Vortex Spin Liquid with Fractional Quantum Spin Hall Effect in Moiré Chern Bands

Ya-Hui Zhang
William H. Miller III Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland, 21218, USA

(Received 24 February 2024; revised 14 May 2024; accepted 30 July 2024; published 4 September 2024)

Recently there is a report of the experimental signatures of a fractional quantum spin hall (FQSH) state at hole filling $n = 3$ in a twisted $MoTe_2$ bilayer. Previous theories of FQSH phases simply considered a decoupled pair of a fractional quantum Hall phase and its time reversal partner. Here, we show the first construction of an FQSH phase beyond the decoupling picture. We consider a pair of half-filled $C = \pm 1$ Chern bands in the two valleys, similar to the well-studied quantum Hall bilayer, but now with opposite chiralities. Because of the strong intervalley repulsion, we expect a charge gap to open with low-energy physics dominated by the neutral intervalley excitons. However, the presence of an effective "flux" frustrates exciton condensation by proliferating vortices. Here, we construct a vortex liquid of excitons dubbed a vortex spin liquid, formed from exciton pairing of the composite fermions in the decoupled composite Fermi liquid phase. This insulator is a quantum spin liquid with gapless spin excitations carried by the flux of an emergent $U(1)$ gauge field. Additionally, there exist neutral and spinless Fermi surfaces formed by fermionic vortices of a nearby intervalley-coherent order. Unlike a conventional Mott insulator, the vortex spin liquid phase also exhibits a quantized FQSH effect with gapless helical charge modes along the edge. Our work demonstrates the possibility of nontrivial FQSH phases and provides predictions to detect them in future experiments.

DOI: 10.1103/PhysRevLett.133.106502

Introduction—Experimental observations of fractional Chern insulator [1–9] at zero magnetic field in twisted MoTe2 [10–13] and in pentalayer rhombohedrally stacked graphene aligned with hBN [14] are among the most important breakthroughs in condensed matter physics in recent years. Theoretically, quantum anomalous Hall and fractional quantum anomalous Hall phases were expected in the moiré systems with nearly flat Chern bands following spontaneous valley polarization [15,16]. For the twisted $MoTe_2$ system with the twist angle $\theta \approx 3.7°$, it is now widely believed that the first moiré band mimics the lowest Landau level quite well and thus supports familiar quantum Hall physics [17–28]. The quantum anomalous Hall effect in the pentalayer graphene likely arise from a different mechanism, with the Chern band itself generated from an interaction-driven spontaneous crystal formation [29–33].

More recently, experimental signatures of the fractional quantum spin Hall (FQSH) effect were reported in the twisted $MoTe_2$ system at the hole filling $n = 3$ with a smaller twist angle $\theta = 2.1°$ [34]. Here, the state has zero Hall resistivity and supports helical edge modes with the edge conductance $G = \frac{3}{2}(e^2/h)$. The $n = 3$ filling can be naturally understood as the filling $n = \frac{1}{2} + \frac{1}{2}$ of the second moiré band on top of the $n = 2$ quantum spin Hall (QSH) insulator with the first moiré band fully filled. Therefore, the experimental result implies a phase with half quantum spin Hall effect at half filling of the second band in each valley. Here, we consider the theoretical possibility of FQSH phases in a pair of $C = \pm 1$ flat bands. While this work is motivated by the experiment in twisted $MoTe_2$, the theory focuses on universal physics and hence may be applied to other systems. We can restrict to the $n = \frac{1}{2} + \frac{1}{2}$ filling of the second moiré band. Given that the two valleys have Chern numbers $C = +1, -1$, respectively, the physics is similar to a quantum Hall bilayer [35] but with opposite chiralities in the two layers. One obvious candidate for a FQSH insulator is a decoupled phase with a pair of quantum Hall states in the two valleys. At half filling, we can have a pair of composite Fermi liquids (CFLs) [36] or a pair of non-Abelian Pfaffian states [37]. But the decoupled phase lacks intervalley correlations to reduce the Coulomb repulsion and is thus not energetically favorable.

