Sparsity-Independent Lyapunov Exponent in the Sachdev-Ye-Kitaev Model

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The saturation of a recently proposed universal bound on the Lyapunov exponent has been conjectured to signal the existence of a gravity dual. This saturation occurs in the low-temperature limit of the dense Sachdev-Ye-Kitaev (SYK) model, N Majorana fermions with q body ($q > 2$) infinite-range interactions. We calculate certain out-of-time-order correlators (OTOCs) for $N \leq 64$ fermions for a highly sparse SYK model and find no significant dependence of the Lyapunov exponent on sparsity up to near the percolation limit where the Hamiltonian breaks up into blocks. This provides strong support to the saturation of the Lyapunov exponent in the low-temperature limit of the sparse SYK. A key ingredient to reaching $N = 64$ is the development of a novel quantum spin model simulation library that implements highly optimized matrix-free Krylov subspace methods on graphical processing units. This leads to a significantly lower simulation time as well as vastly reduced memory usage over previous approaches, while using modest computational resources. Strong sparsity-driven statistical fluctuations require both the use of a much larger number of disorder realizations with respect to the dense limit and a careful finite size scaling analysis. The saturation of the bound in the sparse SYK points to the existence of a gravity analog that would enlarge substantially the number of field theories with this feature.

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The exponential growth of certain out-of-time-order correlation (OTOC) functions up to the Ehrenfest time in the semiclassical limit, at a rate given by the leading classical Lyapunov exponent, is an early signature of quantum chaotic dynamics. Their calculation in simple single-particle problems such as a particle in a random potential [1] or kicked rotors [2] were landmarks in the early development of the theory of quantum chaos. However, they are notoriously difficult to compute quantitatively in many-body systems because the region of exponential growth is relatively short and can be easily overshadowed by other contributions unless the system is strictly within the semiclassical limit.

A resurgence of interest in OTOCs in quantum chaos came quite unexpectedly from quantum gravity. Heuristic arguments [3,4] suggested that the dynamics of a particle close to a black hole horizon is quantum chaotic. Later, these ideas were put on a much firmer ground by showing that a universal bound on the Lyapunov exponents in quantum chaotic systems at thermal equilibrium was saturated in field theories with a gravity dual [5]. Shortly afterward, Kitaev [6] demonstrated analytically that, in the low-temperature limit, this universal bound on chaos was saturated in a simple model, now termed the Sachdev-Ye-Kitaev (SYK) model [6–11], consisting of N Majoranas [6,12] with random q-body interactions in zero spatial dimensions. The quantum chaotic nature of the SYK model for longer timescales was confirmed by a level statistics analysis [13,14], and its gravity dual was identified to be Jackiw-Teitelboim gravity [15–17]. The analytical tractability of the SYK model is one of its most appealing features. Unfortunately, generalizations of the model with finite range [18] or sparsified [19–23] interactions do not inherit this property. This begs the question: is the saturation of the bound, which indicates the possible existence of a gravity dual, a particularity of the dense SYK, or is it present in more general settings? For the dense SYK, a recent numerical calculation [24] of the Lyapunov exponent based on the Krylov subspace method [25,26] for up to $N = 50$ on a graphical processing unit (GPU) system [and $N = 60$ on a central processing unit (CPU) only system] confirmed the analytical results [6,12]. An important benefit of the sparsified SYK model is that it may be easier to simulate on a quantum computer [27–29], which potentially facilitates addressing questions that cannot be answered with classical computers.

