Statistics of Stochastic Entropy for Recorded Transitions between ENSO States

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(Received 29 December 2023; revised 20 April 2024; accepted 11 June 2024; published 27 August 2024)

We analyze the transitions between established phases of the El Niño Southern Oscillation (ENSO) by surveying the daily data of the southern oscillation index from an entropic viewpoint using the framework of stochastic statistical physics. We evaluate the variation of entropy produced due to each recorded path of that index during each transition as well as taking only into consideration the beginning and the end of the change between phases and verified both integral fluctuation relations. The statistical results show that these entropy variations have not been extreme entropic events; only the transition between the strong 1999–2000 La Niña to the moderate 2002–2003 El Niño is at the edge of being so. With that, the present work opens a long and winding avenue of research over the application of stochastic statistical physics to climate dynamics.

DOI: 10.1103/PhysRevLett.133.094201

From chit-chat in a lift to ground breaking science paving the way to the Nobel prize [1], climate is a hot topic. Besides the climatic issues linked to humankind (in)actions that have been in the spotlight, climate dynamics has its own regular phenomena that create important distress in our lives [2]. Among them, the El Niño (EN) and La Niña (LN) episodes are certainly at the first tier [3]. Each event corresponds to the opposing phases of the ENSO first conveyed by Walker on his study over the "southern oscillation" [4]: large-scale changes of the sea level pressure across Southeast Asia and the tropical Pacific. Nonetheless, it took 45 years [5] to assert a connection between ENSO and changes in the ocean, namely, the perception by South American fishermen of the warming up of coastal waters that occurred every so often around Christmas.

The EN corresponds to a warming or above-average temperature of the ocean surface in the central and eastern tropical Pacific [6]; the precipitation over Southeast Asia decreases while rainfall increases over the tropical Pacific. At the same time, easterly low-level surface winds along the equator weaken or even turn westerly instead. Conversely, during the LN, the ocean surface experiences a cooling or below-average temperature and the weather effect over Southeast Asia and the central tropical Pacific is opposite to that of EN; in addition, the typical equatorial easterly winds become stronger. In between, there is the so-called neutral phase in which the tropical Pacific surface temperature is close to its average or, more often than not, ocean conditions match a given "child" (El Niño and La Niña literally translate from Castilian into "the boy" and "the girl," respectively.) state, but the not the atmosphere or the other way around.

Despite the fact that EN/LN are coupled (ocean and atmosphere) phenomenon [6–10], the identification climate is getting in or out of one of them is widely related to the southern oscillation index (SOI) that measures the intensity of the Walker circulation: the driven tropical atmospheric circulation in the longitudinal direction [5]. It defines a measure of the difference in the surface air pressure (anomaly) between Tahiti and Darwin expressed in standard deviation units [11]. Although commonly presented in a monthly time frame, SOI daily values are accessible since 1991. The rule-of-thumb linking SOI and ENSO phases goes as follows [12]: prolonged (average) positive SOI values above $+8$ indicate a LN event, whereas continuing negative values below -8 indicate an EN phase. In Table I, we indicate the 11 fully established transitions according to the World Meteorological Organization [13] since daily records are available.

The dynamics of ENSO/SOI has been mimicked manifold: from assuming a nonlinear systems approach [14–16]

TABLE I. Transitions between cataloged El Niño ↔ La Niña ENSO events comprising the neutral phases as well.

No.: direction	Start date	End date
1: EN → LN	31st August 1992	1st September 1995
2: LN → EN	31st March 1996	1st March 1997
3: EN → LN	31st August 1998	1st June 1999
4: LN → EN	31st May 2000	1st March 2002
5: EN → LN	28th February 2003	1st June 2007
6: LN → EN	28th February 2008	1st June 2009
7: EN → LN	31st May 2010	1st June 2010
8: LN → EN	31st May 2011	1st March 2014
9: EN → LN	31st May 2016	1st September 2017
10: LN → EN	31st May 2018	1st September 2018
11: EN → LN	31st May 2019	1st June 2020

