "where did the momentum go?" The answer is that the lack of scattering (quantum jumps) allows coherent evolution to the new state, and the backaction is also transferred coherently to the environment, which is usually not part of the model. The force applied to the beam splitters accumulates coherently, and the momentum is transferred unnoticed through the holders of the beam splitters to the optical table and Earth. This also solves our problem: the backaction [due to the momentum stored in the atomic ensemble in stage (b)] causes the V-polarized photons at stage (c) to become very slightly elliptical. If we analyze these photons by a polarizing beam splitter, we get a similar situation: even though, for $N_{\rm at} \gg 1$, none of the photons is reflected to the H port, a force is applied on that beam splitter until the momentum is coherently transferred from the atoms to Earth, and the atoms return to their initial dark state (a). With that intuition in mind, we now present a more rigorous treatment.

Model and theoretical framework—Photon interaction with multilevel atoms, including Λ atoms, can be treated directly using the generalized input-output formalism [26]. However, this brute-force technique is relatively computationally expensive because of the large Hilbert space size. It is appealing to develop analytical tools tailored to Λ-atom arrays. We draw inspiration from the scattering matrix approaches for two-level atoms [27–29]. The photon scattering calculations for the Λ-atoms turn out, however, to be more involved because of the 2^N degeneracy of the ground states. This may explain why despite the considerable recent progress, theoretical efforts still primarily focus on relatively simple states of the Λ-atom ensembles [30–34], except for only a few recent studies of squeezing [35,36] and superradiant bursts [37]. Another notable exception is Ref. [38], where multiple photon scattering problem on $N_{\rm at} = 1$ atom has been solved using the Green's function approach and thus generalizing Refs. [27–29]. For $N_{\rm at} > 1$ atoms, the multiple scattering problem remains unsolved.

In this Letter, we introduce a rigorous theoretical approach for calculating the consecutive scattering of multiple photons on Λ-atom ensembles. This approach is naturally suited to the description of the photon switching protocols where the photons are incident upon the system one by one rather than simultaneously. In this simplified case, we obtain explicit analytical results for the photon scattering amplitudes.

We assume that the linewidth of the atoms is much smaller than the cavity linewidth. In this case, the Markovian approximation is valid and the photonic degrees of freedom can be traced out [26]. We introduce the following effective Hamiltonian $\hat{H} = \omega_0 \sum_{n=1}^{N_{\rm at}} |0,n\rangle\langle 0,n| + \sum_{\nu=\pm}\sum_{m,n=1}^{N_{\rm at}} \sigma_{\nu,n}^{\dagger}\sigma_{\nu,m}D_{nm}$. Here, ω_0 is the resonance frequency of the atomic transitions, and $\sigma_{\nu,n}^{\dagger}$ are the corresponding raising operators. Since each of the $N_{\rm at}$ atoms has two ground states $|\pm,n\rangle$ and a single excited state $|0,n\rangle$, the nonzero matrix elements are $\langle 0,n|\sigma_{+,n}^{\dagger}|+,n\rangle = \langle 0,n|\sigma_{-,n}^{\dagger}|-,n\rangle = 1$. The matrix D_{nm} is proportional to the Green's function that describes a photon emitted at atom m and reabsorbed at atom n. We focus on the purely dissipative coupling between the atoms, $D_{nm} = -{\rm i}\gamma_{\rm 1D}/2$. This corresponds to either atoms at distances smaller than the light wavelength λ but still far enough so that short-range dipole-dipole interactions could be ignored. It is also possible to put the atoms further apart at different antinodes of the standing wave, which can be now also realized experimentally [39–42]. The Hamiltonian reduces to

$$\hat{H} = \omega_0 \sum_{n=1}^{N_{\rm at}} |0,n\rangle\langle 0,n| - {\rm i}\left(\frac{\gamma_{\rm 1D}}{2} + \gamma\right)\sum_{\nu=\pm} \sigma_{\nu}^{{\rm tot},\dagger}\sigma_{\nu}^{\rm tot}. \quad (2)$$

Here, just like for two-level atom ensembles described by the celebrated Dicke model [23,43], the collective interaction with photons involves only the total spin operators $\sigma_{\nu}^{\rm tot} = \sum_{n=1}^{N_{\rm at}} \sigma_{\nu,n}$, with $\nu = +$ or $-$. The parameter $\gamma_{\rm 1D}$ is the radiative decay rate into the cavity mode and the phenomenological decay rate γ accounts for emission into nonresonant photon modes and other decay processes. A more general Hamiltonian, including local disorder and does not conserve total spin, is analyzed in [15].

