Rigorous Holographic Bound on AdS Scale Separation

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(Received 25 March 2024; accepted 4 June 2024; published 5 August 2024)

We give an elementary proof of the following property of unitary, interacting four-dimensional $\mathcal{N}=2$ superconformal field theories: At large central charge c, there exist at least $\sqrt{c}$ single-trace, scalar superconformal primary operators with dimensions $\Delta \lesssim \sqrt{c}$ (suppressing multiplicative logarithmic corrections). This follows from a stronger, more refined bound on the spectral density in terms of the asymptotic growth rate of the central charge. The proof employs known results on the structure of Coulomb branch operators. Interpreted holographically, this bounds the possible degree of scale separation in semiclassical AdS_5 half-maximal supergravity. In particular, the bulk must contain an infinite tower of charged scalar states of energies parametrically below the large black hole threshold $E_{\mathrm{BH}} \sim c$. We address the extreme case of AdS_5 pure supergravity, ruling it out as the asymptotic limit of certain sequences in theory space, though the general question remains open.

DOI: 10.1103/PhysRevLett.133.061601

The problem—The question of AdS scale separation is whether a consistent theory of quantum gravity can, in the semiclassical limit, admit an AdS vacuum without parametrically large extra dimensions.

Recent years have seen significant efforts to scrutinize and extend earlier proposals for scale-separated solutions of string theory [1–3]. This uptick was spurred in part by the AdS distance conjecture [4] and other swampland considerations and the pressing cosmological implications of this question *vis-à-vis* possible de Sitter uplift.

A direct bulk approach to the problem is inherently limited: To tuck the extra dimensions away at a parametrically small scale, one must grapple with the full machinery of string or M theory, not just its ten- or 11-dimensional supergravity limit. For this reason, it seems fair to say that gravity constructions are unlikely to definitively answer this question beyond a reasonable doubt, unless and until nonperturbative string and M theory are well understood.

Boundary CFT approaches, however, offer hope for unambiguous resolution. There is a growing appetite for the conformal bootstrap to solve this problem, using axiomatic constraints to rigorous ends. In particular, one might realistically hope to exclude, or bound the degree of, AdS scale separation holographically using the large-N bootstrap (as opposed to explicitly constructing a candidate CFT). Some older [5,6] and newer [7–11] works explore the holographic dictionary for putative scale-separated string backgrounds, but fundamental constraints are missing. A collage of constructions, claims, counterclaims, and conjectures is well-reviewed in [12].

For better or worse, the conformal bootstrap is not magic: The CFT avatar of excluding AdS scale separation is still a hard problem. As emphasized in [13,14], this question differs fundamentally from the canonical bootstrap endeavor of maximizing the spectral gap to the first primary operator: Large (i.e., AdS-sized) extra dimensions in gravity are manifest as *infinite towers* of operators in CFT. Their characteristic energy Δ and asymptotic density $\rho(\Delta)$ encode the size and number of bulk dimensions, respectively, quantities which can be read off from a positive sum rule for CFT correlators of light operators [14].

Even supersymmetric versions of this problem, as articulated in [15], remain open, providing compelling targets for the superconformal bootstrap. It is not known whether continuous R symmetry of a superconformal field theory (SCFT) is always geometrized in AdS, despite familiar examples under the lamppost. Does quantum gravity admit, say, a maximally supersymmetric "pure AdS_5" background without a large S^5? There are arguments both for [16] and against [17] this possibility in the literature. The analogous questions on the CFT side concern the existence of "exotic" SCFTs without infinite towers of Bogomol'nyi-Prasad-Sommerfield (BPS) operators, which are a boundary hallmark of a large compact manifold in the bulk.

This Letter takes a step forward on the supersymmetric version of this problem, proving a universal result for 4D $\mathcal{N}=2$ SCFTs at large central charges. Our proof uses nothing more than established facts about Coulomb branch geometry. The main result is in (21) and (22), a constraint on the spectrum of Coulomb branch primaries. Extremizing over theory space gives the absolute bound (22), valid for all $\mathcal{N}=2$ SCFTs (subject to widely-held assumptions detailed below), as sketched in the abstract.