The above observations motivate us to look for an alternative explanation of the experiment. At filling $n = \frac{1}{2} + \frac{1}{2}$, we can start from a pair of CFLs in the two valleys with opposite chiralities. This conjugate CFL has also been discussed in Ref. [38]. Here, we are interested in the possible proximate insulators due to a large intervalley repulsion. In the conventional quantum Hall bilayer with the same chirality, the ground state is the exciton condensation phase [35], which can be conveniently described by Cooper pairing of the composite fermions [39]. In our case there is a frustration to exciton condensation due to an effective flux. Instead, we show that a vortex liquid of the exciton can be reached by exciton pairing of the composite fermions. The resulting phase has a charge gap, but hosts neutral Fermi surfaces and gapless spin excitations, resembling a quantum spin liquid in a Mott insulator. We dub this new fractional insulator as vortex spin liquid (VSL) because the spin is carried by the internal flux of a $U(1)$ gauge field and the neutral fermions are vortices of a nearby intervalley-coherent (IVC) order. Meanwhile, the VSL phase still supports the half QSH effect and helical charged edge modes in agreement with the experiment [34]. We propose future experiments to test this phase by detecting the separation of the spin and charge gaps, metallic spin susceptibility and spin transport, quantum oscillations, and thermal Hall effect under Zeeman field. We restrict to the sector of total $S_z = 0$ or partially spin polarized sector. In most of the parameter space, we actually expect the fully valley-polarized state with maximal S_z as the ground state [16]. The $S_z = 0$ or partially spin polarized state may be stabilized only in a small parameter regime, which we comment on at the end of the Letter.

Conjugate composite Fermi liquid—We consider a moiré superlattice with valley Chern number $C = 1$. For example, in twisted $MoTe_2$ bilayer, the valley K (locked to spin up) and valley K' (locked to spin down) electrons are in a narrow Chern band with Chern numbers $C = 1$ and $C = -1$, respectively. At filling $n = \frac{1}{2} + \frac{1}{2} = 1$, we restrict to the valley-unpolarized states. First, let us turn off the intervalley repulsion by hand. Then we have two half-filled Chern bands with opposite chiralities. One obvious phase is a pair of CFLs in the two valleys. We will start from the field theory of this conjugate CFL phase and then add intervalley correlations. We describe each CFL by the standard Halperin-Lee-Read theory from flux attachment [36]. In the following we label the K, K' valley as $+, -$. The Halperin-Lee-Read theory of the conjugate CFL (cCFL) is

$$\mathcal{L}_{\mathrm{cCFL}} = L_{\mathrm{FS}}[f_\pm, a_\pm] + \frac{1}{8\pi}(a_+ da_+ - a_- da_-) - \frac{1}{4\pi}(A_+ da_+ - A_- da_-) + \frac{1}{8\pi}(A_+ dA_+ - A_- dA_-), \tag{1}$$

where $A_\pm$ is the abbreviation of the probing $U(1)$ gauge fields $A_\mu^\pm$ for the $U(1)$ global symmetry corresponding to the charge conservation of the two valleys, respectively. $a_\pm$ is the abbreviation of the emergent dynamical $U(1)$ gauge fields $a_\mu^\pm$. adb is the abbreviation of the Chern-Simons term $\epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho$ with the ϵ as the antisymmetric tensor.

The composite fermions f_τ just form decoupled Fermi surfaces (FSs) coupled to the internal U(1) gauge fields:

$$\mathcal{L}_{\mathrm{FS}}[f_\pm, a_\pm] = \sum_{\tau=\pm} f_\tau^\dagger(t,\mathbf{x})(i\partial_t + a_0^\tau + \mu) f_\tau(t,\mathbf{x}) + \frac{\hbar^2}{2m}\sum_{\tau=\pm} f_\tau^\dagger(t,\mathbf{x})(-i\partial_\mu - a_\mu^\tau)^2 f_\tau(t,\mathbf{x}), \tag{2}$$