In this Letter, we aim to calculate the Lyapunov exponent for a sparse variant [19–23] of the SYK model where a

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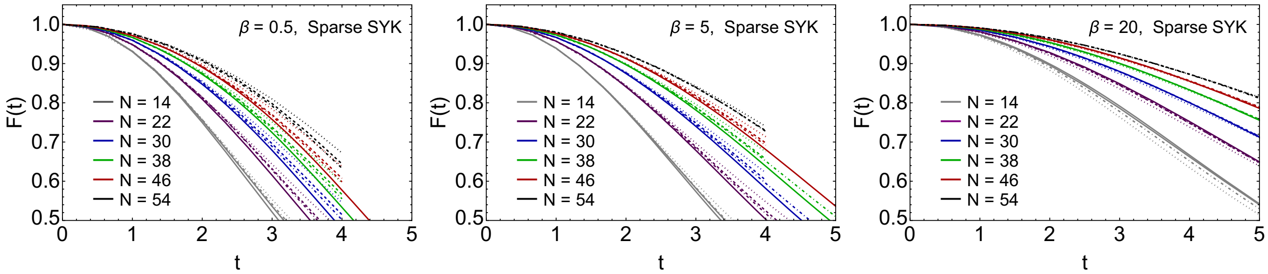

FIG. 1. The OTOC Eq. (2) versus time t for $N = 14$, 22, 30, 38, 46, and 54 as indicated in the legend. Results are given for $\beta = 0.5$ (left), $\beta = 5$ (middle), and $\beta = 20$ (right) and sparseness parameter $k = 3$ (dotted), $k = 6$ (dashed), $k = 9$ (dot-dashed), and dense (solid). The dependence on k in the region where OTOC decreases exponentially is quite weak.

large (to be defined shortly) number of random couplings are set to zero. A key ingredient in our study is the development of a highly optimized GPU computing code, which implements the Krylov-based algorithm for computing time evolution of qubit systems. This allows us to reach up to $N = 64$ Majoranas on single GPU systems. Our main result is that, for all temperatures investigated, the Lyapunov exponent of the sparse SYK model has no significant dependence on sparsity, and agrees with the one in the dense SYK case, all the way up to close to the percolation limit. From the holographic implications of the saturation [6] of the Lyapunov exponent bound [5] in the low-temperature limit of the dense SYK, this demonstrates that the sparse SYK has a gravity dual.

Sparse SYK model—Our Hamiltonian describes N strongly interacting Majorana fermions in zero spatial dimensions [6–10,12,30–32] with sparse [19–22,33] random interactions of infinite range:

$$H = \sum_{0\le i<j<k<l<N} p_{ijkl} J_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l. \quad (1)$$

The Majorana operators γ_n satisfy the Clifford algebra $\{\gamma_m, \gamma_n\} = \delta_{mn}$, and can be expressed as a tensor product of Pauli matrices. The J_{ijkl} are random numbers with a Gaussian distribution of zero average and variance $\langle J^2 \rangle = 3/(8pN^3)$. The sparseness of the Hamiltonian is modeled by the stochastic variable p_{ijkl}, which is sampled from the Bernoulli distribution $B(p)$ with probability $p > 0$. When $p = 1$, we recover the dense SYK model. Models with $0 < p < 1$ are called sparse SYK models. In principle, $p \sim N^{-\alpha}$ with $\alpha > 0$ a real parameter. It was shown in Refs. [19,20], $\alpha = 3$ is the relevant scaling to study the effect of sparsity because for $\alpha > 3$ connectivity in Fock space is broken in the large-N limit while the effect of sparsity is largely irrelevant for $\alpha < 3$. Therefore, it is natural to define the sparsity strength $k = (p/N)\binom{N}{4}$. For the comparison with the dense case, we will focus on the $k \ge 3$ region only because, for sufficiently large N, it is computationally expensive to impose a regularity condition on the vertex connectivity. The latter is necessary for $k \sim 1$ in order to prevent the Hilbert space from splitting into separate invariant subspaces of the Hamiltonian.