The result is modest, and its proof elementary; but its consequences for gravity are significant. By AdS/CFT, these theories are dual to asymptotically AdS_5 vacua of semiclassical gravity with (at least) half-maximal supersymmetry. The result establishes the necessity of an infinite (Planckian) number of U(1)-charged, scalar bulk states which are parametrically below the large AdS_5 black hole threshold $E_{BH}\sim c$. For $a\approx c$, this is a quantitative bound on the degree of scale separation in AdS_5 Einstein supergravity.

The ingredients—We consider unitary, local $\mathcal{N}=2$ SCFTs in four dimensions; this is a vast subject (see, e.g., [18,19]), but we will keep our presentation to the point. We are interested in the BPS spectra of these theories at large central charge. An $\mathcal{N}=2$ SCFT is partially characterized by its Coulomb branch, of rank $r\in\mathbb{N}$. The Coulomb branch may be parametrized by the vacuum expectation values of r superconformal primaries, the generators of the chiral ring, which are the bottom components of protected $\mathcal{E}_{r(0,0)}$-type superconformal multiplets in the notation of [20] ($L\overline{B_1}[0;0]^{(0;r)}$ multiplets in the notation of [21]). These operators, call them $\{\mathcal{O}_i\}$, have vanishing $SU(2)_R$ charge and $U(1)_r$ charge equal to their conformal dimensions, $r_i=\Delta_i$. In all known examples, the $\{\mathcal{O}_i\}$ are Lorentz scalars, which we have assumed here.

Let us state up front that we are making the oft-used assumption that the Coulomb branch is freely generated, i.e., that the generators satisfy no nontrivial relations [22]. This allows for an unambiguous assignment of dimensions Δ_i to operators $\mathcal{O}_i$. There is a whole story here, with possible violations of this property if discrete gaugings are allowed, leading to complex singularities of the Coulomb branch geometry [23–26]. Having emphasized this, we henceforth take the Coulomb branch to be freely generated while keeping in mind the limitations of this assumption at the most abstract level of $\mathcal{N}=2$ SCFT.

We will need two ingredients for our proof.

(i) *Central charge sum rule.*—Recent work on the intricacies of Coulomb branch geometry has established the following central charge formulas in $\mathcal{N}=2$ SCFTs [27]:

$$2a=-\frac{r}{12}+\frac{\mathbb{S}(r)}{2}+f\ \text{(other Coulomb branch data)},$$
$$c=\frac{r}{6}+f\ \text{(other Coulomb branch data)},\tag{1}$$

where

$$\mathbb{S}(r)\coloneqq\sum_{i=1}^{r}\Delta_i.\tag{2}$$

Happily, we will not need the unspecified function of other Coulomb branch data in order to prove the desired result at large central charge. Instead, we take a simpler approach. First, these results imply the following formula, previously established in a more limited context by Shapere and Tachikawa [28]:

$$2a-c=\frac{\mathbb{S}(r)}{2}-\frac{r}{4},\quad \text{where } \mathbb{S}(r)\coloneqq\sum_{i=1}^{r}\Delta_i.\tag{3}$$

Noting the unitarity bound $\Delta_i\geq 1$, saturated by a free vector multiplet, an interacting unitary $\mathcal{N}=2$ SCFT obeys $\mathbb{S}(r)>r$ and, hence, $2a-c>(r/4)$. Besides finding the independent formulas (1), one achievement of the work since [28] is that it establishes the Shapere-Tachikawa formula (3) in fuller generality, without the assumptions of [28] (e.g., that there is a Lagrangian point on the SCFT moduli space) [29]. We will use this formula in conjunction with the Hofman-Maldacena bound [31] for local $\mathcal{N}=2$ SCFTs,

$$\frac{1}{2}\leq\frac{a}{c}\leq\frac{5}{4}.\tag{4}$$

The lower and upper bounds are realized by free hypermultiplets and free vector multiplets, respectively.