where we use the Minkowski metric. We assume simple $(k^2/2m)$ dispersion for the fermion and ignore the lattice effect for now. The chemical potential is introduced to fix the density of f_τ to be half filling per moiré unit cell. Within each unit cell there is, on average, one flux for each valley. In the theory, now f_τ should be viewed as a composite fermion in the valley $\tau = \pm$. It is also useful to redefine $A_\mu^c = \frac{1}{2}(A_\mu^+ + A_\mu^-)$ and $A_\mu^s = A_\mu^+ - A_\mu^-$. The corresponding charges are $(Q_c, Q_s) = \{Q_+ + Q_-, [(Q_+ - Q_-)/2]\}$. One can see that they are just the charge and the spin S_z, while Q_+, Q_- are charges in each valley. In the following we also use (Q, S_z) to indicate (Q_c, Q_s).

VSL from exciton pairing of composite fermions—The cCFL phase is justified only in the decoupling limit. Now let us add intervalley repulsion $H' = \sum_{\mathbf{q}} V(\mathbf{q})\rho_+(\mathbf{q})\rho_-(-\mathbf{q})$, where $\rho_\tau(\mathbf{q})$ is the density operator from the valley τ and $V(\mathbf{q})$ is the Coulomb repulsion. With intervalley repulsion comparable to the intralayer repulsion as in the experiment, there should be an instability of the conjugate CFL phase. Because we are at total filling $n = 1$, it is natural to expect a metal to insulator transition when increasing the intervalley repulsion. One way to obtain a charge gap is to add intravalley pairing of the composite fermions, leading to a pair of non-Abelian Pfaffian phases [37,40]. However, this decoupled phase does not reduce the intervalley repulsion. Instead, we are interested in searching for an insulator with nontrivial intervalley correlations.

The cCFL phase is compressible and metallic with the internal fluxes carrying charges $(Q_+, Q_-) = (1/4\pi)(da_+, -da_-)$. So one can create a flux da_τ to induces a physical charge in the valley $\tau = \pm$. We also have $(Q, S_z) = (1/2\pi)(da_s, \frac{1}{2}da_c)$. So internal flux of da_s induces equally occupied physical charges in the two valleys. Actually $da_s = 4\pi$ leads to a Cooper pair of electrons. On the other hand, $da_c = 4\pi$ corresponds to a neutral intervalley exciton with $Q = 0$, $S_z = 1$. Because of the interlayer repulsion, the double occupancy of the two valleys at the same position should be suppressed. It is thus energetically favorable to gap out a_s. One natural choice is to consider an exciton pairing of the composite fermions in the mean field Hamiltonian

$$H'_M = -\Phi(f_+^\dagger f_- + f_-^\dagger f_+). \tag{3}$$

The above term higgses the two internal $U(1)$ gauge fields down to $a_\mu^+ = a_\mu^- = a_\mu^c$. Meanwhile, a_μ^s is gapped out due to the Meissner effect, leading to a physical charge gap. Now the internal Chern-Simons terms are cancelled. Also, the term Φ hybridizes the two Fermi surfaces and we can now define $f_R = (1/\sqrt{2})(f_+ + f_-)$ and $f_L = (1/\sqrt{2})(f_+ - f_-)$. In the following we use $a_\mu = a_\mu^c$ for simplicity. The action of composite fermion is now

$$\mathcal{L}_{\mathrm{FS}}[f, a] = \sum_{a=R,L} f_a^\dagger(t,\mathbf{x})(i\partial_t + a_0 + \mu \pm \Phi) f_a(t,\mathbf{x}) - \frac{\hbar^2}{2m}\sum_{a=R,L} f_a^\dagger(t,\mathbf{x})(-i\partial_\mu - a_\mu)^2 f_a(t,\mathbf{x}) + \sum_{a=R,L}\sum_{j=1,\ldots,6} V_\tau e^{i\theta_j^a} f_a^\dagger(\mathbf{k}+\mathbf{G}) f_a(\mathbf{k}), \tag{4}$$

