Estimate for the Bulk Viscosity of Strongly Coupled Quark Matter Using Perturbative QCD and Holography

Jesús Cruz Rojas ^{1,[2,*](#page-0-0)} Tyler Gorda ^{3,4,5,[†](#page-0-1)} Carlos Hoyos ^{6,[‡](#page-0-2)} Niko Jokela ^{[7,§](#page-0-3)} Matti Järvinen ^{0,1,8,[∥](#page-0-4)}

Aleksi Kurkela [,](https://orcid.org/0000-0002-5173-7171)^{[9,¶](#page-0-5)} Risto Paatelainen $\mathbb{Q}^{7,*}$ $\mathbb{Q}^{7,*}$ $\mathbb{Q}^{7,*}$. Saga Säppi,^{10,11,[††](#page-0-7)} and Aleksi Vuorinen $\mathbb{Q}^{7,\ddagger\ddagger}$

 $\frac{1}{1}$ Asia Pacific Center for Theoretical Physics, Pohang 37673, Korea

¹[Asia Pacific Center for Theoretical Physics](https://ror.org/011hxwn54), Pohang 37673, Korea
²Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares, [Universidad Nacional Autónoma de M](https://ror.org/01tmp8f25)éxico, Apartado Postal 70-543, CDMX 04510, Mexico
³Institut für Theoretische Physik, Coethe Universität, Max von Laue Str. 1, 6043

Institut für Theoretische Physik, Goethe Universität, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany ⁴ 4 [Technische Universität Darmstadt,](https://ror.org/05n911h24) Department of Physics, 64289 Darmstadt, Germany

⁵ExtreMe Matter Institute EMMI, [GSI Helmholtzzentrum für Schwerionenforschung GmbH](https://ror.org/02k8cbn47), 64291 Darmstadt, Germany

 6 Departamento de Física and Instituto de Ciencias y Tecnologías Espaciales de Asturias (ICTEA), [Universidad de Oviedo](https://ror.org/006gksa02),

c/ Leopoldo Calvo Sotelo 18, ES-33007 Oviedo, Spain
⁷ Department of Physics and Helsinki Institute of Physics, P.O. Par 64, EL00014 U Department of Physics and [Helsinki Institute of Physics,](https://ror.org/01x2x1522) P.O. Box 64, FI-00014 [University of Helsinki,](https://ror.org/040af2s02) Finland
⁸Department of Physics, Pohane University of Science and Technology, Pohane 27673, Kones

 8 Department of Physics, [Pohang University of Science and Technology](https://ror.org/04xysgw12), Pohang 37673, Korea 9 Faculty of Science and Technology, University of Stavanger, 4036 Stavanger, Norway

 10 [Technical University of Munich TUM School of Natural Sciences Department of Physics,](https://ror.org/02kkvpp62)

James-Franck-Str. 1, 85748 Garching, Germany
¹¹[Excellence Cluster ORIGINS,](https://ror.org/010wkny21) Boltzmannstrasse 2, 85748 Garching, Germany

(Received 13 February 2024; revised 22 May 2024; accepted 10 July 2024; published 13 August 2024)

Modern hydrodynamic simulations of core-collapse supernovae and neutron-star mergers require knowledge not only of the equilibrium properties of strongly interacting matter, but also of the system's response to perturbations, encoded in various transport coefficients. Using perturbative and holographic tools, we derive here an improved weak-coupling and a new strong-coupling result for the most important transport coefficient of unpaired quark matter, its bulk viscosity. These results are combined in a simple analytic pocket formula for the quantity that is rooted in perturbative quantum chromodynamics at high densities but takes into account nonperturbative holographic input at neutron-star densities, where the system is strongly coupled. This expression can be used in the modeling of unpaired quark matter at astrophysically relevant temperatures and densities.

DOI: [10.1103/PhysRevLett.133.071901](https://doi.org/10.1103/PhysRevLett.133.071901)

Introduction—During the last ten years, neutron stars (NSs) and their binary mergers—observable through both electromagnetic and gravitational waves (GW) [[1,](#page-5-0)[2\]](#page-5-1)—have established themselves as the leading laboratory for dense quantum chromodynamics (QCD) matter. While the

[*](#page-0-9) Contact author: jesus.cruz@correo.nucleares.unam.mx

[‡](#page-0-9) Contact author: hoyoscarlos@uniovi.es

observable properties of single quiescent NSs and even the inspiral parts of NS mergers are mostly determined by the equation of state (EoS) of the constituent matter, the ringdown phase of a NS merger constitutes a considerably more complicated out-of-equilibrium system. In preparation for the eventual observation of a ringdown GW signal, extensive hydrodynamic simulations of NS mergers are currently being carried out, with one crucial challenge being to correctly account for energy dissipation and transport in NS matter [\[3\]](#page-5-2).

Among the different transport coefficients, the bulk viscosity ζ, which quantifies energy dissipation during a rapid compression or expansion of matter, stands out as particularly important [\[4](#page-5-3)–[12](#page-5-4)]. For isolated NSs, it affects the emission of continuous GWs [[13](#page-5-5)], expected to be detectable in next-generation GW observatories such as the Einstein Telescope [[14](#page-5-6)] and Cosmic Explorer [[15](#page-5-7)], and determines the maximal rotation frequencies of pulsars in a temperature-dependent fashion, giving rise to the so-called r-mode stability window in the $1-100$ keV range $[16-18]$ $[16-18]$ $[16-18]$ (for a review of NS oscillatory modes, see [\[19](#page-5-10)]). In NS

[[†]](#page-0-9) Contact author: gorda@itp.uni-frankfurt.de

[[§]](#page-0-9) Contact author: niko.jokela@helsinki.fi

[[∥]](#page-0-9) Contact author: matti.jarvinen@apctp.org

[[¶]](#page-0-10) Contact author: aleksi.kurkela@uis.no

[^{**}C](#page-0-10)ontact author: risto.paatelainen@helsinki.fi

[^{††}](#page-0-10)Contact author: saga.saeppi@tum.de

[^{‡‡}](#page-0-10)Contact author: aleksi.vuorinen@helsinki.fi

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

mergers, the bulk viscosity on the other hand provides damping for density oscillations, affecting both the inspiral [\[20\]](#page-5-11) and post-merger dynamics, of which the latter involves temperatures up to tens of MeVs. The bulk viscosity may indeed leave a detectable imprint on the post-merger GW waveform [\[21](#page-5-12)–[26](#page-5-13)], the magnitude of which is however still under discussion [\[27\]](#page-5-14).

The dominant contribution to the bulk viscosity comes about when weak interactions cannot keep pace with the compression rate, leading to deviations from beta equilibrium and a nonequilibrium contribution to the pressure, against which work can be done. This effect peaks when the timescales of macroscopic oscillations and microscopic flavor-changing rates match. In the nuclear matter phase, the value of ζ depends on multiple factors, such as whether direct Urca processes are allowed or if hyperons or Cooper pairing between nucleons are present, each affecting in particular the temperature scale where ζ reaches its maximal value (see Ref. [\[28\]](#page-5-15) for a review).

