Comment on "Excitation Spectrum and Superfluid Gap of an Ultracold Fermi Gas"

At zero temperature, as any three-dimensional superfluid with short-range interactions, a gas of paired fermionic atoms exhibits an acoustic excitation branch of low wave-number expansion $\omega_q = cq[1 + \zeta q^2/k_F^2 + O(q^4 \ln q)]$ with c the speed of sound and ζ the curvature parameter (scaled by the Fermi wave number k_F). Reference [1] claims to have experimentally determined whether the branch has a convex $\zeta > 0$ or concave $\zeta < 0$ start, depending on the interaction strength. This is crucial information that dictates the nature of the gas relaxation mechanisms at low temperatures (the well studied three-phonon Beliaev-Landau or the yet unobserved four-phonon Landau-Khalatnikov mechanism [2,3]). However, in this Comment, we argue that the high wave numbers q and temperature T used in the experiment introduce a large bias capable of turning a convex branch into a concave one, so that further measurements at lower q and T are required to give a definitive answer.

To fix ideas, we consider the unitary limit $1/k_F a = 0$ with a the s-wave scattering length, where the interactions are strongest and no solid argument can predict the sign of ζ. Theoretically, one has $\zeta = -\pi^2 (2\xi_B)^{1/2} [c_1 + (3/2)c_2]$ where Bertsch's parameter gives the chemical potential in units of the Fermi energy $\mu = \xi_B E_F$ and $c_{1,2}$ quantify gradient corrections to quantum hydrodynamics [4]. Only ξ_B is well known, $\xi_B \simeq 3/8$ [5]. The dimensional expansion in powers of $\epsilon = 4 - d = 1$ gives $c_1 \simeq -0.0624(1 - 2\epsilon/3) + O(\epsilon^2)$ and $c_2 = O(\epsilon^2)$ [6], so $\zeta > 0$ to subleading order. Anderson's random-phase approximation (RPA), spectrally equivalent to the Gaussian fluctuations approximation of [7], also predicts a positive value $\zeta_{RPA} \simeq 0.0838$ [8] (for $c_1 \simeq -0.021$ [6] this gives $c_2 \simeq 0.0073 \ll |c_1|$). The experimental value $\zeta_{exp} = -0.085(8)$ [1] is negative. However, assuming that the RPA is correct and that the branch start is convex, as we will do, actually has no clear incompatibility with the experiment, because the analysis in [1] suffers from two serious limitations.

First, the value ζ_{exp}, obtained by cubic fitting of ω_q [1], could strongly depend on the fitting interval if too wide. In the RPA, fitting ω_q, e.g., to the interval $0.22 \le q/k_F \le 1.08$ of Fig. 1 in [1] gives $\zeta_{RPA}^{fit} \simeq -0.026$, which even has the wrong sign. Since ω_q^{RPA} has an inflection point at $q/k_F \simeq 0.5$ [8], the fit blindly mixes convex and concave parts, which also explains the erroneous (negative) value of ζ_{RPA} in [9].

Second, the high temperature $T \simeq 0.13 T_F \simeq 0.8 T_c$ (T_c is the superfluid transition temperature) in [1] could modify the curvature of the acoustic branch by a non-negligible amount $\delta\zeta^{\phi\phi}$ via interaction with thermal phonons ϕ. Treating the cubic phonon-phonon coupling $H_3^{\phi\phi}$ to second order and the quartic coupling $H_4^{\phi\phi}$ to first order, then taking the limit $k_B T/mc^2 \to 0$ (m is the fermion mass), [10] obtains an expression for the thermal shift of ω_q. This gives $\delta\zeta^{\phi\phi} \sim -[\pi^2/(3\xi_B)^{3/2}](T/T_F)^2$, or $\delta\zeta^{\phi\phi} \simeq -0.140$ at the experimental temperature. Since the small parameter used $k_B T/mc^2 \simeq 0.5$ is not $\ll 1$, we abandon the $T \to 0$ limit and add corrective curvature factors $(1 \pm \alpha q^2/k_F^2)$ to the amplitudes ρ_q and ϕ_q of the superfluid density and phase quantum fluctuations [$\alpha = \pi^2(\xi_B/2)^{1/2} [c_1 - (3/2)c_2] \simeq -0.136$ [3]]. We find $\delta\zeta^{\phi\phi} \simeq -0.110$, still negative enough to change the sign of curvature in the RPA. Furthermore, thermal pair dissociation creates fermionic quasiparticles γ (another gas excitation branch) that interact with the phonons. Treating to second order the coupling $H_3^{\phi\gamma}$ and to first order the coupling $H_4^{\phi\gamma}$ given in [11] with curvature factors in ω_q, ρ_q, and ϕ_q, we find $\delta\zeta^{\phi\gamma} \simeq -0.052$. This is a rough estimate: [11] uses a simple local-density approximation whose small parameter $(k_B T/m_* c^2)^{1/2}$ is ≈ 1 here (we use the γ dispersion relation of [12] with the effective mass $m_* \simeq 0.56m$ and an energy minimum $\Delta = 0.44 E_F$ located at wave number $k_0 = 0.92 k_F$ [13]). Summing the thermal corrections gives $\zeta_{RPA}^{th} \simeq -0.078$ to compare with $\zeta_{exp} = -0.085(8)$.