We are interested in the weak excitation limit, when the array is illuminated by photons one by one and is never doubly excited. The optical transitions will then occur only between the 2^N ground states $|\pm,n\rangle$ and the $N2^{N-1}$ single-excited states $|\psi_1^{(\mu)}\rangle$, where exactly one atom is excited to $|0,n\rangle$. We also introduce the Green's function

$$G(\omega) = {\rm i}\gamma_{\rm 1D}\sum_{\mu\mu'} |\psi_1^{(\mu)}\rangle \left[\frac{1}{H_1 - \hat{1}\omega}\right]_{\mu\mu'} \langle\psi_1^{(\mu')}|, \quad (3)$$

where $[H_1]_{\mu\mu'} \equiv \langle\psi_1^{(\mu)}|H|\psi_1^{(\mu')}\rangle$. The calculation, detailed in [15], yields the state of the atomic ensemble after scattering the photon:

$$|\psi_{{\rm scat},\nu\to\nu'}\rangle = [\delta_{\nu,\nu'} + \sigma_{\nu'}^{\rm tot} G(\omega)\sigma_{\nu}^{{\rm tot},\dagger}]|\psi_{\rm ground}\rangle, \quad (4)$$

where $|\psi_{\rm ground}\rangle$ is the state before the scattering and ν,ν' are incident and scattered photon polarizations. The first term describes a photon reflected directly from the left cavity mirror, without interacting with atoms, while the second term involves photon interaction with the single-excited states. The corresponding photon reflection coefficient is given by the expectation value $R_{\nu\to\nu'} \equiv \langle\psi_{{\rm scat},\nu\to\nu'}|\psi_{{\rm scat},\nu\to\nu'}\rangle$. The atomic state transformation Eq. (4) is unitary for $\gamma = 0$: $\sum_{\nu'} R_{\nu\to\nu'} = 1$. In order to obtain the total number of reflected photons $N_{\rm tot}^{H}$ after consecutive interaction with $N_V > 1$ V photons we apply Eq. (4) iteratively as detailed in [15].

Spin dynamics and QZE—By considering multiple V-polarized photons, we have verified that the state Eq. (1) is indeed the equilibrium state of ensemble after step (a) of the protocol in Fig. 2, see also [15].

Next, we discuss the steps in Figs. 2(b) and 2(c). We introduce a (pseudo)spin-1/2 $\boldsymbol{s}^{(j)}$ operator acting in the space spanned by the two ground states of the atom, e.g., $s_z^{(j)}|\pm\rangle = \pm|\pm\rangle/2$. Because of the symmetry of the problem, incoming photons are coupled only to the collective spin of the array $\boldsymbol{S} = \sum_{j=1}^{N_{\rm at}} \boldsymbol{s}^{(j)}$. After the initialization step $\boldsymbol{S} = N_{\rm at}\boldsymbol{e}_x/2$, see Fig. 2(a). Scattering of the circularly polarized photon transfers the angular momentum to the ensemble, and the collective spin is rotated in the x–z plane towards z [Fig. 2(b)]. Indeed, as follows from Eq. (4), this scattering process is described by the operator $\sigma_{-}^{\rm tot}G(\omega)\sigma_{+}^{{\rm tot},\dagger} = {\rm i}\alpha(\omega)(S_x - {\rm i}S_y)$, where $\alpha(\omega) = -\gamma_{\rm 1D}/[\omega - \omega_0 + ({\rm i}/2)\gamma_{\rm 1D}(N_{\rm at}+1) + {\rm i}\gamma]$ is the effective polarizability [15]. The imaginary term $({\rm i}/2)\gamma_{\rm 1D}(N_{\rm at}+1)$ increases linearly with $N_{\rm at}$, reflecting collective enhancement of the atom-photon coupling. The ratio between the radiative and nonradiative decay rates in the denominator gives the collective cooperativity $C_{N_{\rm at}} = \gamma_{\rm 1D}(N_{\rm at}+1)/(2\gamma)$. The spin operator $S_x - {\rm i}S_y$ describes the increase of the absolute value of the z projection of the collective pseudospin by 1 during the scattering of the first circularly polarized photon.