(ii) *Coulomb branch operator dimensions.*—Our second ingredient is a fascinating feature of $\mathcal{N}=2$ SCFT operator spectra: At a given rank r, the Coulomb branch dimensions $\{\Delta_i\}$ are drawn from a finite, r-dependent set of rationals. As proven in [32,33], this set admits a remarkably concise characterization:

$$\Delta_i\in\left\{\frac{n}{m}\middle|\varphi(n)\leq 2r, 0<m\leq n\ (m,n)=1\right\},\tag{5}$$

where $\varphi(n)$ is Euler's totient function. Most of the number-theoretic structure of this result will not be used here. All we need is the existence of $\Delta_{\max}$, the largest possible dimension at rank r:

$$\Delta_{\max}\coloneqq\max\{n|\varphi(n)\leq 2r\}.\tag{6}$$

For finite r, $\Delta_{\max}$ is finite. In what follows, we order the dimensions in a given r-tuple as

$$1\leq\Delta_1\leq\Delta_2\leq\cdots\leq\Delta_r\leq\Delta_{\max}.\tag{7}$$

Large r, a, c limits: Consider the large rank limit $r\to\infty$. We first establish the following warmup result [34]: $\mathbb{S}(r)\sim r$ is inconsistent with the existence of the SCFT at large rank. More precisely, $\mathbb{S}(r)\sim r$ is incompatible with a finite partition function on the spatial sphere. The proof is simply that a unitary theory can achieve linear scaling of $\mathbb{S}(r)$ only if $\mathcal{O}(r)$ of the r Coulomb branch primaries have dimensions bounded above by $\Delta_*\sim\mathcal{O}(1)$; but this violates finiteness of $Z_{S^1_\beta\times S^3}=\mathrm{tr}_{\mathcal{H}}(e^{-\beta H})$, which requires that

$$\int_0^{\text{finite}}d\Delta\rho(\Delta)<\infty,\tag{8}$$

where $\rho(\Delta)$ is the spectral density of local operators on S^3. Therefore, the large r limit does not exist [35]. One can rephrase this conclusion as the absence of "large c $\mathcal{N}=2$ vector models"; we suggest that this may also be true of $\mathcal{N}=1$ SCFTs, for a suitable replacement of the Coulomb branch rank [36].

So, at $r\to\infty$ we have $\mathbb{S}(r)\gg r$, and, hence,

$$2a-c\approx\frac{\mathbb{S}(r)}{2}. \tag{9}$$

Therefore, $r\to\infty$ implies $2a-c\to\infty$. Combining this with (4) implies that a and c both grow large independently:

$$r\to\infty\Rightarrow a,\qquad c\to\infty. \tag{10}$$

That is, *large rank implies large central charges*.

The implication runs in reverse as well, modulo two small caveats. The first is if $2a-c$ is finite to leading order: In particular, r can remain finite at large central charge if and only if

$$\frac{a}{c}\approx\frac{1}{2}+\mathcal{O}\left(\frac{1}{c}\right) \tag{11}$$

as $c\to\infty$ (with a strictly positive correction term). The case $a/c=\frac{1}{2}$ is only known to be realized by free hypers; adopting this perspective, the possibility (11) can be phrased as the theory "becoming free" at large central charge. In addition, one must note the logical possibility that the SCFT at large central charge could, in principle, have no Coulomb branch, i.e., $r=0$; but it is widely believed that there are no interacting $\mathcal{N}=2$ SCFTs with $r=0$.

All told, we can summarize as follows: Assuming the existence of a Coulomb branch,

$$a,c\to\infty\Leftrightarrow r\to\infty, \tag{12}$$

with the only possible exception to the $\Rightarrow$ direction being the edge case $(a/c)\approx\frac{1}{2}$. This case is, at any rate, maximally far from the Einstein regime in which $a\approx c$, to which we soon turn.

As for the Coulomb branch dimensions at $r\to\infty$, the upper bound $\Delta_{\max}$ scales asymptotically as [32]

$$\Delta_{\max}\approx 2e^{\gamma_E}r\ \log\ \log\ r, \tag{13}$$

where $\gamma_E\approx 0.577$ is Euler's constant.

The proof—The key observation is that the upper-boundedness of Coulomb branch dimensions, which, in turn, determine the central charges, implies a spectral bound of the former when expressed in terms of the latter.