where the dispersions from f_R and f_L now have energy shift $-\Phi$ and $+\Phi$, respectively. We also add a superlattice potential for the composite fermions from the moiré lattice. Here, $G_1 = [(4\pi/\sqrt{3}a_M), 0]$ is a reciprocal vector with a_M the lattice constant. Other G_j with $j = 2, 3, 4, 5, 6$ can be generated by C_6 rotation. Note time reversal now acts as $f_R(\mathbf{k}) \to f_R(-\mathbf{k})$, $f_L(\mathbf{k}) \to -f_L(-\mathbf{k})$. V_a and θ_j^a are the amplitude and phase of the lattice potential for $a = R, L$. We have $\theta_1^a = \theta_3^a = \theta_5^a = -\theta_2^a = -\theta_4^a = -\theta_6^a = \theta_a$. Φ, V_a, θ_a should be viewed as variational parameters.

The total Fermi surface volumes should be 1 in units of the moiré Brillouin zone for f_R, f_L together. When the lattice effect is not large, such as in the perfect Landau level, even a large Φ can only gap out f_L and leave a single Fermi surface for f_R [see Figs. 1(a) and 1(b)]. When there is large variation of effective magnetic field inside the moiré unit cell, the composite fermions f_R, f_L feel a superlattice potential V_R, V_L. When we increase V_R, initially the Fermi surface in f_R cannot be gapped and we expect a compensated semimetal with equal size of electron and hole pockets as shown in Fig. 1(c). When the lattice effect is large enough, f_R is also fully gapped as in Fig. 1(d). When the fermions are fully gapped as in Fig. 1(d), we have a different phase, a superfluid phase of excitons (or equivalently an intervalley-coherent insulator), which we will discuss later.

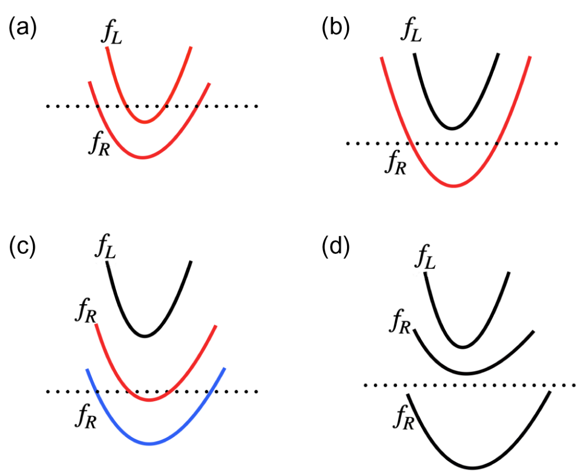

FIG. 1. Illustration of the dispersions of the neutral fermion f_R, f_L in Eq. (4). Horizontal dashed line labels the Fermi level. Black, red, and blue colors mean that the corresponding bands cross the Fermi level without any Fermi surface and with electron pocket and hole pocket, respectively. (a) Φ and $V_{R/L}$ are small; we have two split Fermi surfaces. (b) Φ is large and V_R is small. There is one electron pocket from f_R with volume $A_{\mathrm{FS}} = 1$ in units of the moiré Brillouin zone. (c) Φ is large; V_R is intermediate. We have equal size of electron and hole pockets for f_R due to minibands from the lattice potential. (d) Φ is large and V_R is large. Now f_R is also gapped due to the band gap opening from the lattice effect.

The final theory is

$$\mathcal{L}_{\mathrm{VSL}} = \mathcal{L}_{\mathrm{FS}}[f, a] - \frac{1}{4\pi} A_s da + \frac{1}{4\pi} A_c dA_s, \tag{5}$$

where we have used the definition $A_c = \frac{1}{2}(A_+ + A_-)$ and $A_s = A_+ - A_-$. $\mathcal{L}_{\mathrm{FS}}[f, a]$ can be found in Eq. (4) with a_μ as an internal $U(1)$ gauge field from the a_μ^c component of the conjugate CFL. In the Supplemental Material [41], we show that the same theory can also be derived from the Dirac theory description of the CFL.