OTOC calculation and results—We now define the following regularized out-of-time-order correlation (OTOC) function for the Hamiltonian Eq. (1):

$$F(t) = \frac{1}{\mathcal{N}} \mathrm{Tr}\Big[e^{itH}\gamma_{N-1} e^{-(it+\beta/4)H}\gamma_{N-2} \times e^{(it-\beta/4)H}\gamma_{N-1} e^{-(it+\beta/4)H}\gamma_{N-2} e^{-\beta H/4}\Big], \quad (2)$$

with $\mathcal{N} = \mathrm{Tr}(e^{-\beta H/4}\gamma_{N-1} e^{-\beta H/4}\gamma_{N-2} e^{-\beta H/4}\gamma_{N-1} e^{-\beta H/4}\gamma_{N-2})$ so that $F(0) = 1$. Different regularizations may lead to slightly different prefactors in the $1/N$ expansion of the OTOC, which may be time-dependent but the Lyapunov exponent was recently shown [34,35] to be independent of the regularization.

A strong hint of what will be the main result of this Letter, the independence of the Lyapunov exponent on the sparsity, can already be seen from Fig. 1 depicting the OTOC dependence on the sparseness parameter k for different $N \in [14, 54]$: in the low-temperature limit, the OTOC depends only weakly on k even for relatively small $k = 3$ close to the percolation limit where the Hamiltonian breaks up into blocks for most disorder realizations. We restrict ourselves to $N \bmod 8 = 6$ because the approach to the large-N limit may be more uniform taking advantage of the Bott periodicity of the SYK model [36].

GPU-based numerical optimizations—The calculation of the OTOC in the large-N limit of interest requires the development of a novel quantum time evolution library [37] called REAPERS, short for “a REAsonably PERformant Simulator for qubit systems,” that implements highly optimized matrix-free [38] Krylov subspace methods [25,26,39,40] on NVIDIA GPUs. Written in C++20, it provides a programming interface similar to that of DYNAMITE [41], but does not depend on low-level libraries such as the Portable, Extensible Toolkit for Scientific Computation [42–44] for matrix operations. Instead, we start from scratch and implement optimized Computer Unified Device Architecture kernels that compute spin operator actions on quantum states more efficiently than those provided by the Portable, Extensible Toolkit for Scientific Computation, and carefully manage object allocations and deallocations to use as little video memory as possible. The result of these performance optimizations is that we are able to simulate sparse SYK systems with $N = 62$ fermions in double precision floating point (or $N = 64$ fermions in single precision) on single-GPU systems, and $N = 64$ fermions in double precision (or $N = 66$ in single precision) on dual-GPU systems [45] with the 80 GB version of the NVIDIA A100 graphics card. Comparing this with the previous state of the art CPU-based calculation of an $N = 60$ (albeit dense) SYK system using a 500-node supercluster [24], our hardware cost is far less and we consume far less energy. We refer the reader to the Supplemental Material [46] for the technical details of our optimization techniques and further benchmark data.

Finite size scaling analysis—The expectation for quantum chaotic systems [1,2] is that for sufficiently short times below the Ehrenfest time the OTOC decreases exponentially. At least for low temperatures, an analysis based on the Schwarzian action [47–50] in the dense case shows that the decay only remains exponential up to around the Ehrenfest time, after which it approaches zero with a decreasing exponent before finally turning into a powerlike decay for very long times. In the exponentially decaying domain, the dependence on t and N is only through the combination $\exp(\lambda_L t)/N$ [12,51,52] so that

$$F(t) = g_0(t) - g\left(\frac{e^{\lambda_L t}}{N}\right) \tag{3}$$

with

$$g\left(\frac{e^{\lambda_L t}}{N}\right) = c_1 \frac{e^{\lambda_L t}}{N} + c_2 \left(\frac{e^{\lambda_L t}}{N}\right)^2 + \cdots \tag{4}$$

and λ_L the Lyapunov exponent. In order to extract λ_L, we have to restrict the numerical calculation of $F(t)$ to the region in which the decay obeys Eq. (4). For that purpose, we largely follow the method of Ref. [24] for the dense SYK based on the rescaling symmetry, $t \to t + \log r/\lambda_L$ and $N \to Nr$, where $r > 0$. In a first step, we determine the time $t^*(N)$ for which $F(t)$ drops to a certain value $F_0 < 1$. The value of F_0 cannot be too large because that would not capture the exponential growth but it also cannot be too small because the OTOC no longer decays exponentially. We shall see that for values of F_0 between 0.75 and 0.85 the results are consistent with an exponential growth. For the scaling behavior Eq. (4) we find to leading order in $1/N$,

$$t^*(N) = \frac{\log N}{\lambda_L} + \frac{1}{\lambda_L} (\log(g_0(t^*(N)) - F_0)/c_1). \tag{5}$$

