Editors' Suggestion

Runaway Gravitational Production of Dark Photons

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We demonstrate that gravitational particle production of a massive, Abelian, vector (Proca) field during inflation in the presence of nonminimal coupling to gravity may suffer from an instability which leads to runaway production of high-momentum modes. This is untenable unless there is some mechanism to regulate the runaway. We discuss the parameter space of the particle mass and nonminimal couplings where such a runaway occurs and possible ways to tame the runaway. We find that there is no obvious way to resolve the runaway in a UV completion or with kinetic mixing to the standard model.

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Introduction—Dark matter is known to exist and yet the identity of the dark matter particle(s) remains elusive. A natural possibility, in light of the tremendously successful theory of electroweak symmetry breaking, is that dark matter includes a massive vector field, analogous to the W and Z bosons of the standard model (SM),which interacts weakly (or not at all) with SM particles. The case of a massive U(1) gauge boson is colloquially referred to as a "dark photon."

The theory of a massive U(1) gauge boson ("massive photon") dates back to work of Proca [1]. In a modern context, gauge boson masses are understood to arise from either the Higgs mechanism or the Stueckelberg mechanism. In string theory, where U(1) gauge bosons are ubiquitous, both the Higgs and Stueckelberg mechanisms are realized [2,3].

The Proca action for a dark photon is necessarily but the first few terms in a low-energy effective field theory. One expects higher-derivative terms like $(F_{\mu\nu}F^{\mu\nu})^2$ and $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$ that appear in QED at energy scales below the electron mass (the famous Euler-Heisenberg Lagrangian [4]), but also terms containing A_μ, such as a quartic self-interaction $(A_\mu)^4$, and derivative interactions such as $A_\mu\Box^2A^\mu$. The Proca theory has a natural portal to the standard model, namely kinetic mixing with the standard model photon $F^{(\mathrm{dark})}_{\mu\nu}F^{\mu\nu(\mathrm{vis})}$ (see, e.g., [5–7]).

The dark photon effective field theory can also contain *nonminimal* couplings to gravity, such as $Rg^{\mu\nu}A_\mu A_\nu$ and $R^{\mu\nu}A_\mu A_\nu$ (see, e.g., [8–17]). These are dimension-4 operators consistent with the symmetries of the Proca theory, and thus should be included in the effective field theory. Moreover, even if neglected at tree level, analogous to the $\xi\phi^2R$ coupling of a scalar required for self-consistent quantization of a self-interacting field in curved space [18–20], the vector nonminimal couplings are expected as soon as self-interactions of A_μ are included, such as quartic terms $(A_\mu)^4$, or from loop corrections to the interaction vertex with gravity, namely the mass term $m_A^2g^{\mu\nu}A_\mu A_\nu$ of Proca.

In this Letter, we demonstrate that the nonminimal couplings, despite being perfectly consistent with symmetries of the Proca effective field theory, when considered in a cosmological context can induce a runaway production of arbitrarily high momentum particles, thereby causing a breakdown of the theory. We discuss the implication and interpretations of this in the context of different UV completions of the Proca theory.

The dark photon in curved spacetime—On a generic spacetime background $g_{\mu\nu}$, the Proca action can be written as [14]

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{1}{2}m^2g^{\mu\nu}A_\mu A_\nu -\frac{1}{2}\xi_1Rg^{\mu\nu}A_\mu A_\nu-\frac{1}{2}\xi_2R^{\mu\nu}A_\mu A_\nu\right), \quad (1)$$

where A_μ is the dark photon, R the Ricci scalar, $R_{\mu\nu}$ the Ricci tensor, and ξ_1 and ξ_2 dimensionless constants which couple the Proca field to gravity. The dark photon field

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strength $F_{\mu\nu}$ is defined as $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. Notice that the interaction terms are the only dimension-four operators which can appear with the vector field coupled to curvature. We would like to focus on cosmological production, so we now specialize to the spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, $g_{\mu\nu} = a^2(\eta)\mathrm{diag}(1, -1, -1, -1)$, where $a(\eta)$ is the scale factor as a function of conformal time. In terms of the components A_i and A_0, the action becomes

$$S = \int \mathrm{d}^4x \left[\frac{1}{2}a(\partial_0 A_i - \partial_i A_0)^2 - \frac{1}{4}a^{-1}(\partial_i A_j - \partial_j A_i)^2 + \frac{1}{2}a^3 m_t^2 A_0^2 - \frac{1}{2}a m_x^2 A_i^2\right], \tag{2}$$