The theory in Eq. (5) should be viewed as a composite Fermi liquid of the neutral excitons. Actually the structure of the theory resembles Son's Dirac theory of the CFL in half-filled Landau level [42]. In this phase, the double monopole operator $\mathcal{M}_a^2$ of the gauge field a_μ now corresponds to the physical exciton creation operator with $S_z = 1$. There are gapless spin excitations carried by the internal flux: $da = 2\pi$ flux carries spin $S_z = -\frac{1}{2}$. On the other hand, the fermion f is neutral and spinless and should be viewed as fermionic vortex of a nearby exciton condensation phase, an intervalley-coherence order in the context of the twisted $MoTe_2$ system. For this reason we dub this spin liquid as vortex spin liquid. For excitons, the physical time reversal symmetry acts as an antiunitary particle-hole symmetry and forbids the internal Chern-Simons term. Note that the monopole operator does not cause confinement of a_μ here because it is forbidden by the S_z conservation and its scaling dimension is infinite due to the coupling to the Fermi surface [46].

Bulk property of the VSL phase—Similar to the familiar $U(1)$ spin liquid with spinon fermi surface, the VSL phase has a finite spin susceptibility χ_s. Unlike the spinon Fermi surface state, only the correlation function of S_z, $\langle S_z(\mathbf{x}) S_z(0)\rangle \sim (1/|\mathbf{x}|^\alpha)$, has a finite exponent α. On the other hand, $\langle S^\dagger(\mathbf{x}) S^-(0)\rangle$ has an infinite power-law exponent because $S^\dagger$ corresponds to the monopole operator of the gauge field a_μ, which has infinite scaling dimension due to coupling to the Fermi surface [46]. Another unique property of the VSL phase is that its spin transport resistivity $\rho_s = (4/\rho_f)$ (see the Supplemental Material) is inverse to ρ_f, the resistivity of the neutral fermions. As we expect ρ_f increases with the temperature T as in a usual metal, the spin resistivity ρ_s increases when decreasing the temperature until it saturates at a value $[4/\rho_f(T = 0)]$ at zero temperature. Actually if there is no disorder, the residual resistivity $\rho_f(T = 0) = 0$ and then ρ_s is infinite at zero temperature like an insulator. In this sense, disorder can decrease the resistivity of the spin (exciton) transport and enhance the spin (exciton) mobility.

Another interesting property of the VSL phase is that it generates an internal flux da under Zeeman field. Because of the finite uniform spin susceptibility χ_s, we have $\langle S_z\rangle = g\chi_s B$ with B the Zeeman field and g the renormalized g

factor. In the VSL phase the internal flux $b = da = \partial_x a_y - \partial_y a_x = 4\pi S_z = 4\pi g\chi_s B$. Therefore the neutral fermions feel an effective magnetic field $b_{\mathrm{eff}} = \alpha B$ with $\alpha = 4\pi g\chi_s$ and thus there should be quantum oscillations in this insulator. Note that α in principle can be much larger than 1. From this logic, we may expect large effective flux at small external magnetic field and quantum oscillations under the Zeeman field. For the same reason, there should be a finite thermal Hall conductivity κ_{xy} proportional to the Zeeman field. As the VSL phase has a compressible spin S_z, time reversal symmetry is not important and one may naturally deform the state with a finite valley polarization, while preserving the half quantized FQSH. One interesting scenario is that the FQSH insulator at zero magnetic field may already spontaneously develop a finite S_z and then doping can naturally lead to a finite charge Hall resistivity as observed in the experiment.

FQSH effect and helical edge mode—Because of the background term $(1/4\pi)A_c dA_s$, the VSL phase also hosts a quantized fractional quantum spin Hall effect. Note here the coefficient is half compared to that of the QSH insulator at $n = 2$. One immediate consequence is that there must be helical edge mode carrying charge Q_c. To see this, we can follow the Laughlin argument on an annulus. By threading a flux $dA_s = 4\pi$, a total charge $Q_c = 1$ must be transported from the inner edge to the outer edge. Given that the bulk has a charge gap, the flux-threading process can be done adiabatically in the charge sector, meaning no gapped charge excitations are created in the bulk. Then the edge must host gapless charge mode to make the transportation of the charge possible. And due to the time reversal symmetry, there must be helical edge mode as in the integer QSH insulator.