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to a stochastic perspective of the problem [17–21]. Praising all the others, we rank the latter as particularly suited to set forward an analysis within stochastic statistical physics (SSP) [22,23]. Therein, the description of physical laws hinges on the probability distribution, $p(\{\mathcal{O}_t\})$, of a trajectory assumed by the system—related to the observable $\mathcal{O}$—in going from a state into another and recurrently presented in the form of (integral) fluctuation relations [24]. An instance of that is the Jarzynski relation [25], which can be understood as the probabilistic version of the Clausius inequality [26]. Moreover, this fluctuation oriented description lifted the veil over the different components of the entropy identifying its informational contribution associated with the intrinsic modification of the probability and those related to energetic quantities, viz, heat [27–29]. Although by heeding the thermodynamic limit, we would expect such fluctuations were only meaningful for small systems [30], the truth is that it is possible to set a probabilistic description, namely, for entropy production, of systems as big as wind-tunnel experiments under fully developed turbulence [31] or rogue wave statistics [32]. Thence, even if we are treating the quintessential macroscopic system (the atmosphere), it is plausible that in carrying out a SSP analysis of the SOI we can still obtain relevant insights, namely, the probabilistic features of the trajectories taken by that index when ENSO goes from state into the other. At this point, natural questions arise: Owing that EN and LN are the contrasting phases of ENSO, does the blatant difference in the respective typical weather (i.e., heat or thermal entropy) and patterns have found quantitative correspondence in the entropy variations derived from the SOI dynamics? Do they correspond to extreme events as one is readily prone to assert since they imply extreme weather and heat phenomena? What further can we learn by employing SSP to climate? The answer to these questions constitutes our goal.

To learn over the dynamics of SOI, $\{s_t\}$, we use its daily series recorded between the 157th day of 1991 and 212th day of 2023 [33]. These data are jointly listed and computed from the pressure measurements used to get the monthly SOI, which is more convenient to appraise trends due to the natural reduction of the fluctuations. Still, in the trajectory approach, we aim at being as close as possible to the actual SOI path; thus, we focus on the analysis of the smallest available sampling rate the monthly form would wipe out valuable knowledge. For all that, within SPP [30], it is often assumed a standard differential formulation containing a time-dependent contribution, $\Pi(t)$, representing a driving process, $d\mathcal{O}/dt = f(\mathcal{O}) + \Pi(t) + \sqrt{2g(\mathcal{O})}\eta_t$, where $\langle \eta_t \eta_{t'} \rangle = \delta(t-t')$ in a Stratonovich representation. Milestone findings in SSP [34] considered, $\Pi(t) = \text{const}\, t$, yielding, $\mathcal{O}_t = \text{const}\, t + \xi_t$, which concentrates the stochasticity on ξ_t. For the SOI, a similar description is possible and relates to the usual geophysical procedure of describing a trend ruled by some nonlinear $\Pi(t)$ though. Willing to keep close to the methods usually employed in ENSO analysis, a candidate to infer the SOI protocol, $\Phi(t)$, is the use of the multitaper method [35,36] used in several other areas as well [37]. Following that [36,38,39], we applied the method for $K = 5$ tapers and the subsequent spectral analysis indicated as statistically significant the set of frequencies yielding, $\{T\} = \{24, 28, 36, 74, 102, 365, 2168\}$ days, a set that matches several geophysical and astronomical phenomena related to ENSO [40]. The inversion to the time domain of the so-called background dynamics—i.e., protocol Φ—carried out conforming to the geophysical signal processing technique [36,38] of a nearly optimal reconstruction by means of a mean-square minimization of the numerical adjustment of the reconstructed signal with respect to $\Phi(t) = \Phi_0(t) + \sum_{i=1}^{7} A_i(t) \cos[(2\pi/T_i)t + \theta_i]$; the details and further values are presented in [40]. Thence, we define, $\xi_t \equiv s_t - \Phi_t$, as stationary following the generic stochastic equation, $d\xi = D_1'(\xi)dt + \sqrt{2D_2'(\xi)}dW_t$, where W_t is a standard Wiener process. From the data we evaluate the empirical Kramers-Moyal coefficients [49], $\tilde{D}_n'(\xi,t) \equiv 1/(tn!)\langle(\xi(t)-\xi)^n\rangle_{ss}$, ($ss$ stands for averages assuming a stationary state). For $t \to 0$, we have $\tilde{D}_n'(\xi,t) = D_n'(\xi)$ ($D_n'(\xi) = 0$, for $n \geq 3$). Following Ref. [50], we incorporate the SOI sampling rate, $\tau = 1$ day, and in the Itô definition we get (details in Supplemental Material [40])