with (time-dependent) effective masses, m_t^2 and m_x^2 as

$$m_t^2 = m^2 - \xi_1 R - \frac{1}{2}\xi_2 R - 3\xi_2 H^2, \tag{3a}$$

$$m_x^2 = m^2 - \xi_1 R - \frac{1}{6}\xi_2 R + \xi_2 H^2, \tag{3b}$$

where H is the Hubble parameter and $R = -6(\ddot{a}/a + H^2)$. From Eq. (2), we see that A_0 is not a dynamical variable and can thus be integrated out. The next steps are standard [14]: decompose the action in terms of mode functions, integrate out A_0, and introduce an orthonormal set of transverse and longitudinal mode functions. The action then separates into two pieces. Here, we focus on the longitudinal component of the spin-1 field, with action [14,21,22]

$$S^L = \int \mathrm{d}\eta \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{1}{2}\frac{a^2 m_t^2}{k^2 + a^2 m_t^2}|\partial_0 A_{\boldsymbol{k}}^L|^2 - \frac{1}{2}a^2 m_x^2 |A_{\boldsymbol{k}}^L|^2\right]. \tag{4}$$

To ensure that the kinetic term is canonically normalized, we perform the field redefinition:

$$A_{\boldsymbol{k}}^L(\eta) = \kappa_k(\eta)\chi_{\boldsymbol{k}}(\eta) \quad \text{with} \quad \kappa_k^2(\eta) = \frac{k^2 + a^2 m_t^2}{a^2 m_t^2}, \tag{5}$$

where we now suppress L superscripts on χ. Notice that because m_t^2 is time dependent and not necessarily positive definite, κ_k^2 can potentially be negative, and a ghost can be propagated. If we demand a healthy theory that does not propagate ghosts, we must demand $\kappa^2(\eta) > 0$. Requiring that this condition must be satisfied for arbitrarily large k necessitates $m_t^2 > 0$.

The action for the longitudinal component is then

$$S^L = \int \mathrm{d}\eta \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\left(\frac{1}{2}|\partial_\eta \chi|^2 - \frac{1}{2}\omega_k^2 |\chi|^2\right), \tag{6}$$

where the longitudinal frequency is given by [17]

$$\omega_k^2 = k^2\frac{m_x^2}{m_t^2} + a^2 m_x^2 + \frac{3k^2 a^4 m_t^2 H^2}{(k^2 + a^2 m_t^2)^2} + \frac{k^2 a^2 R}{6(k^2 + a^2 m_t^2)} + \frac{Hak^2 m_t^{2\prime}}{m_t^2}\frac{(-k^2 + 2a^2 m_t^2)}{(k^2 + a^2 m_t^2)^2} - \frac{k^2 m_t^{2\prime\prime}}{2m_t^2(k^2 + a^2 m_t^2)} + \frac{k^2 (m_t^{2\prime})^2}{4(m_t^2)^2}\frac{(k^2 + 4a^2 m_t^2)}{(k^2 + a^2 m_t^2)^2}, \tag{7}$$

where prime denotes ∂_η and the wave number $k = |\boldsymbol{k}|$. The action for the transverse component is similar to Eq. (6) but with $\omega_k^2 = k^2 + a^2 m_x^2$. In each case the equation of motion of the mode function χ is given by $\chi_k'' + \omega_k^2 \chi_k = 0$.

In the high momentum (large-k) limit, the longitudinal frequency, Eq. (7), is dominated by the first term, $\omega_k^2 \to k^2 m_x^2/m_t^2$. The evolution of high-momentum modes is therefore dictated by the evolution of the effective sound speed m_x^2/m_t^2. Since we have established that $m_t^2 > 0$ for a ghostless theory, it follows that if $m_x^2 < 0$, then ω_k^2 will be *negative*, leading to an instability to particle production of arbitrarily large k modes. This is similar but distinct from the instability of spin-3/2 particles observed in [23,24]. In what follows, we refer to this phenomenon as *runaway production*.