We expect that at the next step, Fig. 2(c), after the interaction with V-polarized photons the collective spin will rotate in the x-z plane back to its original direction along x, described by Eq. (1), but some light will be reflected in H polarization during this relaxation stage. This directly follows from the matrix elements of atom-photon interaction. The relevant operators describing spin evolution are $\sigma_V^{\rm tot} G \sigma_V^{{\rm tot},\dagger} = {\rm i}\alpha[(N_{\rm at}/2) - S_x]$, and $\sigma_H^{\rm tot} G \sigma_V^{{\rm tot},\dagger} = {\rm i}\alpha(S_z - {\rm i}S_y)$. Hence, application of a large number of V photons should drive the spin to the eigenstate of S_x operator. After the first σ_+ photon is scattered we obtain $\langle S_z\rangle \neq 0$, $\langle S_y\rangle = 0$, and as long as $\langle S_z\rangle$ remains nonzero the expectation value of $\sigma_H^{\rm tot} G \sigma_V^{{\rm tot},\dagger}$ will also be nonzero and the array will be able to reflect photons in H polarization. In particular, the reflection coefficient of the first V photon is given by the matrix element of $\sigma_H^{\rm tot} G \sigma_V^{{\rm tot},\dagger}$ between the states with $S_z = 0$ and $S_z = 1$, that is $R^{(1)}_{V\to H} = |\alpha|^2$. The final state, after a large number N_V of incident V photons, will be the state with $\boldsymbol{S}\|x$, $S_x = N_{\rm at}/2$ and $S_z = 0$, so that $V \to H$ photon scattering process will be no longer possible. It is this pinning of the spin to the x axis by subsequent photon scattering events that we interpret as a quantum Zeno effect [24,25,44–46]. Indeed, the spin dynamics is slowed down by the observer trying to optically probe the spin state, which is the essence of QZE.

We have performed a detailed simulation of the photon reflection process. We calculate the average projection of the total spin $\langle S_z\rangle$ and the total number of reflected H photons $N^H_{\rm tot}(N_V) \equiv \sum_{j=1}^{N_V} R^{(j)}_{V\to H}$ depending on the number of incident V-polarized photons N_V. Here, the reflection coefficient $R^{(j)}_{V\to H}$ is the probability to reflect the jth incident V-polarized photon with an H polarization. Figure 3 shows the calculation results, described by [15] $N^H_{\rm tot}(N_V) = N^H_{\rm tot}(\infty)(1 - |\chi|^{2N_V})$, with $N^H_{\rm tot}(\infty) = N_{\rm at} C_{N_{\rm at}}/[(N_{\rm at} C_{N_{\rm at}} + N_{\rm at} + 1])(N_{\rm at\,+1})]$ and $\chi = 1 + {\rm i}\alpha$. In particular, Fig. 3(a) shows how $N^H_{\rm tot}$ in the limit $N_{\rm V} \to \infty$ depends on the number of atoms $N_{\rm at}$ and on the single-atom cooperativity C_1. One could expect dependence only on the collective cooperativity $C_{N_{\rm at}} = N_{\rm at} C_1$. This would mean the same outcome for a large $N_{\rm at}$ with low C_1 and a single atom with high C_1, provided that $C_{N_{\rm at}}$ stays the same. However, this is not the case. The value of $N^H_{\rm tot}$ monotonously decreases with $N_{\rm at}$ along the line of constant $C_{N_{\rm at}}$ [dotted curve in Fig. 3(a)]. The same effect is illustrated in Fig. 3(b) that shows $N_{\rm at}$ versus N_V at the four points of constant $C_{N_{\rm at}}$, indicated by triangles in Fig. 3(a). Not only does the limit $N^H_{\rm tot}(N_V \to \infty)$ decrease for larger $N_{\rm at}$, but also the value of N_V required to approach this limit increases. In other words, if the V photons arrive periodically in time, the dynamics of $N^H_{\rm tot}$ is slowed down. The slowdown is somewhat counterintuitive, since the photons are coupled only to collective Dicke states of the array, typically associated with faster dynamics. Here, however, the larger $N_{\rm at}$ and the stronger the interaction with the Dicke state, the slower the dynamics. This happens because the relevant photon reflection process is quenched by the collectively enhanced spontaneous decay rate. It is this collective slowdown of the spin evolution that we interpret as a Zeno effect responsible for the suppression of the single-photon nonlinearity for large $N_{\rm at}$.