$$D_1'(\xi) = -a\xi + b, \qquad D_2'(\xi) = \alpha^2\xi^2 + \beta\xi + \gamma^2, \tag{1}$$

where $a = 0.15 \pm 0.01$, $b = 0.08 \pm 0.01$, $\alpha^2 = 0.010 \pm 0.002$, $\beta = -0.14 \pm 0.03$, and $\gamma^2 = 31.65 \pm 0.8$ [see Figs. 3(a) and 3(b) in [40]]. To further probe the reliability of ξ dynamics, we generated ξ series numerically and compared $\tilde{D}_3'(\xi)$ and $\tilde{D}_4'(\xi)$ with that found to the SOI ξ component. The agreement corroborates the stochastic model defined by Eq. (1). Blending both ξ_t and Φ_t dynamics we get

$$ds = D_1(s,t)dt + \sqrt{2D_2(s,t)}dW_t, \tag{2}$$

(Itô representation of the noise) with

$$D_1(s,t) = -as + a\Phi(t) + b, \quad D_2(s,t) = \alpha_s^2 s^2 + \beta_s s + \gamma_s^2, \tag{3}$$

where, $\beta_s = \beta - 2\alpha^2\Phi(t)$ and $\gamma_s^2 = \gamma^2 + \alpha^2\Phi^2(t) - \beta\Phi(t)$.

As previously stated, we primarily want to evaluate the variation of entropy, $\Delta S(\vec{s} = \{s_{t_i}, \ldots, s_{t_f}\})$ [51],

$$\begin{aligned}\Delta S(\vec{s}) &\equiv -\ln\frac{p(s_{t_f};\{s_{t_i},\ldots,s_{t_{f-1}}\})}{p(s_{t_i};\{s_{t_{i+1}}\ldots s_{t_f}\})} \\ &= -\ln\frac{p^*_{t_i}(s_{t_i})p^F(s_{t_{i+1}}|s_{t_i},\Phi)\ldots p^F(s_{t_f}|s_{t_{f-1}},\Phi)}{p^*_{t_f}(s_{t_f})p^R(s_{t_{f-1}}|s_{t_f},\Phi)\ldots p^R(s_{t_i}|s_{t_{i+1}},\Phi)},\end{aligned} \tag{4}$$

(F and R stand for forward and reverse trajectory) produced in the evolution of SOI along the trajectory segment starting at day t_i and ending at day t_f that took ENSO from a given phase into its opposite with realized final and initial probability distributions. Using the framework of SSP, the southern oscillation *protocol* $\Phi(t)$ acts on the system and its outcome is its evolution from a given (local) stationary state, $p^*_{t_i}(s)$ into a new state $p^*_{t_f}(s)$ [51], where $p^*_t(s)$ is obtained by considering that at time t the probability current $j_t(s)$ vanishes:

$$j^*_t(s) = D_1(s,t)p^*_t(s) - \frac{\partial}{\partial x}D_2(s,t)p^*_t(s) = 0, \quad (5)$$

or, $[\partial j^*_t(s)/\partial x] = -[\partial p^*_t(s)/\partial t] = 0$, which yields,

$$p^*_t(s) = \frac{1}{\mathcal{Z}^*_t}\exp\left[\frac{a\beta+2b\alpha^2}{\alpha^2\sqrt{4\alpha^2\gamma^2-\beta^2}}\arctan\left[\frac{\beta+2\alpha^2(s-\Phi(t))}{\sqrt{4\alpha^2\gamma^2-\beta^2}}\right]\right] \times[\gamma^2+\beta(s-\Phi(t))+\alpha^2(s-\Phi(t))^2]^{-(1+\frac{a}{2\alpha^2})}. \quad (6)$$