In particular, bounding spectra at large central charge becomes an extremization problem to be solved in the asymptotic limit $r\to\infty$: *Given an ordered* r-tuple $\{\Delta_1,\ldots,\Delta_r\}$, *maximize this set as a function of central charges, where the latter are determined by* $\mathbb{S}(r)$ *via* (3) and (4). We are using "maximize this set" as a placeholder for different choices of extremization, depending on the problem of interest. For instance, to extremize the spectral gap *à la* bootstrap, one solves a *maximin problem*, choosing to maximize Δ_1 among allowed r-tuples.

The main result now follows quickly. We established earlier that $\mathbb{S}(r)\gg r$, but we now note the obvious upper bound as well:

$$\mathbb{S}(r)\leq r\Delta_{\max}\approx 2e^{\gamma_E}r^2\ \log\ \log\ r. \tag{14}$$

In fact, strictly maximizing all r operator dimensions is disallowed by more subtle number-theoretic aspects of Coulomb branch geometry—when $\Delta_i=\bar{\Delta}$ for some $\bar{\Delta}$, the latter must actually be drawn from the allowed set at $r=1$ [32]—so, in the $r\to\infty$ limit, one ought to interpret the upper bound as

$$\mathbb{S}(r)\approx r\Delta_{\max} \tag{15}$$

with negative splittings that are subleading in r. For simplicity, and our ultimate interest in the Einstein gravity regime, we specialize to $a\approx c$. Then, plugging (13) and (15) into (9) gives

$$c\approx e^{\gamma_E}r^2\log\log r. \tag{16}$$

In terms of central charge,

$$\Delta_{\max}\approx 2e^{\gamma_E/2}\sqrt{c}\sqrt{\log\log c}. \tag{17}$$

Therefore, *there are* $r\approx e^{-\gamma_E/2}\sqrt{c}/\sqrt{\log\log c}$ *operators with dimensions* $\Delta_i\leq\Delta_{\max}$. Simplifying this exact statement by dropping the logs and prefactors gives the result quoted in the abstract. This clearly generalizes away from $a\approx c$ to any asymptotic ratio of a/c at large rank: Thanks to Hofman-Maldacena, a and c will have identical r scaling but different numerical prefactors determined by their $\mathcal{O}(1)$ ratio.

More generally, we can classify theories by the growth of central charge with the rank:

$$\mathbb{S}(r)\sim r^z,\qquad 1\leq z\leq 2. \tag{18}$$

Multiplicative logarithmic enhancements—required at $z=1$ [cf. (8)] and possible at $z>1$ [e.g., cf. (16)]—are left implicit to avoid clutter. Then, for any finite $2a-c>0$, whereupon $a\sim c\sim r^z$, the extremal spectrum consistent with this scaling, i.e., the sparsest possible low-lying spectrum, is obtained by taking the lowest $\approx r$ operator dimensions to all approach:

$$\Delta_*(z)\sim r^{z-1}. \tag{19}$$

This corresponds to parametric solution of a constrained optimization problem: Maximize Δ_r, then maximize Δ_{r-1}, and so on until maximization of Δ_1, all subject to the scaling (18). In other words, defining a spectral function $\mathbb{N}(\Delta_-, \Delta_+)$ counting primaries in an interval,

$$\mathbb{N}(\Delta_-, \Delta_+) := \int_{\Delta_-}^{\Delta_+} d\Delta \rho(\Delta), \quad \mathbb{N}_*(z) := \mathbb{N}[1, \Delta_*(z)], \tag{20}$$

where $\rho(\Delta)$ is the spectral density of Coulomb branch generators, the scaling (18) requires that $\mathbb{N}_*(z) \approx r$, and the extremal spectrum has these states "piled up" near $\Delta_*(z)$ [39]. Therefore, and restating in terms of central charge $c \to \infty$,

$$\begin{aligned} & c \sim r^z \Rightarrow \exists\ \mathbb{N}_*(z) \sim c^{1/z} \text{ operators} \{\mathcal{O}_i\} \\ & \text{with} \quad \Delta_i \lesssim \Delta_*(z) \sim c^{1-1/z}. \end{aligned} \tag{21}$$