The first milliseconds of a binary NS merger are known to involve baryon densities up to several nuclear saturation densities $n_{\text{sat}} \approx 0.16/\text{fm}^3$ as well as temperatures up to several tens of MeVs (see, e.g., [\[29\]](#page-5-16)). Such conditions may also lead to the creation of deconfined QM [\[30](#page-5-17)–[34](#page-6-0)], the transport properties of which differ significantly from those of nuclear matter. While the value of the QM bulk viscosity is expected to strongly depend on the presence and details of quark pairing, differences between various partially paired configurations are expected to be smaller than between quark and nuclear matter [[28](#page-5-15)]. This makes the bulk viscosity an interesting quantity for tracking the possible creation of QM during mergers.

Despite the phenomenological importance of the bulk viscosity, our ability to predict its behavior remains limited owing to the unavailability of controlled first-principles quantum-field-theory methods at NS densities. The leading first-principles tools include perturbative QCD (pQCD), available only at very high densities (see, e.g., [\[35](#page-6-1)–[37\]](#page-6-2)), and holography, which describes the strong-coupling limit of a class of QCD-like theories [\[38](#page-6-3)–[42\]](#page-6-4). For QM, leadingorder perturbative results for several transport coefficients were derived some thirty years ago [\[43](#page-6-5)[,44](#page-6-6)] and improved to next-to-leading order (NLO) later [\[45,](#page-6-7)[46\]](#page-6-8), whereas at strong coupling, the shear viscosity and the electrical and thermal conductivities were first evaluated only recently in two holographic models [[47](#page-6-9),[48](#page-6-10)]. For the bulk viscosity, only the minuscule purely QCD contribution has been considered in recent literature [[47](#page-6-9),[49](#page-6-11)], but for the dominant contribution stemming from an interplay between the electroweak and strong sectors, no strong-coupling prediction is currently available at all.

In this work, we derive state-of-the-art results for the thermodynamic response of QM to a change in its flavor content, thus providing novel predictions for the bulk viscosity. We do so using both perturbative and holographic

FIG. 1. The bulk viscosity ζ of NS matter, evaluated at rotation frequency $\omega = 2\pi \times 1$ kHz and given as a function of T for a baryon density $n_B \approx 5n_{\text{sat}}$. The uncertainty bands of the holographic results are assessed via their matching to QCD: The D3-D7 result is matched to pQCD quark densities including their uncertainty bands, while the uncertainty of the V-QCD result is estimated by varying the parameters of the model within limits set by lattice-QCD results. Finally, nuclear and hyperonic matter results (labeled Nucl. and Hyperons) from Refs. [\[24,](#page-5-18)[50\]](#page-6-12) are shown for comparison. Note that for technical reasons, the V-QCD result is shown for $7n_{sat}$ and the hyperonic one for 4.5 n_{sat} . We observe that our QM results always peak within the r-mode stability window 1–100 keV, but are strongly suppressed at the $O(10 \text{ MeV})$ temperatures involved in NS mergers. This may, however, be related to the absence of quark pairing in our setup (see [\[51](#page-6-13)] for a counterexample in the color-flavor-locked case).

methods, and in particular derive the first strong-coupling predictions for the quantity. Our results are applicable for unpaired QM and serve as a starting point for any partially unpaired phase [[28](#page-5-15)].

The main result of our work is shown in Fig. [1,](#page-1-0) where we display the bulk viscosity of NS matter as a function of temperature for a baryon density of roughly $5n_{\text{sat}}$. For QM, we include results corresponding to the free-theory limit, evaluated at a fixed strange quark mass ($m_s = 93.4$ MeV), as well as our two holographic models, D3-D7 and V-QCD, but not the pQCD result, which is not under quantitative control at intermediate densities. For the confined phase, we display results corresponding to both nuclear [\[24\]](#page-5-18) and hyperonic [[50](#page-6-12)] matter. As we discuss in detail in the remaining sections of this Letter, our results paint a consistent picture of the behavior of the QM bulk viscosity that displays a stark qualitative difference to that witnessed in the confined phases of QCD. Furthermore, we observe that for astrophysically relevant densities and temperatures, nearly all temperature dependence in the QM result originates from the flavor-changing interactions. For our D3-D7 computation, this leads to a simple analytic result for ζ , given in Eq. [\(5\)](#page-4-0) below, that we suggest for use as an approximation for the bulk viscosity of unpaired QM in future phenomenological applications.

Setup—For unpaired three-flavor QM in the neutrinotransparent regime, the leading contribution to the bulk viscosity arises from W-boson exchange in the process $u + d \leftrightarrow u + s$. Outside beta equilibrium, i.e., when the d and s quark chemical potentials differ $\mu_d \neq \mu_s$, the quark densities n_d and n_s change with rates proportional to an electroweak rate λ_1 [\[52](#page-6-14)–[54\]](#page-6-15), so that

$$
\frac{\mathrm{d}n_d}{\mathrm{d}t} = -\frac{\mathrm{d}n_s}{\mathrm{d}t} \approx \lambda_1(\mu_s - \mu_d). \tag{1}
$$

Neglecting quark masses, the leading low-T contribution to the rate becomes [[54](#page-6-15),[55](#page-6-16)]

$$
\lambda_1 = \left(1 + \sigma \log \frac{\Lambda}{T}\right)^4 \frac{64}{5\pi^3} G_F^2 \sin^2 \theta_c \cos^2 \theta_c \mu_d^5 T^2, \quad (2)
$$

where G_F is the Fermi constant and θ_c the Cabibbo angle. The quartic prefactor on the right-hand side represents the only known $O(\alpha_s)$ correction to the rate, which is moreover logarithmically enhanced at low temperatures as it originates from a so-called non-Fermi-liquid (nFL) contribution to the specific heat of QM [[57\]](#page-6-17) (see also [[58](#page-6-18)[,59\]](#page-6-19)). As discussed in detail around Fig. 5 of the Supplemental Material [[60\]](#page-6-20), this correction allows us to gauge the importance of the (partially unknown) $\mathcal{O}(\alpha_s)$ corrections to the rate: for $\sigma = 0$, the result reduces to the leading-order rate, while for $\sigma = 4\alpha_s/(9\pi)$ and $\Lambda \approx 0.158 \sqrt{\alpha_s} \sqrt{\mu_u^2 + \mu_d^2 + \mu_s^2}$ one recovers the result derived in [\[57](#page-6-17)].

While the unknown QCD corrections to the rate may be sizable, we note that the qualitative behavior of the rate likely remains the same at strong coupling: In holography, the QCD contribution to the rate, replacing the leadingorder multiplicative factor $\mu_d^5 T^2$ above, is available from the convolution of two flavor-current correlators. For these correlators, calculations at nonzero quark densities in the D3-D7 model show a linear dependence on the temperature at low frequencies [\[61](#page-6-21)–[63\]](#page-6-22), consistent with the formula we use. Furthermore, the normalization of the correlators depends on the number of colors and flavors but not on the 't Hooft coupling, thus keeping the rate constant in the strong-coupling limit.