In the Supplemental Material [41] we show that the edge theory is the same as that from a pair of $\nu = \frac{1}{2}$ bosonic Laughlin state and its time reversal partner,

$$\mathcal{S}_{\mathrm{edge}} = \int dxdt \frac{2}{4\pi}\partial_x\phi_+\partial_t\phi_+ - \frac{2}{4\pi}\partial_x\phi_-\partial_t\phi_- - \frac{2}{4\pi}v(\partial_x\phi_+)^2 - \frac{2}{4\pi}v(\partial_x\phi_-)^2 - \frac{g}{\pi}\partial_x\phi_+\partial_x\phi_-, \quad (6)$$

where $\phi_\pm$ are two counterpropagating modes from the valley $\tau = \pm$, respectively. All of the backscattering terms such as $\cos(\phi_+ \pm \phi_-)$ are forbidden by the charge and S_z conservation. There are couplings to the bulk gapless spin degree of freedom. In the Supplemental Material, we argue that these couplings are irrelevant.

Connection to IVC insulator—At filling $n = \frac{1}{2} + \frac{1}{2}$, another obvious time reversal invariant insulator is an excitonic insulator with intervalley-coherent order. However, the intervalley exciton feels an effective two flux per moiré unit cell and its condensation is frustrated. If the superlattice effects are strong, we can imagine a vortex crystal state of the exciton condensation [48]. But it contains nodes in the momentum space and has been shown to be energetically not very good [48]. IVC insulator has been reported in Hartree-Fock study of the twisted $MoTe_2$ system [49], but is from exciton pairing of the bands with the same Chern number. To our best knowledge, IVC order between opposite Chern number was never found to be the ground state in the Hartree-Fock study. Our theory actually shows a new perspective on when the unusual IVC order between opposite Chern number may be stabilized. In the VSL phase, if the effective superlattice potential felt by the neutral fermions f [the V term in Eq. (4)] is strong, the neutral Fermi surface can be fully gapped [see Fig. 1(d)] and we reach the IVC insulator with action $\mathcal{L}_{\mathrm{IVC}} = -(1/4\pi)A_s da$, where the vortices f (charged under a_μ) are gapped. But at smaller effective superlattice potential V, the fermionic vortices f cannot be fully gapped, leading to a vortex liquid phase. The physical picture is quite clear: we need a strong superlattice potential to localize the vortices; otherwise the vortices are mobile and disorder the IVC order. The moiré Chern band is known to resemble the Landau level quite well, thus the effective superlattice potential V felt by the composite fermion is weak. In the Landau level limit, there is approximately a continuous translation symmetry and the vortex lattice is expected to be unstable to a vortex liquid phase.

Discussion on energetics—Here, we briefly discuss the energetics of the VSL phase. As S_z is conserved, we can focus on each subspace labeled by the total S_z. Within the sector of $S_z = 0$, we expect that the VSL phase is energetically better than the decoupled phase due to its nontrivial intervalley correlations to suppress intervalley repulsion. It is also favored over the IVC order when the underlying band is close to a Landau level. However, in the flat band limit a strong competing state should be the fully valley-polarized state in the maximal S_z sector [16]. The sectors of zero and maximal S_z are disconnected and their competition is simply a comparison of numbers. We point out two potential sources to stabilize the $S_z = 0$ state in a small parameter region: (1) the phonon at momentum K mediates attractive intervalley interaction, which lowers the state of $S_z = 0$. (2) A small bandwidth increases the energy of the maximal S_z state and there may be a small window to stabilize the VSL phase before a conventional Fermi liquid dominates. We leave it to future work to identify the ideal parameter regime in a microscopic model.