Representative cosmological model—For numerical examples of runaway production we assume an inflationary model with a quadratic potential. The salient features of this model are common to a wide range of inflationary models, namely, a quasi–de Sitter (qdS) phase followed by a matter-dominated (MD) phase driven by the coherent oscillations of the inflaton field. For an FLRW cosmology with fixed equation of state w (i.e., ignoring any oscillations), we have $r \equiv R/H^2 = -3 + 9w$. In the qdS phase $r \simeq -12$ and in the MD phase r oscillates between $r = -12$ (when the inflaton field is at the maximum of the potential and momentarily in a de Sitter phase with $w = -1$) and $r = 6$ (when the inflaton is at the minimum of the potential and momentarily in a kination phase with $w = +1$). The average value of w in the MD phase is zero.

Let us first examine the conditions for a ghostless theory, $m_t^2 \geq 0$, in the limit $m/H \to 0$. (Dark photons of recent phenomenological interest include ultralight, sub-eV [25], and $\mathcal{O}(10)$ MeV [26–29], while H is as large as 10^{13} GeV, making $m \ll H$ a well motivated assumption). In this case the ghostless requirement is $-r(\xi_1 + \frac{1}{2}\xi_2) - 3\xi_2 \geq 0$. Using $-12 < r < 6$, this leads to $-\xi_2 > \xi_1 > -\xi_2/4$. This can only be satisfied if $\xi_2 < 0$. Now, consider the requirement $m_x^2 > 0$ to avoid runaway particle production: $-r(\xi_1 + \frac{1}{6}\xi_2) - \xi_2 > 0$. Runaway will be avoided if $0 > \xi_1 > -\frac{1}{4}\xi_2$. This is impossible to satisfy for $\xi_2 < 0$. Thus, in the limit $m \ll H$, the requirements of ghostless and no runaway are incommensurate unless $\xi_1 = \xi_2 = 0$.

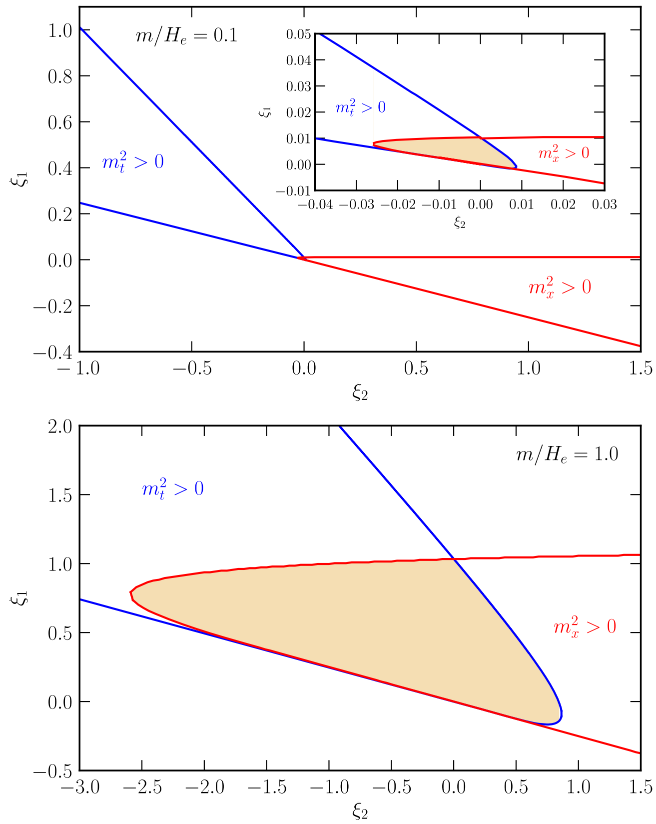

FIG. 1. In the ξ_2-ξ_1 plane, the interior of the blue contour has $m_t^2 > 0$, hence ghostless, and the interior of the red contour has $m_x^2 > 0$, hence the intersection of the two regions has no runaway GPP. The shaded region is ghostless and GPP is well behaved for large k. The upper panel is for $m/H_e = 0.1$, while the lower figure is for $m/H_e = 1$. The inset in the top panel is a blowup of the region surrounding $(\xi_2, \xi_1) = (0, 0)$ showing the small region without ghost or runaway.

For finite m/H it is possible to find values of (ξ_2, ξ_1) that are both ghostless and runaway safe. Clearly from Eq. (3) for sufficiently large m the right hand side of the equations will be positive. This is illustrated in Fig. 1 for two choices of m/H_e, where H_e is the Hubble parameter at the end of inflation. The choice $m/H_e = 0.1$ approximates the $m/H \to 0$ case and there is just a small shaded region where the theory is ghostless and UV safe. As m is increased, the safe region grows, bounded by $|\xi_{1,2}| \sim \mathcal{O}(m^2/H_e^2)$.