This is the general result. Scanning over possible SCFTs by varying z, the spectral gap as a function of c is maximized at $z = 2$ dressed with doubly logarithmic corrections (16), giving the absolute bound derived earlier, valid for all $\mathcal{N} = 2$ SCFTs:

$$\begin{aligned} & \exists\ \mathbb{N}_* \approx e^{-\gamma_E/2} \frac{\sqrt{c}}{\sqrt{\log \log c}} \text{ operators } \{\mathcal{O}_i\} \\ & \text{with} \quad \Delta_i \lesssim \Delta_{\max} \approx 2e^{\gamma_E/2} \sqrt{c} \sqrt{\log \log c}. \end{aligned} \tag{22}$$

The bulk—In summary, unitary interacting $\mathcal{N} = 2$ SCFTs at large central charge contain an infinite tower of $U(1)_r$-charged, scalar, single-trace, superconformal primary local operators $\{\mathcal{O}_i\}$ with dimensions $\Delta_i < \Delta_*$, where

$$\Delta_* \sim c^{1-1/z} \ll c, \qquad 1 \leq z \leq 2, \tag{23}$$

with z defined by the asymptotic scaling $c \sim r^z$ at large rank r of the Coulomb branch. The number of these operators scales as $\sim c^{1/z}$. The absolute bound for all $\mathcal{N} = 2$ SCFTs (obeying the assumptions of [32,33]) is in (22). These are special (protected) operators, namely, the generators of the Coulomb branch chiral ring, of vanishing $SU(2)_R$ charge and $U(1)_r$ charge $r_i = \Delta_i$. We have emphasized in (23) the sublinear growth of Δ_* in the central charge.

Translating to the semiclassical bulk and taking $a \approx c \sim 1/G_N$, the inverse five-dimensional Newton's constant, (23) is a bound on the degree of scale separation possible in AdS_5 supergravity with at least half-maximal supersymmetry: *There must exist an infinite tower of U(1)-charged scalar states $\{\phi_i\}$ with energies parametrically below the large black hole threshold.* The latter is set by the mass of large AdS_5-Schwarzschild black holes:

$$E_{\mathrm{BH}} \sim \mathcal{O}(1/G_N). \tag{24}$$

In particular, all ϕ_i obey $E_i < E_*$, where

$$E_* \sim (E_{\mathrm{BH}})^{1-1/z} \ll E_{\mathrm{BH}}. \tag{25}$$

It is perhaps clearer to discuss the absolute bound (22) for all such theories, which is (up to a doubly-logarithmic prefactor) the $z = 2$ specialization of the above: *There must exist $\sim\sqrt{E_{\mathrm{BH}}}$ U(1)-charged scalar states $\{\phi_i\}$ with energies $E_i < E_* \sim \sqrt{E_{\mathrm{BH}}}$.*

We emphasize once again that this furnishes a bound on AdS scale separation, because it implies the existence of an *infinity* of such excitations, a total number that scales with the Planck scale, in the semiclassical limit of weak gravitational coupling. Note that a Planckian number of states in an $\mathcal{O}(1)$ window around Planckian energies is compatible with the existence of the large c limit, because the states are not fixed-energy states [40].

What are these $U(1)_r$-charged bulk states? In canonical AdS-CFT dual pairs, of course, Coulomb branch generators are dual to Kaluza-Klein (KK) modes on a large internal space, with energies of order one in AdS units; should the states $\{\phi_i\}$ be realized geometrically, the characteristic scale of extra dimensions is bounded below as

$$L_{\mathrm{KK}} \gtrsim \ell_p^{3/2}, \tag{26}$$

where ℓ_p is the five-dimensional Planck scale. But, remaining steadfastly agnostic in the bootstrap spirit, there are many other possibilities, particularly if the states $\{\phi_i\}$ have Planckian energies. In known AdS_5 compactifications of string and M theory, the spectrum also contains small black holes, with horizon radii obeying $\ell_p \lesssim r_h \ll L_{\mathrm{AdS}}$; but nonsingular, nonhairy BPS black holes in AdS_5 [41] must carry large angular momenta, in contrast to the scalar states we are considering. Small black holes are far from the only possibility. In string and M theory, there is a rich spectrum of semiclassical bulk configurations with energies $M_p \lesssim E \lesssim E_{\mathrm{BH}}$, including D-branes, giant gravitons [42], small black holes (possibly with hair [43–46]), microstate geometries [47], topological stars [48], grey galaxies [49], and perhaps yet-undiscovered configurations of matter and black holes in thermal equilibrium. Depending on the energies $\{E_i\}$, some of these may be candidate bulk descriptions of $\{\phi_i\}$.