Summary—In conclusion, we propose a new fractional insulator, a vortex spin liquid, at odd integer filling of moiré Chern band with Chern number $C = \pm 1$ in the two valleys. The VSL phase has a charge gap, but hosts gapless spin excitation, resembling the usual Mott insulator. Interestingly it also supports well-isolated helical charged edge modes and a quantized FQSH effect. We propose the VSL phase as an explanation of the potential FQSH state at filling $n = 3$ of twisted $MoTe_2$ at the twist angle around 2.1° [34]. Future experimental measurements of the spin and charge gap separately can potentially distinguish the VSL phase and the simple decoupled FQSH phase in this system. Especially we expect the half quantized FQSH effect to be robust under out of plane magnetic field B in the VSL phase because a partial spin polarization does not affect the charge dynamics. From a theoretical perspective, this new phase offers an example that combines physics of Mott insulator and quantum spin Hall effect, similar to a previous proposal of quantum Hall spin liquid [50], which combines Mott physics and quantum Hall effect. The VSL phase can also be viewed as a parent state to organize various fully gapped descendants with quantized half FQSH, which will be discussed in a subsequent paper [51].

Note added—The current manuscript is a new version of an unpublished preprint in 2018 [52]. The VSL phase here was called composite fermion insulator in the previous version, but the essential theory remains the same.

Acknowledgments—This work was supported by the National Science Foundation under Grant No. DMR-2237031.