Runaway particle production—We numerically solve for the evolution of the mode functions χ_k and from this compute the comoving number density of particles $n_k = k^3|\beta_k|^2/2\pi^2$ with $|\beta_k|^2 = \omega_k|\chi_k|^2/2 + |\partial_\eta\chi_k|^2/2\omega_k - 1/2$ (see Ref. [30]). In Fig. 2 we show the result for the longitudinal components of a vector field of mass $m = 0.01H_e$, where H_e is the expansion rate at the end of inflation, for two values of (ξ_1, ξ_2): (0,0)-minimal gravitational coupling, and a model with nonminimal couplings, $(0.004, -0.006)$. The nonminimal parameters were chosen such that the model is ghostless ($m_t^2 > 0$)

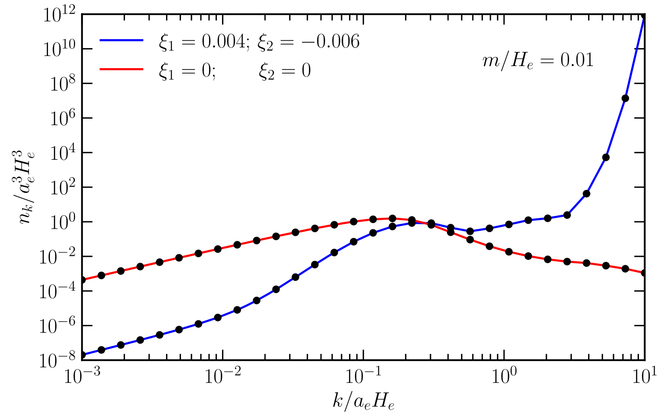

FIG. 2. The spectral density $n_k/a_e^3H_e^3$ as a function of wave number k/a_eH_e for the longitudinal mode for two choices of (ξ_1, ξ_2), with $m/H_e = 0.01$ in both cases. The choice (0,0) is minimal coupling; the nonminimal choice $(0.004, -0.006)$, while ghostless, has $m_x^2 < 0$. The minimal choice results in n_k well behaved at high k, while the nonminimal choice leads to runaway production at large k.

but has a high-k runaway ($m_x^2 < 0$). Here, the physical momentum of a mode is k/a, so k/a_e is the physical momentum of the mode at the end of inflation. Modes with $k/a_eH_e > 1$ were inside the Hubble radius at the end of inflation. The total number density of particles due to gravitational particle production (GPP) is $na^3 = \int n_k \mathrm{d}\log k$ (see Ref. [30] for details).

From Fig. 2 one may appreciate a dramatic amplification of the nonminimally coupled model (blue) at high k. Compared to the minimal coupling model (red), the nonminimal model has $n_k/a_e^3H_e^3$ 40 times larger at $k/a_eH_e = 1$ and 10^{15} times larger at $k/a_eH_e = 10$. This exemplifies runaway production.

Clearly, there is an issue if n_k does not turn over for large k: the Proca theory is itself an EFT, and production of modes at or above the cutoff would necessarily cause a breakdown of the EFT description. The simplest way out is therefore to posit a UV cutoff Λ above which production is tamed. To this end, it is useful to relate a physical momentum Λ to a comoving wave number k: $\Lambda = k/a$. For the example illustrated for $m/H_e = 0.1$ in Fig. 2, the angular frequency ω_k^2 first becomes negative for high-k modes around $a/a_e \sim 1$. Simply taking $a/a_e = \mathcal{O}(1)$, strictly positive $\omega_k^2(k)$ would require a physical cutoff $\Lambda \lesssim H_e$. This is clearly untenable; it would invalidate the whole analysis of quantum fluctuations during inflation.

Either something within the Proca EFT must regularize the high-k behavior, or else there must be a value of k beyond which our calculation is invalid.

We will now discuss three possibilities: (1) the UV completion of the Proca theory could cure the high-k runaway; (2) kinetic mixing with the standard model photon may resolve the instability; (3) the Proca theory becomes strongly coupled at some momentum scale, and beyond this scale our results cannot be trusted.

Dark photon effective field theory—The UV physics is encoded into the EFT by nonrenormalizable operators including derivative terms like $A_\mu \Box^2 A^\mu/\Lambda^2$ which would contribute to ω_k^2 a term like k^4/Λ^2. One might hope this might shut off the instability while maintaining validity of the EFT, namely that the conditions (1) that $k/\Lambda > |m_x/m_t|$ (to make ω_k^2 positive at high k) and (2) $k/\Lambda \ll 1$ to maintain perturbative control, can be satisfied simultaneously. However, there is no hierarchy between m_x and m_t: m_x/m_t oscillates to $\mathcal{O}(1)$ negative values in the instability region. Concretely, for $m \ll H$, one can consider three regimes: $\xi_1 \gg |\xi_2|$, which gives $m_x/m_t \to 1$, $\xi_1 \ll |\xi_2|$ which gives $|m_x/m_t| \to 1/\sqrt{3}$, or $\xi_1 \sim |\xi_2|$ which gives $|m_x/m_t| = \sqrt{|5/3 + 8/(6-r)|} = \mathcal{O}(1)$. Thus, it is not possible for higher-derivative terms in the EFT to shut off the instability at high k while maintaining perturbative control in the EFT. It follows that any hope of resolving the runaway lies in abandoning the EFT in favor of an explicit UV completion.

UV completion—To make more progress we can turn to an explicit UV completion. As a simple UV example we consider the Abelian Higgs model, with Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 + \mu^2|\phi|^2 - \lambda|\phi|^4, \qquad (8)$$

where ϕ is a complex scalar, and D_μ is the gauge covariant derivative $D_\mu = (\partial_\mu - igA_\mu)$ where g is the gauge coupling. We expand ϕ about the minimum at $\phi = \mu/\sqrt{\lambda} \equiv \sqrt{2}v$ as $\phi = (v+h)e^{i\theta}/\sqrt{2}$, where h is a real scalar and fix the U(1) gauge symmetry to unitary gauge where $\theta = 0$ and the photon is massive. The Lagrangian is then simplified to $\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}m_A^2 A_\mu A^\mu[1+(h/v)]^2 + \frac{1}{2}m_h^2 h^2 + 4v\lambda h^3 + \lambda h^4$, corresponding to a Higgs mass $m_h = \sqrt{\lambda/2}v$, a photon mass of $m_A = gv/2$, and cubic and quartic interactions between h and A_μ given by [31]

$$\mathcal{L}_{AAh} = \frac{m_A^2}{v} A_\mu A^\mu h, \qquad \mathcal{L}_{AAhh} = \frac{m_A^2}{v^2} A_\mu A^\mu h^2. \qquad (9)$$

Meanwhile, the gravitational couplings of the gauge field generate interactions with gravitons which at cubic order in the fields (after performing a series expansion of $\sqrt{-g}Rg^{\mu\nu}$ and $\sqrt{-g}R_{\mu\nu}$) are given by

$$\mathcal{L}_{AAg} = \frac{\sqrt{-g}}{M_{pl}}(m_A^2 + \xi_1 R + \xi_2 D_\alpha D^\alpha)\delta g^{\mu\nu} A_\mu A_\nu,$$

where $\sqrt{-g}$, R, and D are defined with respect to the background geometry, $\delta g_{\mu\nu}$ denotes a canonically normalized transverse traceless fluctuation to the metric of mass dimension 1 ($g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}/M_{pl}$). There are additional interactions at quartic order, e.g., from the expansion of $\sqrt{-g}R$, of the form $\mathcal{L}_{AAgg} \sim (m_A^2 + \xi_1 D^2)\delta g_{\mu\nu}\delta g^{\mu\nu} A_\sigma A^\sigma / M_{pl}^2 + \cdots$.

Let us consider the possibility that the nonminimal gravitational coupling constants ξ_1 and ξ_2 vanish at tree level, and are generated as effective interactions via loops in the Abelian Higgs model. With the interactions between h and A given by Eq. (9), loops of ϕ particles renormalize the minimal cubic interaction of A_μ with gravity, which by dimensional analysis generates a nonminimal coupling with $\xi_2 \propto (m_A/m_h)^6$. Similarly, the 1-loop correction to the minimal quartic interaction generates $\xi_1 \sim (m_A/m_h)^6$. Depending on the relative size of the Higgs and gauge couplings, λ and g respectively, the effective couplings $\xi_{1,2}$ can be made small, but are not necessarily so. Meanwhile, the instability depends on the ratio m_A/H, with H the Hubble parameter, whereas the expected size of $\xi_{1,2}$ is independent of H. Thus, even the ostensibly small loop-generated couplings can lead to a runaway production: UV completion into the Abelian-Higgs model does not in itself regulate the instability.

The loop-induced couplings present a similar situation to that with nonminimally coupled scalars: while the coupling may be set to zero at one energy scale, renormalization group (RG) flow will generate nonzero value of the coupling at all other energy scales. This justifies the Wilsonian intuition that all terms allowed by symmetries should be allowed in the low-energy effective field theory, and absent any measurement to anchor the RG flow, the coupling constants should be taken to be free parameters.

With this in mind, we can include the vector nonminimal couplings directly in the Abelian-Higgs model, where they manifest as nonminimal derivative couplings, $\mathcal{L}_1 = [\xi_1/(gv)^2]RD_\mu\phi D^\mu\phi^*$ and $\mathcal{L}_2 = [\xi_2/(gv)^2]D_\mu\phi D_\nu\phi^* R^{\mu\nu}$. This provides a gauge-invariant UV completion of Eq. (1). Couplings of this form have been extensively studied for a neutral scalar field such as [32]. The vector nonminimal couplings now modify the kinetic action of the Higgs scalar field. One may easily appreciate that $\xi_{1,2} \neq 0$ can lead to ghost and/or tachyonic instabilities, just as in the dark photon model that emerges at low energies. Thus, again one sees that the UV completion is not the cure.

Kinetic mixing with the standard model—Another possibility for curing the runway, within the dark photon EFT, is dark-photon interaction with the standard model (SM), namely the kinetic mixing $\mathcal{L}_{\rm int} = \epsilon F_{\mu\nu}^{(\rm dark)} F^{\mu\nu(\rm SM)}$. In the case that the dark photon is massive (as we study here), the kinetic terms can be diagonalized via the field redefinitions [7] $A_\mu^{\rm SM} \to A_\mu^{\rm SM'} \equiv A_\mu^{\rm SM} - \epsilon A_\mu^{\rm dark}$ and $A_\mu^{\rm dark} \to A_\mu^{\rm dark}$. The resulting action has canonical kinetic terms. This endows SM fields with dark-photon charge, i.e., interactions of the form $A_\mu^{\rm dark}\bar{f}\gamma^\mu f$. The nonminimal gravitational terms are unaffected. Thus, in the diagonal basis, the only change generated by the kinetic mixing is the addition of charged fields in the Proca theory. At tree level, the charged fermions do not alter the kinetic sector of the dark photon, and thus have no impact on the k^2 culprit of the runaway production. On the other hand, loops of charged fields can generate higher-derivative terms in the Proca EFT, such as k^4 terms, but these fail to cure the runaway, as described above. Thus kinetic mixing does not prevent or preclude the runaway production.

Strong coupling—The Proca theory nonminimally coupled to gravity should become *strongly coupled* at some scale. Lacking a first principles derivation of the strong-coupling scale we can still consider the implications of the existence of such a scale. Similar but distinct from a breakdown of the EFT, strong coupling would render the evolution of high-k modes impervious to calculation. In practice, the consequences for dark photon phenomenology remain unchanged from the weakly coupled runaway, namely conventional tools of dark matter phenomenology (such as perturbative quantum field theory) no longer apply.

Conclusions—In this Letter we have pointed out the possibility of runaway cosmological production of dark photons with nonminimal couplings to gravity. We studied this phenomenon in the context of inflation with a quadratic potential and late reheating, however, we emphasize that the discussion above is largely generic and independent of both the inflation model and reheating. This is discussed in further detail in [17], in which we show that the particle production remains qualitatively similar in scenarios with rapid-turn multifield inflation (see also Ref. [33]) and/or early reheating. Also, in [17] we point out that even if high-k runaway is stopped at some moderate value of $k/a_e H_e$, values of (ξ_1, ξ_2) where $m_x^2 < 0$ have very much enhanced GPP compared to the minimal model. For instance, from Fig. 2 we see that for $k/a_e H_e = 10$ GPP in the nonminimal model is enhanced by about a factor of 10^{15} compared to the minimal model, even for small values of (ξ_1, ξ_2). If the runaway production can be tamed, while leaving an overall enhanced production, this effect could widen the range of parameters to result in dark photons as dark matter. Finally, it will also be interesting to examine whether the runaway production persists for non-Abelian gauge fields, and for nonminimally coupled pseudovector fields, as realized by the Kalb-Ramond theory of an antisymmetric tensor field, see Ref. [34] for Kalb-Ramond GPP.

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