We emphasize the conceptual point that BPS states are effective tracers of extra dimensions in AdS/CFT. For one, the Heemskerk-Penedones-Polchinski-Sully higher-spin gap condition [50] for emergence of AdS_{d+1} Einstein gravity from CFT is blind to the number of large dimensions $D \geq d + 1$ of the bulk; but BPS sectors can be sensitive to D, because towers of R-charged light states may (and, perhaps, must) be geometrized. More generally, in a D-dimensional bulk gravitational effective field theory, the Planckian states are dual to CFT_d operators with conformal dimensions

$$\Delta \sim M_{p,D} \sim c^{1/(D-2)}. \tag{27}$$

These operators are generically unprotected. For example, we recall the short string states in $AdS_5 \times S^5$ at fixed g_s, dual to Konishi-type operators in $\mathcal{N}=4$ super Yang-Mills with $\Delta \sim \lambda^{1/4} \sim c^{1/8}$. So, although the c scaling of these unprotected operator dimensions encodes the macroscopic dimensionality of the bulk, it is generally easier to access protected states than unprotected states in CFT.

Note that (25) is broadly applicable to semiclassical gravity: It is not specific to Einstein gravity, instead relying only on asymptotic scalings valid for any SCFT with $(a/c) > \frac{1}{2}$. In particular, semiclassical string effects cannot push the compactification scale below (26). In addition, the result applies to SCFTs irrespective of whether they have an independent higher-spin gap scale Δ_{gap} (i.e., it applies to both string and M-theory duals). This generality follows from a mutual compatibility of the known moduli-independence of both the central charges [51] and the Coulomb branch operator dimensions, built into the $\mathcal{N}=2$ central charge formulas.

It is interesting from the gravity point of view that the Coulomb branch dimensions completely fix the strength of the gravitational interaction and the R^2 correction (and, hence, the species scale). This suggests a deeper relation between the Coulomb branch data and BPS black hole sectors that would be nice to understand.

Pure supergravity: Our result also addresses the question of whether AdS_5 *pure* supergravity exists.

It is important to examine what “pure (super)gravity” in more than three bulk dimensions ought to even mean. We give an extended discussion of this topic in an Appendix [52]. Low-energy effective field theory provides a natural and agnostic parametric definition of semiclassical AdS_{d+1} pure gravity: namely, the absence of nongraviton states below Planckian energies $E_* \sim M_p$, where M_p is the $(d+1)$-dimensional Planck scale. Although Planckian degrees of freedom lack a universal abstract characterization, the validity of gravitational effective field theory suggests this threshold. Unlike AdS_3 gravity, this threshold cannot be made sharp with an $\mathcal{O}(1)$ coefficient.

Taking the effective field theory view and applying the holographic relation $c \sim M_p^{d-1}$ to the $d=4$ case, a putative CFT dual to AdS_5 pure (super)gravity has a (super) conformal primary spectrum comprised solely of (super) stress tensor composites up to $\Delta_* \sim c^{1/3}$. Ruling out AdS_5 (super)gravity thus amounts to proving that a dual CFT must have a non-stress tensor single-trace primary parametrically below this threshold. We return now to our result (25). For the range $z < \frac{3}{2}$, we have ruled out AdS_5 pure supergravity as the asymptotic limit, violating the gap condition by an infinite tower of sub-Planckian states. Stated conversely, *a putative CFT dual to* AdS_5 *pure supergravity must have central charge scaling faster than* $c \sim r^{3/2}$ *as* $r \to \infty$ [57].