The rescaling symmetry requires that g_0 does not depend on t in the region of exponential decay. We show in the Supplemental Material [46] that $g_0(t)$ depends only weakly on t. In that case, the second term can be eliminated by differentiation with respect to N, resulting in

$$\lambda_L = \frac{1}{N dt^*(N)/dN} + O(1/N). \tag{6}$$

In principle, the Lyapunov exponent can be obtained from the slope of $t^*(N)$ versus $\log N$, but the slope has a residual N dependence in the time and size window at our disposal. Ideally, we fit observables for which this residual N dependence is minimized. In agreement with Ref. [24], our numerical results suggest that the $1/N$ dependence of the inverse slope is close to linear at low temperatures ($\beta \geq 5$) so that the Lyapunov exponent is determined by

$$\frac{1}{N dt^*(N)/dN} = \lambda_L + \frac{\alpha_1}{N} + \frac{\alpha_2}{N^2} + O(1/N^3), \tag{7}$$

with $\alpha_2 = 0$. At high temperatures (say $\beta = 0.5$), the $1/N$ dependence is fitted by a quadratic dependence with $\alpha_1 = 0$ except in the dense case when the data are sufficiently accurate to use a three parameter fit. An estimate for the Lyapunov exponent is given by the extrapolation of $1/[N dt^*(N)/dN]$ to $1/N \to 0$. Details of the fitting procedure are left to the Supplemental Material [46].

Alternatively, one can integrate Eq. (7), resulting in the equivalent expansion up to logarithmic factors,

$$t^*(N) = \gamma_0 + \frac{\log N}{\lambda_L}\left(1 + \frac{\gamma_1}{N} + \frac{\gamma_2}{N^2}\right) + O\left(\frac{1}{N^3}\right), \tag{8}$$

where λ_L, γ_0, γ_1, and γ_2 are fitting parameters. For $\beta \geq 5$ we set $\gamma_2 = 0$, but for $\beta = 0.5$ we use γ_2/N^2 as the correction term, putting $\gamma_1 = 0$. As can be seen from Fig. 2, this gives an excellent fit of the $t^*(N)$ data for all considered temperatures, sparsity parameters, and cutoff values. Fitting $t^*(N)$ directly has the advantage that the errors are smaller. On the other hand, fitting the numerical derivative $\partial_N t^*(N)$ has the benefit of having one less fitting parameter at the expense of much larger errors (see Supplemental Material [46]). We shall see the fitting results of both methods are consistent though there is a significant systematic error. There is also an issue of overfitting, which trades the $\log N$ dependence for the $1/N$ dependence. For example, using additive $1/N$ corrections instead of multiplicative $1/N$ correction significantly changes the value of the Lyapunov exponent.

Results for the Lyapunov exponent—Our results for the Lyapunov exponents λ_L are shown in Fig. 3. In the range of temperatures we have considered, $\beta = 0.5$, 5, 20, there is no significant dependence of the Lyapunov exponent on the sparsity parameter k. The difference of λ_L for different