The experiment, at first sight at odds with the RPA, could therefore very well be in agreement with it due to large finite-momentum and thermal bias, and the sign of curvature announced in [1] may differ from the zero-temperature low-momentum one relevant for phonon damping.

Acknowledgments—We thank Alice Sinatra for her comments on the text.

Yvan Castin
Laboratoire Kastler Brossel, ENS-Université PSL
CNRS, Université Sorbonne and Collège de France
24 rue Lhomond, 75231 Paris, France

Received 11 December 2023; accepted 30 July 2024; published 4 September 2024
DOI: 10.1103/PhysRevLett.133.109301

[1] H. Biss, L. Sobirey, N. Luick, M. Bohlen, J. J. Kinnunen, G. M. Bruun, T. Lompe, and H. Moritz, Excitation spectrum and superfluid gap of an ultracold Fermi gas, Phys. Rev. Lett. **128**, 100401 (2022).
[2] L. Landau and I. Khalatnikov, Teoriya vyazkosti Geliya-II, Zh. Eksp. Teor. Fiz. **19**, 637 (1949).
[3] H. Kurkjian, Y. Castin, and A. Sinatra, Three-phonon and four-phonon interaction processes in a pair-condensed Fermi gas, Ann. Phys. (Berlin) **529**, 1600352 (2017).
[4] D. T. Son and M. Wingate, General coordinate invariance and conformal invariance in nonrelativistic physics: Unitary Fermi gas, Ann. Phys. (Amsterdam) **321**, 197 (2006).
[5] Mark J. H. Ku, A. T. Sommer, L. W. Cheuk, and M. W. Zwierlein, Revealing the superfluid lambda transition in the

universal thermodynamics of a unitary Fermi gas, Science **335**, 563 (2012).

[6] G. Rupak and T. Schäfer, Density functional theory for non-relativistic fermions in the unitarity limit, Nucl. Phys. **A816**, 52 (2009).

[7] R. B. Diener, R. Sensarma, and M. Randeria, Quantum fluctuations in the superfluid state of the BCS-BEC crossover, Phys. Rev. A **77**, 023626 (2008).

[8] H. Kurkjian, Y. Castin, and A. Sinatra, Concavity of the collective excitation branch of a Fermi gas in the BEC-BCS crossover, Phys. Rev. A **93**, 013623 (2016).

[9] R. Haussmann, M. Punk, and W. Zwerger, Spectral functions and rf response of ultracold fermionic atoms, Phys. Rev. A **80**, 063612 (2009).

[10] Y. Castin, A. Serafin, and A. Sinatra, Amortissement des phonons dans un superfluide 2D: Insuffisance de la règle d'or de Fermi à basse température, C.R. Phys. **24**, 187 (2023).

[11] Y. Castin, A. Sinatra, and H. Kurkjian, Landau phonon-roton theory revisited for superfluid ^{4}He and Fermi gases, Phys. Rev. Lett. **119**, 260402 (2017).

[12] Y. Nishida and D. T. Son, ϵ Expansion for a Fermi gas at infinite scattering length, Phys. Rev. Lett. **97**, 050403 (2006).

[13] A. Schirotzek, Y. I. Shin, C. H. Schunck, and W. Ketterle, Determination of the superfluid gap in atomic Fermi gases by quasiparticle spectroscopy, Phys. Rev. Lett. **101**, 140403 (2008).