There is a sensible analogy to be drawn here with the modular bootstrap in 2D CFTs at large central charge [59]. In unitary 2D CFTs, the state-of-the-art upper bound on the first nonvacuum Virasoro primary dimension at $c \to \infty$ is [60]

$$\Delta_* \lesssim \frac{c}{9.1}. \tag{28}$$

This sits between the classical threshold at $\Delta \approx c/12$ for small Bañados-Teitelboim-Zanelli (BTZ) black holes and the onset of universal Cardy thermodynamics at $\Delta \approx c/6$ [61]. Similarly, our bound (22) for 4D $\mathcal{N}=2$ SCFTs sits between the Planck scale threshold at $\Delta \sim c^{1/3}$ for small black holes and the onset of universal AdS_5-Schwarzschild thermodynamics at $\Delta \sim c$. This comparison (meant only to guide the mind) cannot be made apples-to-apples because of the aforementioned differences in dimensional analysis. Relatedly, we have chosen to phrase the CFT_2 result more optimistically, in terms of thermodynamics instead of horizon size, because the continuum of geometrically large BTZ black holes, which have horizon radius $r_h \gtrsim \mathcal{O}(1)$ in AdS units, has $\Delta \approx (c/12)(1+r_h^2)$, which is well below (28). But the bounds are similarly situated with respect to bulk AdS black hole spectra. We expect both bounds (22) and (28) to be suboptimal.

Indeed, there are early indications of much stronger bounds on Coulomb branch data of $\mathcal{N}=2$ SCFTs at $c \to \infty$ that could rule out pure supergravity unconditionally [62]. As noted earlier, r-tuples are subject to a host of intricate number-theoretic constraints implied by Coulomb branch geometry. Their systematic exploration at $r \to \infty$ is an intriguing and well-defined open problem in $\mathcal{N}=2$ SCFT. As a first step in that direction, one can use these constraints to rule out pure supergravity for a specific sequence of large rank SCFTs. In particular, consider a sequence of SCFTs of rank

$$r = \frac{1}{2}\prod_{i=1}^{N}(p_i - 1) \tag{29}$$

with $N \in \mathbb{N}$, where p_i is the ith prime number. These are the ranks for which $\Delta_{\max}$ is an available Coulomb branch generator dimension. Then one can show that, if such an r-tuple contains $\Delta_r = \Delta_{\max}$, it must also contain a dimension which grows slower than any power of c at $c \to \infty$: Specifically, $\Delta_r = \Delta_{\max} \Rightarrow \Delta_1 \ll c^{\varepsilon}\ \forall\, \varepsilon > 0$. This excludes a pure supergravity limit for this sequence of SCFTs by a wide margin. How representative this is of generic paths out to $c \to \infty$ in the space of $\mathcal{N}=2$ SCFTs touches on a subtle question in general abstract CFT. Regardless, this indicates the potential for a host of hidden Coulomb branch constraints in the generic case, perhaps strong enough to rule out pure supergravity or even any degree of Planckian scale separation [63].

Ultimately, we would like to understand more deeply the physical mechanism underlying our result, in hopes of extending beyond 4D $\mathcal{N}=2$ SCFTs [65]. Analytic bootstrap bounds with a bearing on the AdS scale-separation question may be more accessible than previously thought.

Acknowledgments—We thank Mario Martone, Leonardo Rastelli, and Ben Smith for helpful discussions and related collaboration, as well as the participants of the KITP conference "Gravity from algebra: Modern field theory methods for holography" where this work was first presented. This research was supported by ERC Starting Grant No. 853507 and, in part, by the National Science Foundation under Grant No. NSF PHY-1748958.

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[1] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, De Sitter vacua in string theory, Phys. Rev. D **68**, 046005 (2003).

[2] O. DeWolfe, A. Giryavets, S. Kachru, and W. Taylor, Type IIA moduli stabilization, J. High Energy Phys. 07 (2005) 066.

[3] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, Systematics of moduli stabilisation in Calabi-Yau flux compactifications, J. High Energy Phys. 03 (2005) 007.

[4] D. Lüst, E. Palti, and C. Vafa, AdS and the Swampland, Phys. Lett. B **797**, 134867 (2019).

[5] O. Aharony, Y. E. Antebi, and M. Berkooz, On the conformal field theory duals of type IIA AdS_4 flux compactifications, J. High Energy Phys. 02 (2008) 093.

[6] S. de Alwis, R. K. Gupta, F. Quevedo, and R. Valandro, On KKLT/CFT and LVS/CFT dualities, J. High Energy Phys. 07 (2015) 036.

[7] J. P. Conlon and F. Quevedo, Putting the boot into the Swampland, J. High Energy Phys. 03 (2019) 005.

[8] J. P. Conlon, S. Ning, and F. Revello, Exploring the holographic Swampland, J. High Energy Phys. 04 (2022) 117.

[9] F. Apers, M. Montero, T. Van Riet, and T. Wrase, Comments on classical AdS flux vacua with scale separation, J. High Energy Phys. 05 (2022) 167.

[10] F. Apers, J. P. Conlon, S. Ning, and F. Revello, Integer conformal dimensions for type IIa flux vacua, Phys. Rev. D **105**, 106029 (2022).

[11] F. Apers, Aspects of AdS flux vacua with integer conformal dimensions, J. High Energy Phys. 05 (2023) 040.

[12] T. Coudarchet, Hiding the extra dimensions: A review on scale separation in string theory, Phys. Rep. **1064**, 1 (2024).

[13] J. Polchinski and E. Silverstein, Dual purpose landscaping tools: Small extra dimensions in AdS/CFT, in *Strings, Gauge Fields, and the Geometry Behind: The Legacy of Maximilian Kreuzer*, edited by A. Rebhan, L. Katzarkov, J. Knapp, R. Rashkov, and E. Scheidegger (World Scientific, Singapore, 2009), pp. 365–390.

[14] L. F. Alday and E. Perlmutter, Growing extra dimensions in AdS/CFT, J. High Energy Phys. 08 (2019) 084.

[15] R. Gopakumar, E. Perlmutter, S. S. Pufu, and X. Yin, Snowmass white paper: Bootstrapping string theory, arXiv:2202.07163.

[16] L. F. Alday and S. M. Chester, Pure anti–de Sitter supergravity and the conformal bootstrap, Phys. Rev. Lett. **129**, 211601 (2022).

[17] M. Montero, M. Rocek, and C. Vafa, Pure supersymmetric AdS and the Swampland, J. High Energy Phys. 01 (2023) 094.

[18] C. Beem, M. Lemos, P. Liendo, L. Rastelli, and B. C. van Rees, The $\mathcal{N}=2$ superconformal bootstrap, J. High Energy Phys. 03 (2016) 183.

[19] M. Akhond, G. Arias-Tamargo, A. Mininno, H.-Y. Sun, Z. Sun, Y. Wang, and F. Xu, The hitchhiker's guide to 4d $\mathcal{N}=2$ superconformal field theories, SciPost Phys. Lect. Notes **64**, 1 (2022).

[20] F. A. Dolan and H. Osborn, On short and semi-short representations for four-dimensional superconformal symmetry, Ann. Phys. (Amsterdam) **307**, 41 (2003).

[21] C. Cordova, T. T. Dumitrescu, and K. Intriligator, Multiplets of superconformal symmetry in diverse dimensions, J. High Energy Phys. 03 (2019) 163.

[22] We thank Mario Martone and Leonardo Rastelli for discussions about the status of this assumption.

[23] P. C. Argyres, Y. Lü, and M. Martone, Seiberg-Witten geometries for Coulomb branch chiral rings which are not freely generated, J. High Energy Phys. 06 (2017) 144.

[24] P. C. Argyres and M. Martone, Coulomb branches with complex singularities, J. High Energy Phys. 06 (2018) 045.

[25] A. Bourget, A. Pini, and D. Rodriguez-Gomez, Gauge theories from principally extended disconnected gauge groups, Nucl. Phys. **B940**, 351 (2019).

[26] P. Argyres and M. Martone, Construction and classification of Coulomb branch geometries, arXiv:2003.04954.

[27] M. Martone, Towards the classification of rank-r $\mathcal{N}=2$ SCFTs. Part I. Twisted partition function and central charge formulae, J. High Energy Phys. 12 (2020) 021.

[28] A. D. Shapere and Y. