Universal Bound on Effective Central Charge and Its Saturation

Andreas Karch,[1,*] Yuya Kusuki,[2,3,4,5,†] Hirosi Ooguri,[2,6,‡] Hao-Yu Sun,[1,∥] and Mianqi Wang[1,§]

[1]*Theory Group, Weinberg Institute, Department of Physics, University of Texas, 2515 Speedway, Austin, Texas 78712-1192, USA*
[2]*Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Interdisciplinary Theoretical and Mathematical Sciences (iTHEMS), RIKEN, Wako, Saitama 351-0198, Japan*
[4]*Institute for Advanced Study, Kyushu University, Fukuoka 812-8581, Japan*
[5]*Department of Physics, Kyushu University, Fukuoka 819-0395, Japan*
[6]*Kavli Institute for the Physics and Mathematics of the Universe (WPI), University of Tokyo, Kashiwa 277-8583, Japan*

(Received 18 April 2024; accepted 5 August 2024; published 30 August 2024)

The effective central charge (denoted by c_{eff}) is a measure of entanglement through a conformal interface, while the transmission coefficient (encoded in the coefficient c_{LR} of the two-point function of the energy-momentum tensor across the interface) is a measure of energy transmission through the interface. It has been pointed out that these two are generally different. In this Letter, we propose the inequalities, $0 \leq c_{LR} \leq c_{\text{eff}} \leq \min(c_L, c_R)$. They have the simple but important implication that the amount of energy transmission can never exceed the amount of information transmission. We verify them using the AdS/CFT correspondence, using the perturbation method, and in examples beyond holography. We also show that these inequalities are sharp by constructing a class of interfaces that saturate them.

DOI: 10.1103/PhysRevLett.133.091604

Introduction and summary—Conformal interfaces play an important role in the study of quantum critical systems. However, our knowledge of their general properties is limited because they break half of the conformal symmetry. The AdS/CFT correspondence is useful in this context because it gives us insight into systems far from free fields. Indeed, our recent work [1] demonstrated a utility of the AdS/CFT correspondence in studying general properties of interfaces. In this Letter we extend this line of research and formulate the conjecture that the effective central charge, which measures the entanglement across a conformal interface in $1+1$ dimensions, is bounded below by the two-point function of the energy-momentum tensor across the interface. Namely, the amount of energy transmitted across the interface cannot exceed the amount of information transmitted. The conjecture is motivated by holographic conformal field theories (CFTs), free field examples, and the defect perturbation theory. We also show that the bound can be saturated by constructing explicit examples.

In $1+1$ dimensions, conformal interfaces are defined by the following boundary condition for the energy-stress tensors across the interface [2–4]:

$$T^{(L)} - \bar{T}^{(L)} = T^{(R)} - \bar{T}^{(R)}, \tag{1}$$

where $T^{(i)}$ and $\bar{T}^{(i)}$ are the holomorphic and antiholomorphic energy-stress tensors of CFT_i, respectively. In the operator formalism, this can be reexpressed using the Virasoro generators $L_n^{(i)}$ and $\bar{L}_{-n}^{(i)}$ in CFT_i as

$$\left(L_n^{(L)} - \bar{L}_{-n}^{(L)}\right)\mathcal{I} = \mathcal{I}\left(L_n^{(R)} - \bar{L}_{-n}^{(R)}\right) \quad \forall\ n. \tag{2}$$

This condition does not fully determine an interface: It demands the interface not to absorb energy while allowing flexibility regarding the amount of energy reflected by the interface.

Conformal interfaces can be characterized by the "effective central charge," which controls the amount of entanglement across them. We focus on the vacuum state, where entanglement entropy between two (possibly different) systems has a simple logarithmic form,

$$S_A = \frac{c_{\text{eff}}}{3} \ln \frac{L}{\pi\epsilon}. \tag{3}$$

Here, the system size for CFT_L and CFT_R is denoted as L, ϵ is the lattice regularization parameter, and c_{eff} is the effective central charge. A convenient way to evaluate the entanglement entropy is given by the replica trick,

*Contact author: karcha@utexas.edu
†Contact author: kusuki.yuya@phys.kyushu-u.ac.jp
‡Contact author: ooguri@caltech.edu
§Contact author: mqwang@utexas.edu
∥Contact author: hkdavidsun@utexas.edu

$$S_A = \lim_{n\to 1} S_A^{(n)}, \qquad S_A^{(n)} = \frac{1}{1-n}\ln\frac{Z_n}{(Z_1)^n}, \tag{4}$$

where the replica partition function is defined as

$$Z_n \equiv \mathrm{tr}\left(\mathrm{e}^{-(\beta/2)H^{(L)}}\mathcal{I}^{L\to R}\mathrm{e}^{-(\beta/2)H^{(R)}}\mathcal{I}^{R\to L}\right)^n. \tag{5}$$

The dependence on the subsystem size is encoded in the temperature as $\beta = (1/\pi)\ln(L/\pi\epsilon)$. We can define the interface Hilbert space $\mathcal{H}_n^I$ by the dual-channel expansion of the replica partition function,

$$Z_n = \mathrm{tr}_{\mathcal{H}_n^I}\mathrm{e}^{-[(2\pi)^2/\beta]H_n^{\mathcal{I}}}, \tag{6}$$

where we formally define the Hamiltonian $H_n^{\mathcal{I}}$ in the presence of the interfaces. Then, one can give an alternative definition of the effective central charge in terms of the vacuum energy Δ_n^0 in the interface Hilbert space as

$$c_{\mathrm{eff}} \equiv \lim_{n\to 1}\frac{12n}{1-n^2}\left(n\Delta_1^0 - \frac{\Delta_n^0}{n}\right). \tag{7}$$

The effective central charge has been calculated in some specific models [5–9]. Nevertheless, there is still much unknown about c_{eff} due to the lack of techniques in interface CFT (ICFT) where the conformal symmetry is partially broken by interfaces.

Another quantity known to characterize interfaces is the "transmission coefficient," which measures the transfer of energy across the interface [10]. This quantity is controlled by the two-point function of the stress tensor across the interface,

$$\langle T^{(L)}(z_1)T^{(R)}(z_2)\rangle = \frac{c_{LR}}{2(z_1-z_2)^4}. \tag{8}$$

The weighted average transmission coefficient can be expressed in terms of c_{LR} as

$$\mathcal{T} = \frac{2c_{LR}}{c_L + c_R}, \tag{9}$$

where c_L and c_R are the central charges of the two CFTs connected by the interface. Similar expressions in terms of c_{LR} can be given for transmission from left and right separately [11]. Here, we would like to emphasize that the transmission of energy across the interface is independent of the transmission of information. One will see this independence later in this Letter.

One of our main results is to provide evidence for the following inequality,

$$c_{LR} \le c_{\mathrm{eff}}. \tag{10}$$

It implies that the amount of energy transmitted across the interface cannot exceed the amount of information transmitted, which is directly controlled by c_{eff} [8]. We have confirmed that this inequality holds in general holographic CFTs. Furthermore, it also holds in free CFT beyond holography. We also verify the inequality in the defect perturbation theory. Based on these examples, we propose it to hold in general CFTs.

Using the entropic c theorem, it has been shown that there is an upper bound on c_{eff} [1]:

$$c_{\mathrm{eff}} \le \min(c_L, c_R). \tag{11}$$

In fact, this is consistent with the upper bound $c_{LR} \le \min(c_L, c_R)$ shown in [10,11]. Combining it with our conjectured inequality (10),

$$0 \le c_{LR} \le c_{\mathrm{eff}} \le \min(c_L, c_R). \tag{12}$$

Another result of this Letter is that these inequalities are sharp. For holographic CFTs, we were able to find the conditions under which interfaces saturate the bounds. We also show that $c_{LR} = c_{\mathrm{eff}}$ is satisfied only if c_{eff} is either $\min(c_L, c_R)$ or 0. This means that the amount of energy transmission and information transmission across the interface match only in the case of the simplest interfaces, which are either boundaries or topological interfaces. We expect that these results will contribute to the understanding of nontopological interfaces.

Holographic proof of the bound $c_{LR} \le c_{\mathrm{eff}}$ and its saturation—The relation between c_{eff} and the transmission coefficient $\mathcal{T}$ (or equivalently c_{LR}) in ICFT_2 has been studied in certain one-parameter families of conformal interfaces [5,6], where a monotonous function $c_{\mathrm{eff}}(\mathcal{T})$ was found for free boson as well as certain lattice models. It is tempting to generalize this relation. However, as we will show below, they are generally independent quantities. Instead, we prove that in a holographic ICFT_2, there is an inequality (10) between them. Moreover, the saturation of this bound in both holographic theories and free boson-fermion theories is realized when either $c_{LR} = c_{\mathrm{eff}} = 0$, or $c_{LR} = c_{\mathrm{eff}} = c_L = c_R$.

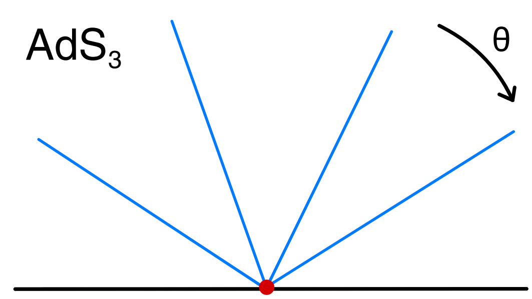

FIG. 1. The foliation of an asymptotic AdS_3 bulk. The black line below corresponds to the AdS asymptotic boundary, and the red dot is the one-dimensional interface. Each blue line represents an AdS_2 slice, and in certain degeneration limit of the warp factor it can be a thin brane across which the effective AdS radius jumps.

Consider bottom-up AdS/CFT where an ICFT_2 is dual to an asymptotic AdS_3 spacetime with sliced AdS_2 leaves and SO(2,1) isometry [12], as shown in Fig. 1. Its metric is

$$ds^2 = a^2(\theta)\left(\frac{dx^2 - dt^2}{x^2} + d\theta^2\right), \tag{13}$$

where the AdS_2 is written in Poincaré patch, and $\theta \in (-\pi/2, \pi/2)$ is the slicing coordinate. $a(\theta)$ is a general function, referred to as the "warp factor."

Below, we will consider a general warp factor $a(\theta)$ that is a C^2 function. Near $\theta \to \pm\pi/2$, we have the asymptotic form $\lim_{\theta\to-\pi/2} a(\theta) = l_L/\cos\theta$ and $\lim_{\theta\to\pi/2} a(\theta) = l_R/\cos\theta$. Recall that the Brown-Henneaux formula relates $l_{L/R}$ to the central charges $c_{L/R}$ of $\mathrm{CFT}_{L/R}$ as $c_{L/R} = 3l_{L/R}/2G_N$ where G_N is the Newton constant in 3D.

For such a continuous AdS domain wall solution, the transmission coefficient of the interface (or equivalently, c_{LR} defined above) is given by [13]

$$c_{LR} = \frac{3}{G_N}\left(\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G_N \sigma\right)^{-1}, \tag{14}$$

where σ characterizes the net brane tension. To calculate the effective net tension, we first define a function $L(\theta)$ as

$$L(\theta) \equiv \frac{a(\theta)}{\sqrt{1 + \left(\frac{a'(\theta)}{a(\theta)}\right)^2}}. \tag{15}$$

$L(\theta)$ represents an effective local AdS curvature radius.

To calculate the brane tension σ we note that the Israel junction condition [14,15] gives the differential change in brane tension needed to support the change in curvature radius [16]

$$8\pi G_N \frac{d\sigma}{d\theta} = \frac{a(\theta)|L'(\theta)|}{L(\theta)^2\sqrt{a(\theta)^2 - L(\theta)^2}}. \tag{16}$$

Set L_j to be the set of solutions for $L'(\theta) = 0$ in $\theta \in (-\pi/2, \pi/2)$ where $j = 1, \ldots, M$. The integration is bounded by [17]

$$\begin{aligned} 8\pi G_N \int_{-\pi/2}^{\pi/2} d\theta \frac{d\sigma}{d\theta} &\geq \int_{-\pi/2}^{\pi/2} d\theta \frac{|L'(\theta)|}{L^2(\theta)} \\ &= \int |dL| \frac{1}{L^2} = \left|\frac{1}{l_L} - \frac{1}{L_1}\right| \\ &\quad + \cdots + \left|\frac{1}{L_M} - \frac{1}{l_R}\right|. \end{aligned} \tag{17}$$

The total net tension σ can be derived by integrating $d\sigma/d\theta$ over its C^2 support of θ [16,18,19]. Let the global minimum of $L(\theta)$ function be $l_{\min}$. By definition, $l_{\min}$ is equal to one of the L_j. Picking out $\theta_{\min}$ in (17) means that

$$8\pi G_N \sigma \geq \left|\frac{1}{l_L} - \frac{1}{l_{\min}}\right| + \left|\frac{1}{l_R} - \frac{1}{l_{\min}}\right|. \tag{18}$$

This leads us to conclude that

$$\begin{aligned} c_{LR} &= \frac{3}{G_N}\left(\frac{1}{l_L} + \frac{1}{l_R} + 8\pi G\sigma\right)^{-1} \\ &\leq \frac{3}{G_N}\left(\frac{1}{l_L} + \frac{1}{l_R} + \left|\frac{1}{l_L} - \frac{1}{l_{\min}}\right| + \left|\frac{1}{l_R} - \frac{1}{l_{\min}}\right|\right)^{-1} \\ &\leq \frac{3}{G_N}\left(\frac{2}{l_{\min}}\right)^{-1} \leq \frac{3a_{\min}}{2G_N} = c_{\text{eff}}, \end{aligned} \tag{19}$$

where the second line is from (18), and the third line is the universal formula for the effective central charge in holographic ICFT_2, indicating the relation between c_{eff} and the minimal value $a_{\min}$ of the warp factor $a(\theta)$ [20,21].

In order to saturate this inequality, from (17), the warp factor has to diverge wherever $|L'(\theta)| > 0$. Hence, there are two ways to realize $c_{LR} = c_{\text{eff}}$. One is when $a_{\min} = l_{\min} = 0$ and the net brane tension diverges. In this case, $c_{LR} = c_{\text{eff}} = 0$, and the two BCFTs are uncorrelated at all. The other is when $L(\theta)$ is constant, and the ICFT_2 is dual to a pure AdS_3 with a topological interface. In particular, $c_{LR} = c_{\text{eff}} = c_L = c_R$.

It is worth mentioning that holographic duals with discontinuity in $a'(\theta)$ are often considered as thin branes anchoring on the AdS boundary [13,19]. It corresponds to a delta function in (16). Upon integrating, it contributes to the net tension a term $8\pi G_N \sigma_t$ that follows the Coleman–De Luccia bound [22]

$$8\pi G_N \sigma_t \geq \left|\frac{1}{L_{\text{left}}} - \frac{1}{L_{\text{right}}}\right|, \tag{20}$$

where L_{left} and L_{right} are the effective AdS_2 radii on the left and right of the thin brane, respectively. The equality holds only when the AdS_2 radius diverges at the brane. It is obvious that our proof follows through in this degenerate limit of the warp factor, and so does the saturation condition.

As a corollary of the above proof, the transmission coefficient c_{LR} depends on an integration of functions on the warp factor $a(\theta)$ over the entire range, while the entanglement entropy proportional to c_{eff} only depends on the minimal value of $a(\theta)$ as in (19) and the comments below. Therefore, in general, there is no strict monotonicity (correlation) between those two quantities.

The free boson-fermion theories across a partially transmissive interface provide additional evidence for the inequality [23]. In both cases, there is a parameter $s \in [0, 1]$ controlling the jumping radii on the two sides that characterize the interface, and the transmission coefficient is $\mathcal{T} = s^2$ [4,5].

$c = 1$ free boson: The entanglement entropy for $c = 1$ free theories across an interface has been derived to be [5]

$$c_{\rm eff}^{\rm bos} = \frac{1}{2} + s + \frac{3}{\pi^2}[(s+1)\log(s+1)\log s + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)], \quad (21)$$

where $\mathrm{Li}_2(s)$ is the dilogarithm function. Algebraically, we always have $c_{\rm eff}^{\rm bos} \geq c_{LR}^{\rm bos} = s^2$ and the equality saturates only when $\mathcal{T} = s^2 = 0, 1$.

$c = 1/2$ free fermion: Similarly, in the presence of an interface, the entanglement entropy in the vacuum for the free fermion is [6]

$$c_{\rm eff}^{\rm fer} = \frac{s-1}{2} - \frac{3}{\pi^2}[(s+1)\log(s+1)\log s + (s-1)\mathrm{Li}_2(1-s) + (s+1)\mathrm{Li}_2(-s)]. \quad (22)$$

Again, we have $c_{\rm eff}^{\rm fer} \geq c_{LR}^{\rm fer} = s^2/2$, and the equality holds iff $\mathcal{T} = s^2 = 0, 1$.

It is obvious that the free theories also saturate this bound only when $c_{LR} = c_{\rm eff}$ is at either end of their spectrum. Therefore, we propose that this saturation condition for $c_{LR} \leq c_{\rm eff}$ is a universal feature among all ICFT_2.

Additional evidence for the inequality comes from the defect perturbation. Consider deforming a topological defect on a line γ by a relevant or marginal defect field ϕ,

$$\delta S = \lambda \int_\gamma dw \phi(w), \quad (23)$$

where λ is the coupling. Under this perturbation, the effective central charge changes as follows up to order λ^2 [24],

$$c_{\rm eff} = c\left(\mathcal{T} + \frac{1}{4}\mathcal{R}\right), \quad (24)$$

where $\mathcal{R}$ is the reflection coefficient $\mathcal{R} = 1 - \mathcal{T}$, which is non-negative because $0 \leq \mathcal{T} \leq 1$. Note that this is consistent with (22), and the point is that the result (24) is not limited to free fermion but holds in general. Since $c\mathcal{T} = c_{LR}$ and $\mathcal{R} \geq 0$, we obtain

$$c_{LR} \leq c_{\rm eff}. \quad (25)$$

Holographic saturation of $c_{\rm eff} \leq \min\{c_L, c_R\}$—The upper bound (11) on $c_{\rm eff}$ has been derived for both holographic theories and general ICFT_2 in [1]. Below, we will write down its saturation condition in holographic theories in terms of conditions on the warp factor, which is much more mathematically tractable compared to the CFT side. In particular, we show that with the possible presence of thin branes in the bulk, there is a much broader family of holographic ICFT_2 that saturates this bound than ICFT_2 with topological (transparent) interfaces.

For a holographic ICFT_2 dual to the bulk (13) with warp factor $a(\theta)$, we construct an auxiliary function [1]

$$F = \frac{1}{L^2} = \left(\frac{a'}{a^2}\right)^2 + \frac{1}{a^2}. \quad (26)$$

The derivative of F gives

$$F' = \frac{2a'(a''a - 2a'^2 - a^2)}{a^5}. \quad (27)$$

The null-energy condition (NEC) on $a(\theta)$ reads

$$a^2(\theta) + 2a'^2(\theta) - a(\theta)a''(\theta) \geq 0. \quad (28)$$

From the above two equations we conclude that at $\theta_{\rm min}$ where the warp factor $a'(\theta) = 0$ and reaches its minimal value $a_{\rm min}$, we have $F'(\theta) = 0$ and reaches its maximal value $F_{\rm max}$. Explicitly, we have [25]

$$F_{\rm max} = \left(\frac{3}{2G_N}\right)^2 \frac{1}{c_{\rm eff}^2},$$
$$\lim_{\theta\to\pm\pi/2} F(\theta) = \left(\frac{3}{2G_N}\right)^2 \frac{1}{c_{R/L}^2}. \quad (29)$$

If we set $c_L \geq c_R$, the saturation of $c_{\rm eff} \leq \min\{c_L, c_R\}$ is then equivalent to.

Case (a) $c_L > c_R$: Saturation of NEC, i.e., $a(\theta) = l_R/\cos\theta$, for $\theta \in (\theta_{\rm min}, \pi/2)$, and *any* $a(\theta)$ for the rest of the region subject to Einstein's equation and boundary conditions, with the possible presence of thin branes.

Case (b) $c_L = c_R$: Pure AdS_3 with a topological interface, or at least two minima for the warp factor at $\theta_{\min 1} < \theta_{\min 2}$. Saturation of NEC, i.e., $a(\theta) = l/\cos\theta$ where $l_L = l_R = l$, for $\theta \in (-\pi/2, \theta_{\min 1}) \cup (\theta_{\min 2}, \pi/2)$, and *any* $a(\theta)$ for $(\theta_{\min 1}, \theta_{\min 2})$ subject to Einstein's equation, with the possible presence of thin branes.

Discussion—Our results inspire various future works and applications. (i) We have proposed a universal relationship between entanglement through the interface and energy transmission, based on the AdS/CFT correspondence. It is desirable to have a proof for general CFTs. In [1], we were able to give a general proof of the upper bound on the effective central charge using the entropic c theorem. A similar approach might be useful for our current purpose as well. It may also be possible to verify our results numerically using the lattice realization of the conformal interface [26,27]. (ii) We have identified holographic interfaces which saturate the bounds. It is important to determine the saturation condition for general CFTs. In fact, there are nonholographic CFTs which saturate the bounds, as we show with explicit examples in Supplemental Material [28]. (iii) It is also desirable to generalize our results to higher-dimensional CFTs. Two potential challenges in higher dimensions are the lack of the Virasoro symmetry and the growth of entanglement, which makes well-known numerical calculation methods like density matrix renormalization group unusable. (iv) The effective central charge plays an important role in the weak measurement and the pseudo entropy (see, for example, [29–31]). The relationship revealed in our Letter can be useful in elucidating the properties of such quantities.

What draws our attention here is the observation that many analytical methods on the gravity side do not depend on the dimension d. The successful generalization of the concept of the effective central charge to higher dimensions has been achieved using the AdS/CFT correspondence [1]. This is precisely because the calculation on the gravity side is not dependent on the dimension d. Based on this insight, it is a very interesting challenge to predict how the results revealed in this Letter would change in higher dimensions using the AdS/CFT correspondence. Additionally, providing proof within CFT for such predictions is also an important challenge.

Acknowledgments—We would like to thank Costas Bachas, Ilka Brunner, Shinsei Ryu, and Yifan Wang for careful reading and valuable comments on a draft of this Letter. A. K., H. S., and M. W. are supported in part by the U.S. Department of Energy under Grant No. DE-SC0022021 and a grant from the Simons Foundation (Grant No. 651440, A. K.). H. O. and Y. K. are supported in part by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Award No. DE-SC0011632. In addition, Y. K. is supported by the Brinson Prize Fellowship at Caltech and the INAMORI Frontier Program at Kyushu University. H. O. is supported in part by the Simons Investigator Award (No. MP-SIP-00005259), the Guggenheim Fellowship, the World Premier International Research Center Initiative, MEXT, Japan, and JSPS Grants-in-Aid for Scientific Research 23K03379. This work was performed in part at the Aspen Center for Physics, which is supported by NSF Grant No. PHY-1607611, and at the Kavli Institute for Theoretical Physics (KITP), which is supported by NSF Grant No. PHY-2309135.

[1] A. Karch, Y. Kusuki, H. Ooguri, H.-Y. Sun, and M. Wang, Universality of effective central charge in interface CFTs, J. High Energy Phys. 11 (2023) 126.

[2] M. Oshikawa and I. Affleck, Defect lines in the Ising model and boundary states on orbifolds, Phys. Rev. Lett. **77**, 2604 (1996).

[3] M. Oshikawa and I. Affleck, Boundary conformal field theory approach to the critical two-dimensional Ising model with a defect line, Nucl. Phys. **B495**, 533 (1997).

[4] C. Bachas, J. de Boer, R. Dijkgraaf, and H. Ooguri, Permeable conformal walls and holography, J. High Energy Phys. 06 (2002) 027.

[5] K. Sakai and Y. Satoh, Entanglement through conformal interfaces, J. High Energy Phys. 12 (2008) 001.

[6] E. M. Brehm and I. Brunner, Entanglement entropy through conformal interfaces in the 2D Ising model, J. High Energy Phys. 09 (2015) 080.

[7] E. M. Brehm, I. Brunner, D. Jaud, and C. Schmidt-Colinet, Entanglement and topological interfaces, Fortschr. Phys. **64**, 516 (2016).

[8] X. Wen, Y. Wang, and S. Ryu, Entanglement evolution across a conformal interface, J. Phys. A **51**, 195004 (2018).

[9] M. Gutperle and J. D. Miller, A note on entanglement entropy for topological interfaces in RCFTs, J. High Energy Phys. 04 (2016) 176.

[10] T. Quella, I. Runkel, and G. M. T. Watts, Reflection and transmission for conformal defects, J. High Energy Phys. 04 (2007) 095.

[11] M. Meineri, J. Penedones, and A. Rousset, Colliders and conformal interfaces, J. High Energy Phys. 02 (2020) 138.

[12] We do not consider (top-down) bulk configurations with dependence on extra dimensions in this Letter (e.g., SUSY Janus).

[13] C. Bachas, S. Chapman, D. Ge, and G. Policastro, Energy reflection and transmission at 2D holographic interfaces, Phys. Rev. Lett. **125**, 231602 (2020).

[14] W. Israel, Singular hypersurfaces and thin shells in general relativity, Nuovo Cimento B **44S10**, 1 (1966).

[15] K. Lanczos, Flächenhafte verteilung der materie in der einsteinschen gravitationstheorie, Ann. Phys. (Berlin) **379**, 518 (1924).

[16] C. Bachas, S. Baiguera, S. Chapman, G. Policastro, and T. Schwartzman, Energy transport for thick holographic branes, Phys. Rev. Lett. **131**, 021601 (2023).

[17] The set of solutions can be empty, in which case there is no L_j, or it can be a collection of intervals, in which case we choose L_j to be the right bound of the intervals. None of these cases affect our results.

[18] C. Bachas, Z. Chen, and V. Papadopoulos, Steady states of holographic interfaces, J. High Energy Phys. 11 (2021) 095.

[19] S. A. Baig and A. Karch, Double brane holographic model dual to 2D ICFTs, J. High Energy Phys. 10 (2022) 022.

[20] A. Karch, Z.-X. Luo, and H.-Y. Sun, Universal relations for holographic interfaces, J. High Energy Phys. 09 (2021) 172.

[21] A. Karch and M. Wang, Universal behavior of entanglement entropies in interface CFTs from general holographic spacetimes, J. High Energy Phys. 06 (2023) 145.

[22] S. Coleman and F. De Luccia, Gravitational effects on and of vacuum decay, Phys. Rev. D **21**, 3305 (1980).

[23] The interface in the free cases discussed here was referred to as a "permeable interface" in [4].

[24] E. M. Brehm, Defects and perturbation, J. High Energy Phys. 04 (2021) 300.

[25] One might worry about the thin branes where $a'(\theta)$ is not well defined, but from the achronal-averaged null energy condition, the minimum of $a(\theta)$ and maximum of $F(\theta)$ are not reached at the locations of thin branes [19].

[26] C. V. Cogburn, A. L. Fitzpatrick, and H. Geng, CFT and lattice correlators near an RG domain wall between minimal models, SciPost Phys. Core **7**, 021 (2024).
[27] Q. Tang, Z. Wei, Y. Tang, X. Wen, and W. Zhu, Universal entanglement signatures of interface conformal field theories, Phys. Rev. B **109**, L041104 (2024).
[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.091604 for an explicit exapmle of c_{eff} saturation in CFT.
[29] T. Nishioka, T. Takayanagi, and Y. Taki, Topological pseudo entropy, J. High Energy Phys. 09 (2021) 015.
[30] H. Kanda, T. Kawamoto, Y.-k. Suzuki, T. Takayanagi, K. Tasuki, and Z. Wei, Entanglement phase transition in holographic pseudo entropy, J. High Energy Phys. 03 (2024) 060.
[31] X. Sun and S.-K. Jian, Holographic weak measurement, J. High Energy Phys. 12 (2023) 157.