A study of energy dissipation during a compressiondecompression cycle near beta equilibrium connects ζ to various susceptibilities $\chi_{ij} \equiv \frac{\partial^2 p}{\partial \mu_i \partial \mu_j}$ and reaction rates (see Supplemental Material Sec. A [[60](#page-6-20)]) [[64](#page-6-23)]. If we only take into account the $u + d \leftrightarrow u + s$ process [\[46\]](#page-6-8), this leads to

$$
\zeta = \frac{\lambda_1 A_1^2}{\omega^2 + (\lambda_1 C_1)^2},\tag{3}
$$

where the coefficients A_1 and C_1 , determined by various susceptibilities and quark densities, are found in Eqs. (33) and (34) of the Supplemental Material [\[60](#page-6-20)], and ω denotes the angular frequency of density oscillations (see [[19](#page-5-10)] for discussion) [[65](#page-6-24)].

The combination of susceptibilities appearing in Eq. (33) vanishes if the d and s quarks are degenerate in mass—a fact most easily verified if (33) is given in terms of the inverse susceptibility matrix (see Supplemental Material [[60\]](#page-6-20) for details). This implies that a nonzero strange quark mass must be implemented in both the weak- and strong-coupling setups, which we briefly introduce below.

Methods—In this section, we review our perturbative and holographic determinations of the susceptibilities that enter Eq. [\(3\).](#page-2-0) In both calculations, we treat electrons as non-interacting and (numerically) solve the corresponding chemical potential μ_e from the charge neutrality condition $2n_u/3 - n_d/3 - n_s/3 = n_e = T^2\mu_e/3 - \mu_e^3/(3\pi^2)$. Together with the beta-equilibrium conditions $\mu_s = \mu_d$, $\mu_u = \mu_d - \mu_e$, this allows us to obtain ζ in terms of μ_d , T, ω . Finally, our results will depend on the parameter $X = \bar{\Lambda}/(2\mu_d)$ which parametrizes our results' dependence on the unphysical renormalization scale $\overline{\Lambda}$ in the $\overline{\text{MS}}$ scheme. It appears directly in our pQCD results and indirectly in the D3-D7 ones, where it enters through the high-density matching of the model to pQCD.

Perturbative QCD: For vanishing quark masses, the perturbative pressure of deconfined unpaired QCD matter is known up to order $\alpha_s^{5/2}$ at nonzero temperatures and densities [\[66](#page-6-25)[,67\]](#page-6-26) and up to partial $\mathcal{O}(\alpha_s^3)$ in the $T = 0$ limit [\[37](#page-6-2)[,68,](#page-6-27)[69](#page-6-28)]. Up to the highest fully known order $\mathcal{O}(\alpha_s^{5/2})$, the result can be split into two distinct terms corresponding to contributions from the hard and soft momentum scales, which for $\mu \gg T$ are of order μ and $\alpha_s^{1/2} \mu$, respectively. We treat the additional mass-dependent contribution to the pressure p_m within the mass-expansion scheme of [[70](#page-7-0)], where m_s is formally treated as a quantity of $\mathcal{O}(\alpha_s^{1/2}\mu)$ and the light quark masses are neglected. This mass expansion is performed to $\mathcal{O}(m_s^4)$ and up to a combined $\mathcal{O}(\alpha_s^{5/2})$ (for the full mass-dependence at $T = 0$, see [\[35\]](#page-6-1)). For the value of the s quark mass, we use the physical \overline{MS} renormalized value $m_s \approx 93.4$ MeV [\[71\]](#page-7-1). We have confirmed that additionally including nonzero m_u and m_d terms would lead to a vanishingly small effect, while the chemical potentials realized in NSs are not large enough to allow for heavier quarks. For the soft contribution, evaluated in the massless limit, we furthermore use an analytic small- T/μ expansion derived in [[67](#page-6-26)] that is valid for $T \lesssim 100$ MeV. Mass corrections to this result start at $\mathcal{O}(\alpha_s^3)$ and can therefore be neglected.

The perturbative pressure described above can be readily differentiated to obtain predictions for the coefficients A_1 , C_1 and eventually for ζ as functions of the three quark chemical potentials and the renormalization scale parameter X. The results constitute lengthy closed-form expressions in terms of standard special functions and their derivatives, allowing for inexpensive evaluation of the necessary quantities.

Holography: The D3-D7 model [\[72](#page-7-2)] is the holographic dual of $\mathcal{N} = 4$ SU(N_c) super Yang-Mills theory with N_f copies of $\mathcal{N} = 2$ hypermultiplets in the quenched approximation $N_f/N_c \ll 1$. It consists of N_f probe D7-branes embedded in the AdS₅ \times S⁵ spacetime, while baryon charge is introduced by turning on an electric field on the D7-branes [\[73](#page-7-3)[,74\]](#page-7-4) and temperature by modifying the geometry to that of a black brane. Following [[75](#page-7-5)], we extrapolate the model to the physically relevant $N_c = N_f = 3$ and fix $\alpha_s \approx 0.285$ so that the pressure matches the Stefan-Boltzmann value at high density, extending the model's validity towards higher densities. Although the field content of the model differs from that of QCD, we note that the thermodynamic coefficients A_1 and C_1 , obtained through chemical-potential derivatives of the pressure, are highly insensitive the additional fields in the D3-D7 model.

At vanishing temperature, the pressure of the D3-D7 model takes the simple form [\[75,](#page-7-5)[76](#page-7-6)]

$$
p = \frac{1}{4\pi^2} \sum_{i=u,d,s} (\mu_i^2 - M_i^2)^2,
$$
 (4)

where M_i are the constituent quark masses that we fix by equating quark densities with pQCD at $\mu_d = 1$ GeV and varying $X \in [1/2, 2]$. Doing so, we obtain $M_u \in (522.5, 1)$ 434.6) MeV, $M_d \in (526.4, 435.9)$ MeV, and $M_s \in (541.8,$ $450.1)$ MeV, within this interval in X. In what follows, in addition to estimating uncertainties by matching to $pQCD$ at different values of X , we also vary this matching density within $\mu_d \in [1, 2]$ GeV. At $T \neq 0$, we finally compute the pressure numerically, following methods introduced in [\[73](#page-7-3)[,74\]](#page-7-4).

The other holographic model we use is V-QCD [[77](#page-7-7)], which is a bottom-up model tuned to reproduce QCD physics as closely as possible (see, e.g., the reviews [\[40](#page-6-29)[,41](#page-6-30),[78](#page-7-8)]). It combines the improved holographic QCD model for pure Yang-Mills theory [[79](#page-7-9),[80](#page-7-10)] to a description of flavors introduced via tachyonic brane actions [[81](#page-7-11)–[83](#page-7-12)], featuring, e.g., a running α_s as reviewed in the Supplemental Material [\[60\]](#page-6-20). Given that quarks are treated as unquenched ($N_f/N_c \sim 1$) in V-QCD, the model should capture their physics more realistically than the D3-D7 model. Indeed, V-QCD by construction agrees with various qualitative properties of QCD (such as confinement and asymptotic freedom), and its parameters are fitted to data, including lattice results for the pressure [\[84,](#page-7-13)[85\]](#page-7-14) and baryon number susceptibilities [[85](#page-7-14)] at $\mu = 0$. The model is consistent with all known astrophysical observations in the NS-matter regime [\[86,](#page-7-15)[87\]](#page-7-16), but eventually becomes inconsistent with pQCD at high densities [[88](#page-7-17)].

In this paper, we otherwise follow the treatment of the above V-QCD papers but relax the assumption of exact chiral symmetry in the QM phase by turning on a nonzero strange quark mass, thus extending the prescription of [[89](#page-7-18)]. The corresponding mass parameter of the model is fixed by demanding that the masses of kaons and η mesons are well reproduced in the vacuum (see Supplemental Material [\[60\]](#page-6-20) and Refs. [\[90](#page-7-19)–[93\]](#page-7-20) for details). We find that this procedure underpredicts the dependencies of quark number susceptibilities on the strange quark mass at zero μ and high T, where the results can be benchmarked against lattice data [[94](#page-7-21)]. This leads us to expect that this model similarly underpredicts the effects of the strange quark mass in physical quantities at high densities.

Finally, we quantify the underlying uncertainty of our results by allowing the V-QCD parameters vary within limits set by the lattice QCD fit in the chirally symmetric phase [\[85,](#page-7-14)[95\]](#page-7-22), but otherwise follow the computational strategy of [[96](#page-7-23)] in determining the quantities appearing in Eq. [\(3\).](#page-2-0) In both holographic setups, the variation procedure we perform thus corresponds to the uncertainties associated with the respective matching procedures.

Results—Our main result for the bulk viscosity of unpaired QM is displayed in Fig. [1.](#page-1-0) It highlights a qualitative contrast between the behavior of ζ in the confined and deconfined phases of QCD, with the more suppressed QM results peaking at lower temperatures, and in addition demonstrates the important effect of interaction corrections in the latter case. Consistently with our expectations for quantities that vanish in the degenerate-mass limit, V-QCD appears to predict somewhat lower values for ζ than our other methods, but nevertheless retains the same qualitative features.

A closer inspection of our results reveals a number of interesting further findings. Explicit calculations show that in all three approaches, the bulk viscosity is insensitive to the T dependence originating from the coefficients A_1 and C_1 of Eq. [\(3\)](#page-2-0). As demonstrated in Fig. 4 of Supplemental Material [\[60\]](#page-6-20), to a good accuracy we can indeed set $T = 0$ in these functions and only keep the T dependence of the electroweak rate λ_1 in Eq. [\(2\).](#page-2-1) Another universal characteristic that all our results exhibit is an approximate quartic dependence on the strange quark mass, which has been noted before in [\[56\]](#page-6-31).

While the full ζ depends on the rate λ_1 , we may construct physical features of the bulk viscosity that are sensitive only to QCD input. For example, the peak value of the viscosity, $\zeta_{\text{peak}} \equiv \zeta(T_{\text{peak}})$, and its rescaled zero-frequency limit $\lambda_1 \zeta(\omega = 0)$ that corresponds to the dc bulk viscosity entering the Israel-Stewart theory [[12](#page-5-4),[97](#page-7-24),[98](#page-7-25)] are completely insensitive to the electroweak rate and can be fully extracted from the coefficients A_1 and C_1 in Eqs. (33)–(34). These two quantities are shown in Fig. [2](#page-4-1), where we observe a good agreement between our pQCD and D3-D7 results for densities where both predictions are available, while V-QCD again appears to underestimate the quantities (see discussion in Supplemental Material [[60](#page-6-20)]).

Setting $T = 0$ in A_1 and C_1 , we find that the D3-D7 calculation leads to a remarkably simple analytic formula as a function of μ_d

$$
\zeta = \frac{4\lambda_1 \mu_d^6 (M_s^2 - M_d^2)^2}{K_d^2 K_s^2 \omega^2 + \pi^4 \lambda_1^2 (K_d + K_s)^2},\tag{5}
$$

where we have defined $K_i \equiv 3\mu_d^2 - M_i^2$. We stress that for the M_i in this formula, one should use the constituent quark-mass ranges listed below Eq. [\(4\)](#page-3-0), leading to the uncertainty ranges visible in Fig. [1.](#page-1-0) To express this as a function of n_B for the small temperatures of relevance to BNS mergers, one can further use the $T = 0$ pressure in Eq. [\(4\)](#page-3-0) to numerically relate n_B to μ_d in beta equilibrium.

Returning finally to the bulk viscosity itself, we note that it is straightforward to compare our NNLO pQCD results to lower perturbative orders, as shown in Fig. [2](#page-4-1) of the Supplemental Material [[60](#page-6-20)]. We find that the difference between the NLO and NNLO results is non-negligible even at $40n_{\text{sat}}$, and that the results diverge rapidly at lower densities, making extrapolation to the NS realm impossible. While the naive free quark expression can, in principle, be extrapolated to low densities, it completely fails to take into account the effects of interactions, which become

FIG. 2. A comparison of the values of two quantities characterizing the bulk viscosity: its zero-frequency limit and peak value, $\zeta(\omega = 0)$ and ζ_{peak} . These quantities are multiplied by different factors so that they depend only on A_1 and C_1 in Eqs. (33) and (34) and are independent of the oscillation frequency ω and the electroweak rate λ_1 , as indicated by the expressions on the right vertical axes. The error bars in these panels capture the variation of model parameters in the different models as described in the main text.

increasingly important much before the hadronic phase is eventually reached. For phenomenological purposes, the compact D3-D7 bulk viscosity of Eq. [\(5\)](#page-4-0) is on the other hand appealing as it is rooted in pQCD but takes into account the strongly coupled nature of the theory at low densities. To this end, despite its limitations discussed above, we recommend the use of this result in the modeling of dense unpaired QM at astrophysically relevant densities and temperatures, and similarly expect the V-QCD result to provide a reasonable lower bound for the bulk viscosity.

An important limitation of our present approach is finally related to the fact that the pairing channel and the magnitude of the superconducting gap in low- and moderate-density QM remains unknown (though see [\[99\]](#page-7-26) for a recent model-independent study bounding the gap at high densities). To obtain estimates for the bulk viscosity in various pairing channels, corrections to both the electroweak rate in Eq. [\(2\)](#page-2-1) and to the thermodynamic functions entering through Eqs. (33)–(34) should be separately considered. While the latter are expected to be subleading, the former may be substantial given that the contribution of gapped quark modes to the reaction rate is exponentially suppressed. While the detailed evaluation of these corrections is left for future work, we note that the electroweak rate receives $O(\alpha_s)$ QCD corrections even in the unpaired phase, some of which are presently known [[57](#page-6-17)]. Their effect is studied in Fig. 5 of the Supplemental Material [[60](#page-6-20)], where we observe that, in agreement with the λ_1 independence of ζ_{peak} , they primarily simply shift the peak of the viscosity to lower temperatures.

Acknowledgments—We thank Arus Harutyunyan for providing us with the data for the nuclear bulk viscosities and V. Sreekanth for the hyperonic bulk viscosities, both included in our Fig. [1,](#page-1-0) and we thank Mark Alford, Christian Ecker, Carlo Musolino, and Andreas Schmitt for helpful comments and suggestions on the manuscript. J. C. R. and M. J. have been supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government and by the Korean Local Governments —Gyeongsangbuk-do Province and Pohang City—and by the National Research Foundation of Korea (NRF) funded by the Korean government (MSIT) (Grant No. 2021R1A2C1010834). T. G. has been supported in part by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Project-ID 279384907– SFB 1245, by the State of Hesse within the Research Cluster ELEMENTS (projectID 500/10.006), and by the ERC Advanced Grant "JETSET: Launching, propagation and emission of relativistic jets from binary mergers and across mass scales" (Grant No. 884631). C. H. is partially supported by the AEI and the MCIU through the Spanish Grant No. PID2021-123021NB-I00 and by FICYT through the Asturian Grant No. SV-PA-21-AYUD/2021/52177.

N. J., R. P., and A. V have been supported in part by the Research Council of Finland Grants No. 322507, No. 345070, No. 347499, No. 353772, and No. 354533 as well as by the ERC Consolidator Grant No. 725369. S. S. acknowledges support of the DFG cluster of excellence ORIGINS funded by the DFG under Germany's Excellence Strategy—EXC-2094-390783311.

- [1] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), GW170817: Observation of gravitational waves from a binary neutron star inspiral, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.119.161101) 119, 161101 [\(2017\).](https://doi.org/10.1103/PhysRevLett.119.161101)
- [2] B. P. Abbott et al. (LIGO Scientific, Virgo, Fermi GBM, INTEGRAL, IceCube, AstroSat Cadmium Zinc Telluride Imager Team, IPN, Insight-Hxmt, ANTARES, Swift, AGILE Team, 1M2H Team, Dark Energy Camera GW-EM, DES, DLT40, GRAWITA, Fermi-LAT, ATCA, ASKAP, Las Cumbres Observatory Group, OzGrav, DWF (Deeper Wider Faster Program), AST3, CAASTRO, VINROUGE, MASTER, J-GEM, GROWTH, JAGWAR, CaltechNRAO, TTU-NRAO, NuSTAR, Pan-STARRS, MAXI Team, TZAC Consortium, KU, Nordic Optical Telescope, ePESSTO, GROND, Texas Tech University, SALT Group, TOROS, BOOTES, MWA, CALET, IKI-GW Follow-up, H.E.S.S., LOFAR, LWA, HAWC, Pierre Auger, ALMA, Euro VLBI Team, Pi of Sky, Chandra Team at McGill University, DFN, ATLAS Telescopes, High Time Resolution Universe Survey, RIMAS, RATIR, and SKA South Africa/MeerKAT Collaborations), Multi-messenger observations of a binary neutron star merger, [Astrophys. J. Lett.](https://doi.org/10.3847/2041-8213/aa91c9) 848, L12 (2017).
- [3] L. Baiotti and L. Rezzolla, Binary neutron star mergers: A review of Einstein's richest laboratory, [Rep. Prog. Phys.](https://doi.org/10.1088/1361-6633/aa67bb) 80, [096901 \(2017\).](https://doi.org/10.1088/1361-6633/aa67bb)
- [4] R. F. Sawyer, Bulk viscosity of hot neutron-star matter and the maximum rotation rates of neutron stars, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.39.3804) 39[, 3804 \(1989\)](https://doi.org/10.1103/PhysRevD.39.3804).
- [5] P. Haensel and R. Schaeffer, Bulk viscosity of hot-neutronstar matter from direct URCA processes, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.45.4708) 45, [4708 \(1992\)](https://doi.org/10.1103/PhysRevD.45.4708).
- [6] P. Haensel, K. P. Levenfish, and D. G. Yakovlev, Bulk viscosity in superfluid neutron star cores. I. Direct urca processes in $npe\mu$ matter, Astron. Astrophys. 357, 1157 (2000).
- [7] M. G. Alford and S. P. Harris, Beta equilibrium in neutron star mergers, Phys. Rev. C 98[, 065806 \(2018\).](https://doi.org/10.1103/PhysRevC.98.065806)
- [8] M. G. Alford and S. P. Harris, Damping of density oscillations in neutrino-transparent nuclear matter, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.100.035803) 100[, 035803 \(2019\).](https://doi.org/10.1103/PhysRevC.100.035803)
- [9] M. Alford, A. Harutyunyan, and A. Sedrakian, Bulk viscosity of baryonic matter with trapped neutrinos, Phys. Rev. D 100[, 103021 \(2019\)](https://doi.org/10.1103/PhysRevD.100.103021).
- [10] M. G. Alford, A. Haber, S. P. Harris, and Z. Zhang, Beta equilibrium under neutron star merger conditions, [Universe](https://doi.org/10.3390/universe7110399) 7[, 399 \(2021\).](https://doi.org/10.3390/universe7110399)
- [11] M. G. Alford, A. Haber, and Z. Zhang, Isospin equilibration in neutron star mergers, Phys. Rev. C 109[, 055803 \(2024\).](https://doi.org/10.1103/PhysRevC.109.055803)
- [12] Y. Yang, M. Hippert, E. Speranza, and J. Noronha, Farfrom-equilibrium bulk-viscous transport coefficients in neutron star mergers, Phys. Rev. C 109[, 015805 \(2024\)](https://doi.org/10.1103/PhysRevC.109.015805).
- [13] M. Sieniawska and M. Bejger, Continuous gravitational waves from neutron stars: Current status and prospects, Universe 5[, 217 \(2019\).](https://doi.org/10.3390/universe5110217)
- [14] M. Punturo et al., The Einstein telescope: A third-generation gravitational wave observatory, [Classical Quantum Gravity](https://doi.org/10.1088/0264-9381/27/19/194002) 27[, 194002 \(2010\).](https://doi.org/10.1088/0264-9381/27/19/194002)
- [15] D. Reitze et al., Cosmic explorer: The U.S. contribution to gravitational-wave astronomy beyond LIGO, Bull. Am. Astron. Soc. 51, 035 (2019).
- [16] M. G. Alford, S. Mahmoodifar, and K. Schwenzer, Viscous damping of r-modes: Small amplitude instability, [Phys. Rev.](https://doi.org/10.1103/PhysRevD.85.024007) D 85[, 024007 \(2012\)](https://doi.org/10.1103/PhysRevD.85.024007).
- [17] M. G. Alford and K. Schwenzer, What the timing of millisecond pulsars can teach us about their interior, [Phys.](https://doi.org/10.1103/PhysRevLett.113.251102) Rev. Lett. 113[, 251102 \(2014\).](https://doi.org/10.1103/PhysRevLett.113.251102)
- [18] M. G. Alford and S. Han, Characteristics of hybrid compact stars with a sharp hadron-quark interface, [Eur. Phys. J. A](https://doi.org/10.1140/epja/i2016-16062-9) 52, [62 \(2016\).](https://doi.org/10.1140/epja/i2016-16062-9)
- [19] C. J. Krüger, K. D. Kokkotas, P. Manoharan, and S. H. Völkel, Fast rotating neutron stars: Oscillations and instabilities, [Front. Astron. Space Sci.](https://doi.org/10.3389/fspas.2021.736918) 8, 736918 (2021).
- [20] J. L. Ripley, A. Hegade K. R., R. S. Chandramouli, and N. Yunes, First constraint on the dissipative tidal deformability of neutron stars, [arXiv:2312.11659.](https://arXiv.org/abs/2312.11659)
- [21] M. G. Alford, L. Bovard, M. Hanauske, L. Rezzolla, and K. Schwenzer, Viscous dissipation and heat conduction in binary neutron-star mergers, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.120.041101) 120, 041101 [\(2018\).](https://doi.org/10.1103/PhysRevLett.120.041101)
- [22] P. Hammond, I. Hawke, and N. Andersson, Impact of nuclear reactions on gravitational waves from neutron star mergers, Phys. Rev. D 107[, 043023 \(2023\)](https://doi.org/10.1103/PhysRevD.107.043023).
- [23] E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford, and J. Noronha, Emergence of microphysical bulk viscosity in binary neutron star postmerger dynamics, [Astrophys. J.](https://doi.org/10.3847/2041-8213/ad454f) Lett. 967[, L14 \(2024\)](https://doi.org/10.3847/2041-8213/ad454f).
- [24] M. Alford, A. Harutyunyan, and A. Sedrakian, Bulk viscosity of relativistic $npe\mu$ matter in neutron-star mergers, Particles 5[, 361 \(2022\)](https://doi.org/10.3390/particles5030029).
- [25] M. Chabanov and L. Rezzolla, Impact of bulk viscosity on the post-merger gravitational-wave signal from merging neutron stars, [arXiv:2307.10464.](https://arXiv.org/abs/2307.10464)
- [26] M. Chabanov and L. Rezzolla, Numerical modelling of bulk viscosity in neutron stars, [arXiv:2311.13027](https://arXiv.org/abs/2311.13027).
- [27] D. Radice, S. Bernuzzi, A. Perego, and R. Haas, A new moment-based general-relativistic neutrino-radiation transport code: Methods and first applications to neutron star mergers, [Mon. Not. R. Astron. Soc.](https://doi.org/10.1093/mnras/stac589) 512, 1499 (2022).
- [28] A. Schmitt and P. Shternin, Reaction rates and transport in neutron stars, [Astrophysics and Space Science Library](https://doi.org/10.1007/978-3-319-97616-7) 457, [455 \(2018\)](https://doi.org/10.1007/978-3-319-97616-7).
- [29] M. Hanauske, K. Takami, L. Bovard, L. Rezzolla, J. A. Font, F. Galeazzi, and H. Stöcker, Rotational properties of hypermassive neutron stars from binary mergers, [Phys. Rev.](https://doi.org/10.1103/PhysRevD.96.043004) D 96[, 043004 \(2017\)](https://doi.org/10.1103/PhysRevD.96.043004).
- [30] E. Annala, T. Gorda, A. Kurkela, J. Nättilä, and A. Vuorinen, Evidence for quark-matter cores in massive neutron stars, Nat. Phys. 16[, 907 \(2020\)](https://doi.org/10.1038/s41567-020-0914-9).
- [31] E. Annala, T. Gorda, E. Katerini, A. Kurkela, J. Nättilä, V. Paschalidis, and A. Vuorinen, Multimessenger constraints for ultradense matter, Phys. Rev. X 12[, 011058 \(2022\).](https://doi.org/10.1103/PhysRevX.12.011058)
- [32] E. Annala, T. Gorda, J. Hirvonen, O. Komoltsev, A. Kurkela, J. Nättilä, and A. Vuorinen, Strongly interacting matter exhibits deconfined behavior in massive neutron stars, [Nat. Commun.](https://doi.org/10.1038/s41467-023-44051-y) 14, 8451 (2023).
- [33] A. Prakash, D. Radice, D. Logoteta, A. Perego, V. Nedora, I. Bombaci, R. Kashyap, S. Bernuzzi, and A. Endrizzi, Signatures of deconfined quark phases in binary neutron star mergers, Phys. Rev. D 104[, 083029 \(2021\).](https://doi.org/10.1103/PhysRevD.104.083029)
- [34] S. Tootle, C. Ecker, K. Topolski, T. Demircik, M. Järvinen, and L. Rezzolla, Quark formation and phenomenology in binary neutron-star mergers using V-QCD, [SciPost Phys.](https://doi.org/10.21468/SciPostPhys.13.5.109) 13[, 109 \(2022\).](https://doi.org/10.21468/SciPostPhys.13.5.109)
- [35] A. Kurkela, P. Romatschke, and A. Vuorinen, Cold quark matter, Phys. Rev. D 81[, 105021 \(2010\).](https://doi.org/10.1103/PhysRevD.81.105021)
- [36] T. Gorda, O. Komoltsev, A. Kurkela, and A. Mazeliauskas, Bayesian uncertainty quantification of perturbative QCD input to the neutron-star equation of state, [J. High Energy](https://doi.org/10.1007/JHEP06(2023)002) [Phys. 06 \(2023\) 002.](https://doi.org/10.1007/JHEP06(2023)002)
- [37] T. Gorda, R. Paatelainen, S. Säppi, and K. Seppänen, Equation of state of cold quark matter to $O(\alpha_s^3 \ln \alpha_s)$, [Phys.](https://doi.org/10.1103/PhysRevLett.131.181902) Rev. Lett. 131[, 181902 \(2023\).](https://doi.org/10.1103/PhysRevLett.131.181902)
- [38] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, and U. A. Wiedemann, Gauge/String Duality, Hot QCD and Heavy Ion Collisions (Cambridge University Press, Cambridge, England, 2014).
- [39] N. Brambilla et al., QCD and strongly coupled gauge theories: Challenges and perspectives, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-014-2981-5) 74, [2981 \(2014\)](https://doi.org/10.1140/epjc/s10052-014-2981-5).
- [40] M. Järvinen, Holographic modeling of nuclear matter and neutron stars, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-022-10227-x) 82, 282 (2022).
- [41] C. Hoyos, N. Jokela, and A. Vuorinen, Holographic approach to compact stars and their binary mergers, [Prog.](https://doi.org/10.1016/j.ppnp.2022.103972) [Part. Nucl. Phys.](https://doi.org/10.1016/j.ppnp.2022.103972) 126, 103972 (2022).
- [42] R. Rougemont, J. Grefa, M. Hippert, J. Noronha, J. Noronha-Hostler, I. Portillo, and C. Ratti, Hot QCD phase diagram from holographic Einstein–Maxwell–Dilaton models, [Prog. Part. Nucl. Phys.](https://doi.org/10.1016/j.ppnp.2023.104093) 135, 104093 (2024).
- [43] J. Madsen, Bulk viscosity of strange quark matter, damping of quark star vibration, and the maximum rotation rate of pulsars, Phys. Rev. D 46[, 3290 \(1992\)](https://doi.org/).
- [44] H. Heiselberg and C. J. Pethick, Transport and relaxation in degenerate quark plasmas, Phys. Rev. D 48[, 2916 \(1993\).](https://doi.org/10.1103/PhysRevD.48.2916)
- [45] B. A. Sa'd, I. A. Shovkovy, and D. H. Rischke, Bulk viscosity of spin-one color superconductors with two quark flavors, Phys. Rev. D 75[, 065016 \(2021\)](https://doi.org/10.1103/PhysRevD.75.065016).
- [46] B.A. Sa'd, I.A. Shovkovy, and D.H. Rischke, Bulk viscosity of strange quark matter: Urca versus non-leptonic processes, Phys. Rev. D 75[, 125004 \(2007\)](https://doi.org/10.1103/PhysRevD.75.125004).
- [47] C. Hoyos, N. Jokela, M. Järvinen, J. G. Subils, J. Tarrío, and A. Vuorinen, Transport in strongly coupled quark matter, Phys. Rev. Lett. 125[, 241601 \(2020\).](https://doi.org/10.1103/PhysRevLett.125.241601)
- [48] C. Hoyos, N. Jokela, M. Järvinen, J. G. Subils, J. Tarrío, and A. Vuorinen, Holographic approach to transport in dense QCD matter, Phys. Rev. D 105[, 066014 \(2022\).](https://doi.org/10.1103/PhysRevD.105.066014)
- [49] A. Czajka, K. Dasgupta, C. Gale, S. Jeon, A. Misra, M. Richard, and K. Sil, Bulk viscosity at extreme limits: From kinetic theory to strings, [J. High Energy Phys. 07 \(2019\) 145.](https://doi.org/10.1007/JHEP07(2019)145)
- [50] O. P. Jyothilakshmi, P. E. S. Krishnan, P. Thakur, V. Sreekanth, and T. K. Jha, Hyperon bulk viscosity and

r-modes of neutron stars, [Mon. Not. R. Astron. Soc.](https://doi.org/10.1093/mnras/stac2360) 516, [3381 \(2022\)](https://doi.org/10.1093/mnras/stac2360).

- [51] M. G. Alford, M. Braby, and A. Schmitt, Bulk viscosity in kaon-condensed color-flavor locked quark matter, [J. Phys.](https://doi.org/10.1088/0954-3899/35/11/115007) G 35[, 115007 \(2008\)](https://doi.org/10.1088/0954-3899/35/11/115007).
- [52] H. Heiselberg, J. Madsen, and K. Riisager, Strange quark decay rates in quark matter at high temperatures, [Phys. Scr.](https://doi.org/10.1088/0031-8949/34/6A/011) 34[, 556 \(1986\).](https://doi.org/10.1088/0031-8949/34/6A/011)
- [53] H. Heiselberg, The weak conversion rate in quark matter, Phys. Scr. 46[, 485 \(1992\)](https://doi.org/10.1088/0031-8949/46/6/002).
- [54] J. Madsen, Rate of the weak reaction $s + u$ to $u + d$ in quark matter, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.47.325) 47, 325 (1993).
- [55] We have checked that the free-quark ζ following from Eq. [\(2\)](#page-2-1) agrees with the unpaired results of [\[56\]](#page-6-31), where the numerical rate was evaluated without the small- T or small m_s approximations.
- [56] M. G. Alford and A. Schmitt, Bulk viscosity in 2SC quark matter, J. Phys. G 34[, 67 \(2007\).](https://doi.org/10.1088/0954-3899/34/1/005)
- [57] K. Schwenzer, How long-range interactions tune the damping in compact stars, [arXiv:1212.5242](https://arXiv.org/abs/1212.5242).
- [58] A. Gerhold, A. Ipp, and A. Rebhan, Non-Fermi-liquid specific heat of normal degenerate quark matter, [Phys.](https://doi.org/10.1103/PhysRevD.70.105015) Rev. D 70[, 105015 \(2004\)](https://doi.org/10.1103/PhysRevD.70.105015).
- [59] T. Schäfer and K. Schwenzer, Non-Fermi liquid effects in QCD at high density, Phys. Rev. D 70[, 054007 \(2004\)](https://doi.org/10.1103/PhysRevD.70.054007).
- [60] See Supplemental Material at [http://link.aps.org/](http://link.aps.org/supplemental/10.1103/PhysRevLett.133.071901) [supplemental/10.1103/PhysRevLett.133.071901](http://link.aps.org/supplemental/10.1103/PhysRevLett.133.071901) for a compact derivation of the main formulas for the bulk viscosity, details of how we have supplemented the V-QCD setup with a nonzero s quark mass parameter, and a quantitative analysis and comparison of our three independent results for ζ.
- [61] J. Erdmenger, M. Kaminski, and F. Rust, Holographic vector mesons from spectral functions at finite baryon or isospin density, Phys. Rev. D 77[, 046005 \(2008\).](https://doi.org/10.1103/PhysRevD.77.046005)
- [62] J. Mas, J. P. Shock, J. Tarrio, and D. Zoakos, Holographic spectral functions at finite baryon density, [J. High Energy](https://doi.org/10.1088/1126-6708/2008/09/009) [Phys. 09 \(2008\) 009.](https://doi.org/10.1088/1126-6708/2008/09/009)
- [63] J. Erdmenger, M. Kaminski, P. Kerner, and F. Rust, Finite baryon and isospin chemical potential in AdS/CFT with flavor, [J. High Energy Phys. 11 \(2008\) 031.](https://doi.org/10.1088/1126-6708/2008/11/031)
- [64] The breaking of β equilibrium in mergers is discussed in [\[7](#page-5-19),[12](#page-5-4)].
- [65] To express the frequency in the high-energy physics units of MeV instead of Hz, we use MeV $\equiv 4.1351 \times 10^{-21}$ rad/s \equiv $2\pi \times 6.58122 \times 10^{-22}$ Hz. Similarly, should one further wish to express ζ in astrophysical units, the relevant conversion factor from the high-energy-physics units is $MeV^3 = 137286 \times g cm^{-1} s^{-1}$.
- [66] A. Vuorinen, The pressure of QCD at finite temperatures and chemical potentials, Phys. Rev. D 68[, 054017 \(2003\).](https://doi.org/10.1103/PhysRevD.68.054017)
- [67] A. Kurkela and A. Vuorinen, Cool quark matter, [Phys. Rev.](https://doi.org/10.1103/PhysRevLett.117.042501) Lett. **117**[, 042501 \(2016\)](https://doi.org/10.1103/PhysRevLett.117.042501).
- [68] T. Gorda, A. Kurkela, P. Romatschke, S. Säppi, and A. Vuorinen, Next-to-next-to-next-to-leading order pressure of cold quark matter: Leading logarithm, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.121.202701) 121, [202701 \(2018\).](https://doi.org/10.1103/PhysRevLett.121.202701)
- [69] T. Gorda, A. Kurkela, R. Paatelainen, S. Säppi, and A. Vuorinen, Soft interactions in cold quark matter, [Phys. Rev.](https://doi.org/10.1103/PhysRevLett.127.162003) Lett. 127[, 162003 \(2021\)](https://doi.org/10.1103/PhysRevLett.127.162003).
- [70] T. Gorda and S. Säppi, Cool quark matter with perturbative quark masses, Phys. Rev. D 105[, 114005 \(2022\)](https://doi.org/10.1103/PhysRevD.105.114005).
- [71] R. L. Workman et al. (Particle Data Group), Review of particle physics, [Prog. Theor. Exp. Phys.](https://doi.org/10.1093/ptep/ptac097) 2022, 083C01 (2022).
- [72] A. Karch and E. Katz, Adding flavor to AdS/CFT, [J. High](https://doi.org/10.1088/1126-6708/2002/06/043) [Energy Phys. 06 \(2002\) 043.](https://doi.org/10.1088/1126-6708/2002/06/043)
- [73] S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers, and R. M. Thomson, Holographic phase transitions at finite baryon density, [J. High Energy Phys. 02 \(2007\) 016.](https://doi.org/10.1088/1126-6708/2007/02/016)
- [74] D. Mateos, S. Matsuura, R. C. Myers, and R. M. Thomson, Holographic phase transitions at finite chemical potential, [J. High Energy Phys. 11 \(2007\) 085.](https://doi.org/10.1088/1126-6708/2007/11/085)
- [75] C. Hoyos, D. Rodríguez Fernández, N. Jokela, and A. Vuorinen, Holographic quark matter and neutron stars, Phys. Rev. Lett. 117[, 032501 \(2016\).](https://doi.org/10.1103/PhysRevLett.117.032501)
- [76] A. Karch and A. O'Bannon, Holographic thermodynamics at finite baryon density: Some exact results, [J. High Energy](https://doi.org/10.1088/1126-6708/2007/11/074) [Phys. 11 \(2007\) 074.](https://doi.org/10.1088/1126-6708/2007/11/074)
- [77] M. Järvinen and E. Kiritsis, Holographic models for QCD in the Veneziano limit, [J. High Energy Phys. 03 \(2012\) 002.](https://doi.org/10.1007/JHEP03(2012)002)
- [78] U. Gürsoy, E. Kiritsis, L. Mazzanti, G. Michalogiorgakis, and F. Nitti, Improved holographic QCD, [Lect. Notes Phys.](https://doi.org/10.1007/978-3-642-04864-7) 828[, 79 \(2011\).](https://doi.org/10.1007/978-3-642-04864-7)
- [79] U. Gürsoy and E. Kiritsis, Exploring improved holographic theories for QCD: Part I, [J. High Energy Phys. 02](https://doi.org/10.1088/1126-6708/2008/02/032) [\(2008\) 032.](https://doi.org/10.1088/1126-6708/2008/02/032)
- [80] U. Gürsoy, E. Kiritsis, and F. Nitti, Exploring improved holographic theories for QCD: Part II, [J. High Energy Phys.](https://doi.org/10.1088/1126-6708/2008/02/019) [02 \(2008\) 019.](https://doi.org/10.1088/1126-6708/2008/02/019)
- [81] F. Bigazzi, R. Casero, A. L. Cotrone, E. Kiritsis, and A. Paredes, Non-critical holography and four-dimensional CFT's with fundamentals, [J. High Energy Phys. 10 \(2005\) 012.](https://doi.org/10.1088/1126-6708/2005/10/012)
- [82] R. Casero, E. Kiritsis, and A. Paredes, Chiral symmetry breaking as open string tachyon condensation, [Nucl. Phys.](https://doi.org/10.1016/j.nuclphysb.2007.07.009) B787[, 98 \(2007\)](https://doi.org/10.1016/j.nuclphysb.2007.07.009).
- [83] O. Bergman, S. Seki, and J. Sonnenschein, Quark mass and condensate in HQCD, [J. High Energy Phys. 12 \(2007\) 037.](https://doi.org/10.1088/1126-6708/2007/12/037)
- [84] U. Gürsoy, E. Kiritsis, L. Mazzanti, and F. Nitti, Improved holographic Yang-Mills at finite temperature: Comparison with data, Nucl. Phys. B820[, 148 \(2009\)](https://doi.org/10.1016/j.nuclphysb.2009.05.017).
- [85] N. Jokela, M. Järvinen, and J. Remes, Holographic QCD in the Veneziano limit and neutron stars, [J. High Energy Phys.](https://doi.org/10.1007/JHEP03(2019)041) [03 \(2019\) 041.](https://doi.org/10.1007/JHEP03(2019)041)
- [86] N. Jokela, M. Järvinen, and J. Remes, Holographic QCD in the NICER era, Phys. Rev. D 105[, 086005 \(2022\).](https://doi.org/10.1103/PhysRevD.105.086005)
- [87] T. Demircik, C. Ecker, and M. Järvinen, Dense and hot QCD at strong coupling, [Phys. Rev. X](https://doi.org/10.1103/PhysRevX.12.041012) 12, 041012 [\(2022\).](https://doi.org/10.1103/PhysRevX.12.041012)
- [88] O. Komoltsev and A. Kurkela, How perturbative QCD constrains the equation of state at neutron-star densities, Phys. Rev. Lett. 128[, 202701 \(2022\).](https://doi.org/10.1103/PhysRevLett.128.202701)
- [89] M. Järvinen, Massive holographic QCD in the Veneziano limit, [J. High Energy Phys. 07 \(2015\) 033.](https://doi.org/10.1007/JHEP07(2015)033)
- [90] A. Amorim, M. S. Costa, and M. Järvinen, Regge theory in a holographic dual of QCD in the Veneziano limit, [J. High](https://doi.org/10.1007/JHEP07(2021)065) [Energy Phys. 07 \(2021\) 065.](https://doi.org/10.1007/JHEP07(2021)065)
- [91] M. Järvinen, E. Kiritsis, F. Nitti, and E. Préau, The V-QCD baryon: Numerical solution and baryon spectrum, [J. High](https://doi.org/10.1007/JHEP05(2023)081) [Energy Phys. 05 \(2023\) 081.](https://doi.org/10.1007/JHEP05(2023)081)
- [92] D. Arean, I. Iatrakis, M. Järvinen, and E. Kiritsis, V-QCD: Spectra, the dilaton and the S-parameter, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2013.01.070) 720, [219 \(2013\)](https://doi.org/10.1016/j.physletb.2013.01.070).
- [93] D. Areán, I. Iatrakis, M. Järvinen, and E. Kiritsis, The discontinuities of conformal transitions and mass spectra of V-QCD, [J. High Energy Phys. 11 \(2013\) 068.](https://doi.org/10.1007/JHEP11(2013)068)
- [94] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, Fluctuations of conserved charges at finite temperature from lattice QCD, [J. High Energy Phys. 01](https://doi.org/10.1007/JHEP01(2012)138) [\(2012\) 138.](https://doi.org/10.1007/JHEP01(2012)138)
- [95] T. Ishii, M. Järvinen, and G. Nijs, Cool baryon and quark matter in holographic QCD, [J. High Energy Phys.](https://doi.org/10.1007/JHEP07(2019)003) [07 \(2019\) 003.](https://doi.org/10.1007/JHEP07(2019)003)
- [96] T. Alho, M. Järvinen, K. Kajantie, E. Kiritsis, C. Rosen, and K. Tuominen, A holographic model for QCD in the Veneziano limit at finite temperature and density, [J. High](https://doi.org/10.1007/JHEP04(2014)124) [Energy Phys. 04 \(2014\) 124;](https://doi.org/10.1007/JHEP04(2014)124) [02 \(2015\) 033\(E\).](https://doi.org/10.1007/JHEP02(2015)033)
- [97] W. Israel and J. M. Stewart, Transient relativistic thermodynamics and kinetic theory, [Ann. Phys. \(N.Y.\)](https://doi.org/10.1016/0003-4916(79)90130-1) 118, 341 [\(1979\).](https://doi.org/10.1016/0003-4916(79)90130-1)
- [98] L. Gavassino, M. Antonelli, and B. Haskell, Bulk viscosity in relativistic fluids: From thermodynamics to hydrodynamics, [Classical Quantum Gravity](https://doi.org/10.1088/1361-6382/abe588) 38, [075001 \(2021\)](https://doi.org/10.1088/1361-6382/abe588).
- [99] A. Kurkela, K. Rajagopal, and R. Steinhorst, Astrophysical equation-of-state constraints on the color-superconducting gap, Phys. Rev. Lett. 132[, 262701 \(2024\)](https://doi.org/10.1103/PhysRevLett.132.262701).