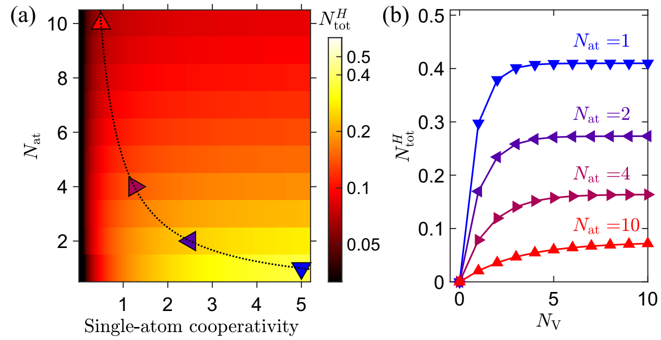

FIG. 3. (a) Total number of reflected H photons $N^H_{\rm tot}$ in the limit $N_{\rm V} \to \infty$ depending on the single-atom cooperativity $\gamma_{\rm 1D}/\gamma$ and the number of atoms $N_{\rm at}$. (b) Dependence of $N^H_{\rm tot}$ on $N_{\rm V}$ for different values of $N_{\rm at}$. For each value of $N_{\rm at}$, we tune the decay γ to keep constant the collective cooperativity, $C_{N_{\rm at}} = \gamma_{\rm 1D}(N_{\rm at}+1)/(2\gamma) = 5$. Corresponding values of single-atom cooperativity $C_1 = \gamma_{\rm 1D}/\gamma$ are shown in (a). The calculation has been performed for $\omega = \omega_0$ and $B = 0$.

The QZE interpretation can be corroborated by analyzing the dependence of the $\langle S_z\rangle$ spin projection on N_V, given by $S_z(N_V) = S_z(0){\rm Re}(\chi^{N_V})$ or $S_z(N_V) = S_z(0){\rm Re}\,{\rm e}^{{\rm i}\alpha N_V}$ for $\alpha \ll 1$. Thus, after a large number of incident photons the spin returns to the stationary value along the x axis. The corresponding dynamics is shown by filled triangles in Fig. 4. It is instructive to study the effect of an external static magnetic field along the y direction. Such a field leads to the rotation of the collective spin in the x-z plane

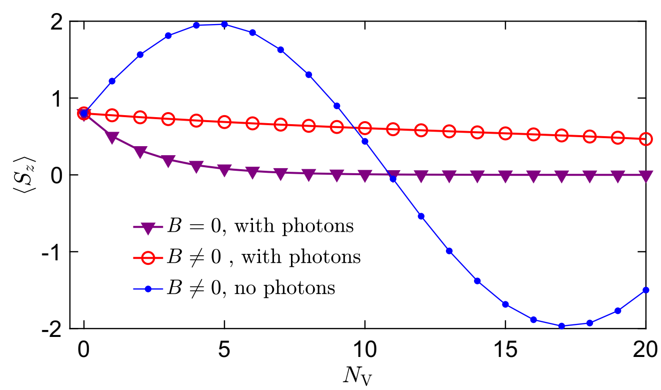

FIG. 4. Average spin projection $\langle S_z\rangle$ calculated with and without applied external magnetic field $\boldsymbol{B}$ and incoming V photons. Calculation has been performed for the magnetic field strength parameter $\varphi = 0.25$, $N_{\rm at} = 4$, and $C_{N_{\rm at}} = 5$.

[see Fig. 2(c)], which competes with the Zeno effect. To describe this we introduce a total spin rotation $\psi' = \exp(i\varphi S_y)\psi$ to the wave function between the photon scattering events. Here, φ is the spin rotation angle during the time between the two incident V photons. This simplified description assumes that the Zeeman splitting is much smaller than the resonance linewidth and optical selection rules are not modified. The calculated spin dynamics is shown in Fig. 4 by the circles. Small filled circles show the free spin rotation induced by the magnetic field without any incoming photons. When the V-polarized photons are sent upon the system (red open circles) the oscillations are replaced by the slow decay of the spin toward the equilibrium state. This further supports our quantum Zeno effect interpretation.

Summary—We have developed an analytical framework to describe the consecutive interaction of the Λ-atom ensemble with multiple incident photons. Our calculations reveal the quantum Zeno effect as the mechanism that suppresses photon switching in large arrays. The conditions required for the demonstration of this effect, namely interacting with a controlled (or post-selected) number of atoms within a single-sided cavity with single-atom cooperativity $C_1 > 1$, should be within the existing capabilities of a number of cavity-QED labs (see for example, Refs. [39–42]). We expect even more interesting physics of collective light-matter interactions in the strong excitation regime, or when the ensemble is driven by nonclassical states of light.

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