[1] K. Sun, Z. Gu, H. Katsura, and S. Das Sarma, Phys. Rev. Lett. **106**, 236803 (2011).
[2] D. Sheng, Z.-C. Gu, K. Sun, and L. Sheng, Nat. Commun. **2**, 389 (2011).
[3] T. Neupert, L. Santos, C. Chamon, and C. Mudry, Phys. Rev. Lett. **106**, 236804 (2011).
[4] Y.-F. Wang, Z.-C. Gu, C.-D. Gong, and D. N. Sheng, Phys. Rev. Lett. **107**, 146803 (2011).
[5] E. Tang, J.-W. Mei, and X.-G. Wen, Phys. Rev. Lett. **106**, 236802 (2011).
[6] N. Regnault and B. A. Bernevig, Phys. Rev. X **1**, 021014 (2011).
[7] E. J. Bergholtz and Z. Liu, Int. J. Mod. Phys. B **27**, 1330017 (2013).
[8] S. A. Parameswaran, R. Roy, and S. L. Sondhi, C. R. Phys. **14**, 816 (2013).
[9] T. Neupert, L. Santos, S. Ryu, C. Chamon, and C. Mudry, Phys. Rev. B **84**, 165107 (2011).
[10] J. Cai, E. Anderson, C. Wang, X. Zhang, X. Liu, W. Holtzmann, Y. Zhang, F. Fan, T. Taniguchi, K. Watanabe, Y. Ran, T. Cao, L. Fu, D. Xiao, W. Yao, and X. Xu, Nature (London) **622**, 63 (2023).
[11] Y. Zeng, Z. Xia, K. Kang, J. Zhu, P. Knüppel, C. Vaswani, K. Watanabe, T. Taniguchi, K. F. Mak, and J. Shan, Nature (London) **622**, 69 (2023).
[12] H. Park *et al.*, Nature (London) **622**, 74 (2023).
[13] F. Xu, Z. Sun, T. Jia, C. Liu, C. Xu, C. Li, Y. Gu, K. Watanabe, T. Taniguchi, B. Tong, J. Jia, Z. Shi, S. Jiang, Y. Zhang, X. Liu, and T. Li, Phys. Rev. X **13**, 031037 (2023).
[14] Z. Lu, T. Han, Y. Yao, A. P. Reddy, J. Yang, J. Seo, K. Watanabe, T. Taniguchi, L. Fu, and L. Ju, arXiv:2309.17436.
[15] Y.-H. Zhang, D. Mao, Y. Cao, P. Jarillo-Herrero, and T. Senthil, Phys. Rev. B **99**, 075127 (2019).
[16] C. Repellin, Z. Dong, Y.-H. Zhang, and T. Senthil, Phys. Rev. Lett. **124**, 187601 (2020).
[17] F. Wu, T. Lovorn, E. Tutuc, I. Martin, and A. H. MacDonald, Phys. Rev. Lett. **122**, 086402 (2019).
[18] H. Yu, M. Chen, and W. Yao, Natl. Sci. Rev. **7**, 12 (2020).
[19] T. Devakul, V. Crépel, Y. Zhang, and L. Fu, Nat. Commun. **12**, 6730 (2021).
[20] H. Li, U. Kumar, K. Sun, and S.-Z. Lin, Phys. Rev. Res. **3**, L032070 (2021).
[21] V. Crépel and L. Fu, Phys. Rev. B **107**, L201109 (2023).
[22] C. Wang, X.-W. Zhang, X. Liu, Y. He, X. Xu, Y. Ran, T. Cao, and D. Xiao, Phys. Rev. Lett. **132**, 036501 (2023).
[23] A. P. Reddy, F. F. Alsallom, Y. Zhang, T. Devakul, and L. Fu, Phys. Rev. B **108**, 085117 (2023).
[24] C. Xu, J. Li, Y. Xu, Z. Bi, and Y. Zhang, Proc. Natl. Acad. Sci. U.S.A. **121**, e2316749121 (2024).
[25] J. Yu, J. Herzog-Arbeitman, M. Wang, O. Vafek, B. A. Bernevig, and N. Regnault, arXiv:2309.14429.
[26] H. Goldman, A. P. Reddy, N. Paul, and L. Fu, Phys. Rev. Lett. **131**, 136501 (2023).
[27] J. Dong, J. Wang, P. J. Ledwith, A. Vishwanath, and D. E. Parker, Phys. Rev. Lett. **131**, 136502 (2023).
[28] N. Morales-Durán, N. Wei, and A. H. MacDonald, Phys. Rev. Lett. **132**, 096602 (2024).
[29] Z. Dong, A. S. Patri, and T. Senthil, arXiv:2311.03445.
[30] B. Zhou, H. Yang, and Y.-H. Zhang, arXiv:2311.04217.
[31] J. Dong, T. Wang, T. Wang, T. Soejima, M. P. Zaletel, A. Vishwanath, and D. E. Parker, arXiv:2311.05568.
[32] Z. Guo, X. Lu, B. Xie, and J. Liu, arXiv:2311.14368 [Phys. Rev. Lett. (to be published)].
[33] Y. H. Kwan, J. Yu, J. Herzog-Arbeitman, D. K. Efetov, N. Regnault, and B. A. Bernevig, arXiv:2312.11617.
[34] K. Kang, B. Shen, Y. Qiu, K. Watanabe, T. Taniguchi, S. J. Shan, and M. K. Fai, arXiv:2402.03294.
[35] J. Eisenstein, Annu. Rev. Condens. Matter Phys. **5**, 159 (2014).
[36] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[37] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[38] N. Myerson-Jain, C.-M. Jian, and C. Xu, arXiv:2311.16250.
[39] I. Sodemann, I. Kimchi, C. Wang, and T. Senthil, Phys. Rev. B **95**, 085135 (2017).
[40] G. Moore and N. Read, Nucl. Phys. **B360**, 362 (1991).
[41] See Supplemental Material, which includes Refs. [42–47], at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.106502 for details of the derivation.
[42] D. T. Son, Phys. Rev. X **5**, 031027 (2015).
[43] C. Wang and T. Senthil, Phys. Rev. B **93**, 085110 (2016).
[44] M. A. Metlitski and A. Vishwanath, Phys. Rev. B **93**, 245151 (2016).
[45] C. Wang and T. Senthil, Phys. Rev. X **5**, 041031 (2015).
[46] S.-S. Lee, Phys. Rev. B **78**, 085129 (2008).
[47] D. L. Maslov and M. Stone, Phys. Rev. B **52**, R5539 (1995).
[48] N. Bultinck, S. Chatterjee, and M. P. Zaletel, Phys. Rev. Lett. **124**, 166601 (2020).
[49] T. Wang, M. Wang, W. Kim, S. G. Louie, L. Fu, and M. P. Zaletel, arXiv:2312.12531.
[50] Y.-H. Zhang and T. Senthil, Phys. Rev. B **102**, 115127 (2020).
[51] Y.-H. Zhang, arXiv:2403.12126.
[52] Y.-H. Zhang, arXiv:1810.03600.