To evaluate Eq. (4) we still need to compute the conditioned probabilities $p^{F(R)}(s_{t'}|s_t,\Phi)$. To that, we employ a path integral formulation [52], but first some points must be handled. First, we tackle the multiplicative noise nature of the SOI dynamics that is also found in other atmospheric quantities [53]. Several techniques were cast at providing a probabilistic solution to multiplicative noise systems, but they fundamentally boil down to transformations of variables [54], parameters, and expansions [52,55–58]. In order to maintain the work as straightforward as possible, we opt to transform the original multiplicative noise dynamics into an additive noise one as a means to retrieve $p(s_{t'}|s_t,\Phi)$ at the end. That is worked at in the Supplemental Material [40] leading to the transformation of variables [49] $y=(1/\alpha)\text{arcsinh}\{(\alpha/\lambda)[s+\beta_s/(2\alpha^2)]\}$, where $\lambda^2 = \gamma^2 - \beta/(4\alpha^2)$. Second, we must remove the impact of the trajectory direction on the stochastic dynamics caused by the Itô interpretation of noise in Eq. (2). That is made by rewriting Eq. (2) into its Stratonovich version [59]. Combining both steps it finally yields

$$\frac{dy}{dt} = \left(\frac{b}{\lambda}+\frac{a\beta}{2\lambda\alpha^2}\right)\text{sech}[\alpha y] - \left(\alpha+\frac{a}{\alpha}\right)\tanh[\alpha y] + \sqrt{2}\eta_t. \quad (7)$$

The derivation of $p(y,t|y_0,t_0)$ from Eq. (7) equals [52]

$$p(y,t|y_0,t_0) = \int \mathcal{D}y\ \exp\left[-\frac{1}{4}\mathcal{S}[y(t)]\right]\det\frac{\delta\eta_t}{\delta y_0}, \quad (8)$$

assuming the optimizing condition, $\delta\mathcal{S}[y(t)]] = 0$, of the stochastic action, $\mathcal{S}[y(t)] \equiv \int_{t_0}^{t}\mathcal{L}(t')dt'$, where the Onsager-Machlup function reads [40]

$$\mathcal{L} = \left\{\dot{y} - \left(\frac{b}{\lambda}+\frac{a\beta}{2\lambda\alpha^2}\right)\text{sech}[\alpha y] + \left(\alpha+\frac{a}{\alpha}\right)\tanh[\alpha y]\right\}^2, \quad (9)$$

allowing the application of a saddle-point approximation.

The optimization is achieved by solving the Euler-Lagrange equation, $[(\partial\mathcal{L}/\partial y) - (d/dt)(\partial\mathcal{L}/\partial\dot{y})]|_{y=y^*} = 0$. Because such equation is not analytically solvable to the best of our efforts, we focus on the parameters in Eq. (2) and understand the SOI fluctuations are dominated by the additive contribution to the noise given by γ^2, whereas the remainder of multiplicative contributions act as perturbations. Expanding the Euler-Lagrange equation in powers of α and β so that $(\alpha\gamma/\beta)^2 \gg 1$—jointly with the inspection of further relations between the parameters—yields, $\ddot{y}^* - c_1 y^* + c_0 = 0$, which is bounded by $y^*(t) = y$ and $y^*(t_0) = y_0$ with $c_0 = (b/\gamma)(a+\alpha^2) + a\beta/(2\gamma)[1+(a/\alpha^2)]$ and $c_1 = a^2 + 2a\alpha^2 - (b\alpha^2 + a\beta)b/\gamma^2$. The explicit solution thereto is presented in the Supplemental Material [40]; utilizing that solution into the stochastic action, $\mathcal{S}$ into Eq. (8), and finally reverting the change of variables $y = y(s)$, we obtain $p^F[s(t)|s(t_0)]$ and $p^R[s(t_0)|s(t)]$. The (endless) formulas of both of them are shown in the Supplemental Material [40].

At last, we compute Eq. (4) considering the trajectories of the transitions in Table II as well as the average $\langle S(\vec{s})\rangle$, and the standard deviation $\sigma_{\Delta S(\vec{s})}$ over trajectories given by Eq. (2). From Table II, we learn that only 3 out of the 11 transitions have $|\Delta S(\vec{s})|$ above 2σ, namely, transitions No. 1, 2, and 4; only the last of these cases—a transition from strong LN to moderate EN [60]—is a 4σ event and could statistically be considered an extreme event. Even so, all the transitions abide by the integral fluctuation theorem, $\langle\exp[-\Delta S]\rangle = 1$, a probabilistic version of the 2nd law of thermodynamics $\Delta S \geq 0$ [26].

Still within SPP, our problem can be surveyed in a slightly different way by strictly looking at the variation of entropy, $\Delta\tilde{S}(t_f,t_i) \equiv \int ds_{t_i+1}\ldots ds_{t_f-1}\Delta S(\vec{s})$. The respective outcomes are exhibited in Table III. Therefrom, we

TABLE II. Realized variation of the entropy $\Delta S(\vec{s})$, average $\langle\Delta S(\vec{s})\rangle$, and standard deviation $\sigma_{\Delta S(\vec{s})}$ over trajectories.

Transition No.	$\Delta S(\vec{s})$	$\langle\Delta S(\vec{s})\rangle$	$\sigma_{\Delta S(\vec{s})}$
1	1.61×10^{-1}	9.6×10^{-5}	8.15×10^{-2}
3	-6×10^{-4}	2.01×10^{-3}	9.05×10^{-2}
5	-5×10^{-4}	-1.17×10^{-3}	8.17×10^{-2}
7	4×10^{-4}	-3.6×10^{-4}	4×10^{-2}
9	-2.9×10^{-3}	-2.8×10^{-4}	8.16×10^{-2}
11	3.1×10^{-2}	8.6×10^{-4}	7.99×10^{-2}
2	-1.553×10^{-1}	-6.9×10^{-4}	8.65×10^{-2}
4	3.30×10^{-1}	3.1×10^{-4}	8.13×10^{-2}
6	8×10^{-4}	1.11×10^{-3}	8.74×10^{-2}
8	-5.3×10^{-3}	2.2×10^{-3}	8.33×10^{-2}
10	-1×10^{-4}	8.6×10^{-4}	7.99×10^{-2}

TABLE III. Realized variation of the entropy $\Delta\tilde{S}(t_f, t_i)$, the average $\langle\Delta\tilde{S}(t_f, t_i)\rangle$, and standard deviation, $\sigma_{\Delta\tilde{S}(t_f,t_i)}$, over trajectories.

Transition No.	$\Delta\tilde{S}(t_f, t_i)$	$\langle\Delta\tilde{S}(t_f, t_i)\rangle$	$\sigma_{\Delta\tilde{S}(t_f,t_i)}$
1	1×10^{-4}	5.8×10^{-4}	9.64×10^{-2}
3	1.78×10^{-3}	-9×10^{-6}	9.85×10^{-2}
5	1.9×10^{-4}	7.3×10^{-4}	9.98×10^{-2}
7	3.6×10^{-4}	-1.0×10^{-3}	5.52×10^{-2}
9	1.75×10^{-3}	5.3×10^{-4}	1.01×10^{-1}
11	5.35×10^{-3}	7.8×10^{-4}	9.34×10^{-2}
2	-1.727×10^{-1}	-6.6×10^{-4}	1.07×10^{-1}
4	2.622×10^{-1}	-5.9×10^{-4}	1.0×10^{-1}
6	3.04×10^{-3}	-1.55×10^{-3}	8.82×10^{-2}
8	2.24×10^{-3}	-1×10^{-5}	1.07×10^{-1}
10	1×10^{-4}	-1.2×10^{-4}	9.11×10^{-2}

perceive all but events No. 2 and No. 4 have $|\Delta\tilde{S}|$ less than $\sigma/10$, whereas for the former $|\Delta\tilde{S}|$ is less than 2σ and for the latter is less than 3σ. Herein, we verify the integral fluctuation theorem as well. Regarding these results, we cannot assert transition No. 4 is an extreme event as in specific trajectory approach.

In this Letter, we have aimed at learning whether the acute difference in the typical weather and patterns of EN and LN phases of ENSO are mirrored in a cornerstone quantity such as the variations of entropy the SOI experiences when climate evolves from one phase into the other, as would be expected in a phenomenon-indicator relation. We have done so considering the SSP framework and asserting to the SOI a stochastic dynamics evolving under a climate protocol that drives the changes in the system. To the best of our knowledge, this work inaugurates the application of such a mindset to climate variables. We have proceeded by assuming the SOI path between phases as provided by its daily records and also considering its values at the boundaries of each transition. The results indicate the EN $\rightarrow$ LN transitions have had quite regular values of entropy variation during the process, usually a less than $\sigma/10$ event. For LN $\rightarrow$ EN transitions we have found more noteworthy values, namely, the transitions No. 2 and No. 4 in Table I, which is a 4σ event in the first approach and a 3σ event in the second one though. All said and done, the transition from the strong 1999–2000 LN to the moderate 2002–2003 EN is the only case on the brink of being an extreme event considering the statistics of entropy variations. Thence, from these SOI data it is not possible to consider the transitions between ENSO phases have been entropic extreme events on the whole. Moreover, we have analyzed the (mean) entropy variation rate, $\Delta S/(t_f - t_i)$, for both approaches and could not find any relation between them and the classification of the phases. This result sheds light on the significant contrast between eminently informational based entropy variations of the indicator, namely, $\Delta S(\vec{s})$ and $\Delta\tilde{S}(t_f, t_i)$, and the thermal or heat-related entropy variations in the transition between the El Niño (hotter) $\leftrightarrow$ La Niña (colder) ENSO phenomenon phases. Ultimately, this raises the question over the extent of the change in weather conditions when SOI extreme entropic events befall (and they statistically will do so) as the index is used to appraise ENSO states. Alternatively, this can demote SOI into a minor role on the description of the phenomenon. Regarding that, we plan this study be enlarged to multivariate approaches of ENSO considering not only the temperature dynamics, but also other quantities like the outgoing long-wave radiation (anomaly) or precipitation [20,61–65]. Furthermore, minding the role of entropy in image analysis [66] and studies on entropy production and fluctuation relations for surface growth and percolative models [67], a SPP methodology can be also implemented on self-organizing maps (som) used to distinguish climate dynamics and its patterns [68] by defining ΔS_{som} for sequence sets. Still, each EN/LN phase can be studied in itself; in this situation, the analysis of (integral) fluctuation relations equivalent to the Speck-Seifert [28] and Hatano-Sasa [29] are among the future problems to be worked at as well.

Acknowledgments—I acknowledge CNPq and the comments presented by the referees from which this work benefitted. C. B. Naves for software assistance.

[1] R. Benzi, G. Parisi, A. Sutera, and A. Vulpiani, Tellus **34**, 10 (1982).
[2] T. A. Carleton and S. M. Hsiang, Science **353**, aad9837 (2016).
[3] P. Cashin, K. Mohaddes, and M. Raissi, Fair weather or foul? The macroeconomic effects of El Niño., IMF working paper 15/89 (2015).
[4] G. T. Walker, Mem. Indian Meteorol. Dep. **24**, 275 (1924).
[5] J. Bjerknes, Mon. Weather Rev. **97**, 163 (1969).
[6] M. L'Heureux, What is the El Niño Southern Oscillation (ENSO) in a nutshell? (2014), https://www.climate.gov/news-features/blogs/enso/what-el-niño?southern-oscillation-enso-nutshell.
[7] D. W. Behringer, M. Ji, and A. Leetmaa, Mon. Weather Rev. **126**, 1013 (1998).
[8] M. Ji, D. W. Behringer, and A. Leetmaa, Mon. Weather Rev. **126**, 1022 (1998).
[9] K. E Trenberth, P. D. Jones, P. Ambenje, R. Bojariu, D. Easterling *et al.*, Observations: Surface and atmospheric climate change, In *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, edited by S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K. B. Averyt, M. Tignor, and H. L. Miller (Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 2007).
[10] A. Navarra, J. Tribbia, and G. Conti, J. Clim. **26**, 9633 (2013).

[11] Tokyo Climate Center—WHO, Explanation of El Niño Monotoring Indices, https://ds.data.jma.go.jp/tcc/tcc/products/elnino/index/Readme.txt.

[12] Australian Bureau of Meteorology, Record-breaking La Niña events (2012), http://www.bom.gov.au/climate/enso/history/La-Nina-2010-12.pdf.

[13] Tokyo Climate Center—WHO, Historical El Niño and La Niña Events, https://ds.data.jma.go.jp/tcc/tcc/products/elnino/ensoevents.html.

[14] P. Chang, L. Ji, H. Li, and M. Flügel, Physica D (Amsterdam) **98**, 301 (1996).

[15] K. Y. Choi, G. A. Vecchi, and A. T. Wittenberg, J. Clim. **26**, 9462 (2013).

[16] J. T. Bruun, J. I. Allen, and T. J. Smyth, J. Geophys. Res. Oceans **122**, 6746 (2017).

[17] F. Petroni and M. Ausloos, Eur. Phys. J. Spec. Top. **143**, 201 (2007).

[18] F.-F. Jin, L. Lin, A. Timmermann, and J. Zhao, Geophys. Res. Lett. **34**, 1 (2007).

[19] F.-F. Jin and L. Lin, J. Atmos. Sci. **64**, 497 (2007).

[20] S. I. An, S. K. Kim, and A. Timmermann, Sci. Rep. **10**, 16282 (2020).

[21] S.-K. Kim and S. I. An, Geophys. Res. Lett. **47**, e2019GL085881 (2020).

[22] K. Sekimoto, *Stochastic Energetics* (Springer, Berlin, 2010).

[23] U. Seifert, Rep. Prog. Phys. **75**, 126001 (2012).

[24] G. Gallavotti, Scholarpedia **3**, 5904 (2008).

[25] C. Jarzynski, Phys. Rev. Lett. **78**, 2690 (1997).

[26] C. Jarzynski, Annu. Rev. Condens. Matter Phys. **2**, 329 (2011).

[27] M. Esposito and C. Van den Broeck, Phys. Rev. Lett. **104**, 090601 (2010).

[28] T. Speck and U. Seifert, J. Phys. A **38**, L581 (2005).

[29] T. Hatano and S. I. Sasa, Phys. Rev. Lett. **86**, 3463 (2001).

[30] *Nonequilibrium Statistical Physics of Small Systems* edited by R. Klages, W. Just, and C. Jarzinsky (Wiley-VCH Verlag, Weinheim, 2013).

[31] A. Fuchs, S. M. Duarte Queirós, P. G. Lind, A. Girard, F. Bouchet, M. Wächter, and J. Peinke, Phys. Rev. Fluids **5**, 034602 (2020).

[32] A. Hadjihoseini, P. G. Lind, N. Mori, N. P. Hoffmann, and J. Peinke, Europhys. Lett. **120**, 30008 (2017).

[33] Data downloaded from The Long Paddock website of Queensland Government (Australia), https://www.longpaddock.qld.gov.au/soi/soi-data-files/.

[34] R. van Zon and E. G. D. Cohen, Phys. Rev. Lett. **91**, 110601 (2003).

[35] D. J. Thomson, Proc. IEEE **70**, 1055 (1982).

[36] M. Ghil, M. R. Allen, M. D. Dettinger, K. Ide, D. Kondrashov, M. E. Mann *et al.*, Rev. Geophys. **40**, 1003 (2002).

[37] S. Karnik, J. Romberg, and M. A. Davenport, IEEE Trans. Inf. Theory **68**, 4864 (2002).

[38] M. E. Mann and J. Park, J. Clim. **9**, 2137 (1996).

[39] F. Petroni and M. Ausloos, Physica (Amsterdam) **387A**, 5246 (2008).

[40] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.094201 for further description of the method and the significance of the frequencies, which includes Refs. [41–46], and a detailed derivation of the result, which includes Refs. [47,48].

[41] D. B. Percival and A. T. Walden, *Spectral Analysis for Physical Applications* (Cambridge University Press, New York, 1993).

[42] S. Slepian, Bell Syst. Tech. J. **57**, 1371 (1978).

[43] M. E. Mann and J. Park, Geophys. Res. Lett. **20**, 1055 (1993); M. E. Mann and J. M. Lees, Clim. Change **33**, 409 (1996).

[44] G. Branstator, J. Atmos. Sci. **44**, 2310 (1987); Y. Kushnir, J. Atmos. Sci. **44**, 2727 (1987).

[45] F. Gasperini, H. Liu, and J. McInerney, J. Geophys. Res. **125**, e2019JA027649 (2020).

[46] https://library.wolfram.com/infocenter/MathSource/789/ based on: J. Steinier, Y. Termonia, and J. Deltour, Anal. Chem. **44**, 1906 (1972).

[47] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. (Elsevier, Amsterdam, 2007).

[48] L. Onsager and S. Machlup, Phys. Rev. **51**, 1505 (1953); H. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1989).

[49] H. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1989).

[50] C. Anteneodo and S. M. Duarte Queirós, Phys. Rev. E **82**, 041122 (2010).

[51] R. Spinney and I. Ford, Fluctuation Relations: A Pedagogical Overview, Chap. 1 in Ref. [30].

[52] H. Wio, *Path Integrals for Stochastic Processes: An Introduction* (World Scientific, Singapore, 2013).

[53] P. Sura, M. Newman, C. Penland, and P. Sardeshmukh, J. Atmos. Sci. **62**, 1391 (2005).

[54] N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (Elsevier, Amsterdam, 2007).

[55] C. Aron, G. Biroli, and L. F. Cugliandolo, J. Stat. Mech. (2010) P11018.

[56] Y. Tang. R. Yuan and P. Ao, J. Chem. Phys. **141**, 044125 (2014).

[57] C. Aron, D. G. Barci, L. F. Cugliandolo, Z. G. Arenas, and G. S. Lozano, J. Stat. Mech. (2016) 053207.

[58] M. V. Moreno, D. G. Barci, and Z. G. Arenas, Phys. Rev. E **99**, 032125 (2019).

[59] C. W. Gardiner, *Handbook of Stochastic Methods* (Springer-Verlag, Berlin, 1997).

[60] Golden Gate Weather Services, El Niño and La Niña Years and Intensities. Classification regarding the impact of the events can be found in Ref. [12].

[61] K. Wolter and M. S. Timlin, Monitoring ENSO in COADS with a seasonally adjusted principal component index, *Proceedings of the 17th Climate Diagnostics Workshop, Norman, OK, NOAA/NMC/CAC, NSSL, Oklahoma Clim. Survey, CIMMS and the School of Meteor* (University of Oklahoma, Norman, 1993), 52–57.

[62] K. Wolter and M. S. Timlin, Int. J. Climatol. **31**, 1074 (2011).

[63] J. Gergis and A. M. Fowler, Int. J. Climatol. **25**, 1541 (2005).

[64] K. Braganza, J. L. Gergis, S. B. Power, J. S. Risbey, and A. M. Fowler, J. Geophys. Res. **114**, D05106 (2009).

[65] P. E. Ardanuy and H. L. Kyle, Mon. Weather Rev. **114**, 415 (1986).

[66] D.-Y. Tsai, Y. Lee, and E. Matsuyama, J. Digit Imaging **21**, 338 (2008).

[67] A. C. Barato, R. Chetrite, H. Hinrichsen, and D. Mukamel, J. Stat. Mech. (2010) P10008.

[68] P. B. Gibson, S. E. Perkins-Kirkpatrick, P. Uotila, A. S. Pepler, and L. V. Alexander, JGR Atmos. **122**, 3891 (2017).