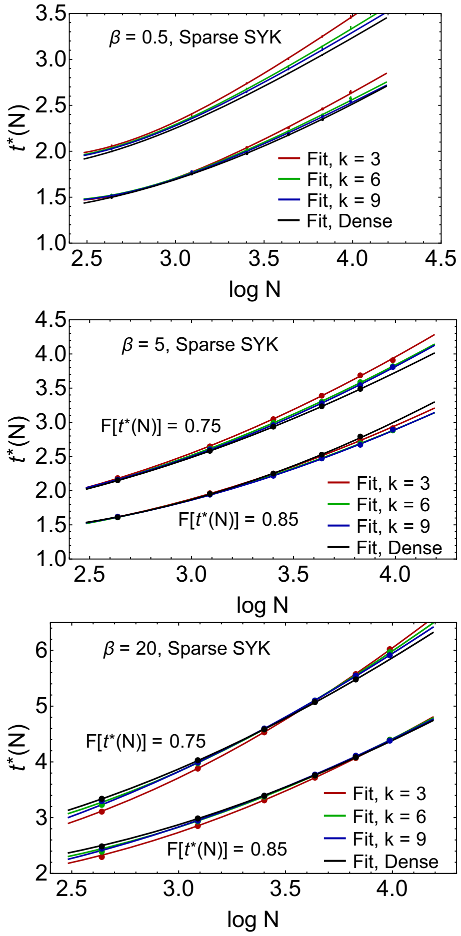

FIG. 2. The time $t^*(N)$ at which F Eq. (2) drops to either 0.75 or 0.85 versus $\log N$ for $\beta = 0.5$ (upper), $\beta = 5$ (middle), and $\beta = 20$ (lower) and values of k as indicated in the legend of the figures. For the numerical fit we employ Eq. (8). For $\beta = 20$, $k = 3$, and $N = 14$, 13 outliers that differ by more than 30 standard deviations from the median have been excluded from the averages [53].

values of k is less than the discrepancies between the two choices of F_0 and the two choices of the fitting method, both of which are a measure of the systematic error. Numerical results for the best fitting lines are given in Table I.

We now compare our results with predictions for the dense case, either analytical or based on the numerical solutions of the Schwinger-Dyson (SD) equations. As shown in the last column of Table I, for $\beta = 0.5$ and $\beta = 5$ our numerical results are in good agreement with the large N prediction obtained by solving the SD equations [24]. For $\beta \geq 20$ (see Fig. 3), we find a Lyapunov exponent that is consistent with the chaos bound of $2\pi/\beta$ [5]. Note that in Fig. 3 (right) the result for $\beta = 40$ only has been obtained for $k = 6$. Taking into account subleading finite temperature corrections (which can be obtained by solving the SD equations) lowers the theoretical large N value. For $\beta = 20$ is reduced to 0.24, which is still above our result but is in agreement with previous numerical calculations in the dense case [24], where for $\beta = 17.8$ the Lyapunov exponent was 0.36 versus 0.26 from solving the SD equations. Therefore this discrepancy is not related to the sparsity of the model. We stress that in order to reach this relatively low level of statistical fluctuations, it is necessary to simulate a number of disorder realizations at least of order 10^4, which is 2 orders of magnitude larger than in the dense case [24]. Surprisingly, unlike the dense case, the fluctuations are not larger for low temperature and they are not reduced as N increases, which prevents us from including $N = 62$ in this analysis despite the fact that it is numerically accessible. This technical difficulty prevented a precise numerical calculation of the Lyapunov exponent in a recent calculation of OTOC in the sparse SYK [33] that employed CPUs instead of GPUs. A comment is in order: it was previously claimed in Ref. [20] that the sparse SYK was maximally chaotic for k of order 1. However, this statement was not supported by any analytic or numerical calculation of OTOC. Moreover, the provided heuristic arguments do

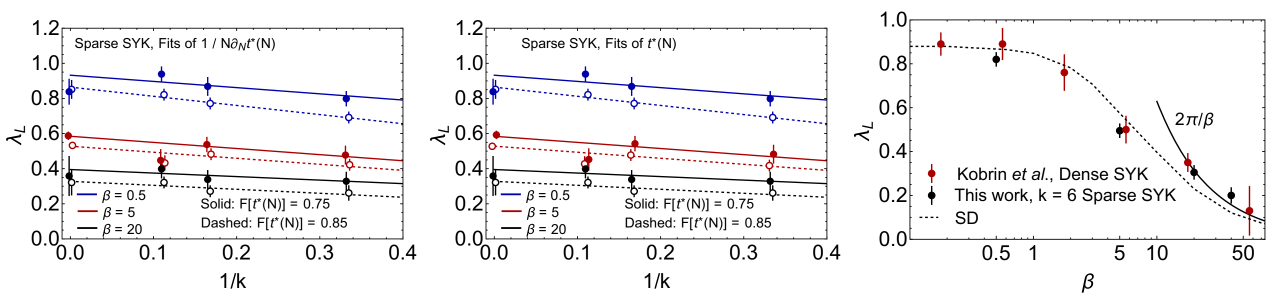

FIG. 3. The Lyapunov exponent, obtained from the derivative of $t^*(N)$ (left) and from $t^*(N)$ (middle), as a function of the inverse sparsity parameter k for $\beta = 0.5$, 5, and 20 using two different cutoffs: $F_0 = 0.75$ (dashed lines and open circles), 0.85 (solid lines and solid disks). The dense case is represented by the point $1/k = 0$. The lines stand for linear fits to the data points. See Table I for a summary of results. For $\beta = 20$, the slope of the line that accounts for the k dependence is consistent with zero. Right: Lyapunov exponent versus inverse temperature. Bound on chaos (solid) [5], result from the solution of the SD equations (dashed) [6,12], numerical dense SYK [24] (red dots), numerical sparse SYK (black dots).

TABLE I. The Lyapunov exponents obtained by a linear fit to the data of Fig. 3. In the first two columns, λ_L is obtained from Eq. (8), in the next two columns λ_L is obtained from Eq. (7). Dense SD stands for the result using the exact solutions of the SD equations in the dense SYK [12].

λ_L	$F_0 = 0.75,\ 1/[N\partial_N t^*(N)]$	$F_0 = 0.85,\ 1/[N\partial_N t^*(N)]$	$F_0 = 0.75,\ t^*(N)$	$F_0 = 0.85,\ t^*(N)$	Dense SD
$\beta = 0.5$	$0.86(3) - 0.52(15)/k$	$0.93(4) - 0.35(20)/k$	$0.84(2) - 0.29(10)/k$	$0.94(2) - 0.20(7)/k$	0.86
$\beta = 5$	$0.52(1) - 0.34(9)/k$	$0.58(2) - 0.19(14)/k$	$0.50(1) - 0.22(4)/k$	$0.60(1) - 0.22(6)/k$	0.59
$\beta = 20$	$0.33(4) - 0.2(2)/k$	$0.39(6) - 0.2(3)/k$	$0.30(2) - 0.16(10)/k$	$0.40(3) - 0.2(2)/k$	0.24

not address whether the $1/N$ corrections needed to compute the Lyapunov exponent are affected by the sparsity.

Conclusions and outlook—We have studied out-of-time-order correlators in a sparse variant of the SYK model. After a careful data analysis, we have shown that the Lyapunov exponent has no significant dependence on the sparsity k for all temperatures we have considered, and agrees with previous [24] numerical results for the dense case. In the low-temperature limit, this is a confirmation that the bound on chaos [5] is still saturated for the sparse SYK model. However, a word of caution is in order: in the low-temperature limit where saturation occurs, the value of the Lyapunov exponent for both the dense and the sparse SYK is above the analytical prediction [12]. We believe that reaching larger sizes will bring agreement between analytic and numerical results.

A crucial part of our work is the development of an optimized GPU-based quantum simulation library, which enables us to reach $N \leq 64$ Majoranas due to drastic improvements in simulation speed and memory usage. The energy consumption and the cost of hardware are both vastly smaller than equivalent simulations on CPU-based systems. Natural extensions of this work include computing OTOCs in non-Hermitian SYK and sparse spin chains, such as those employed in studies of many-body localization [54,55].

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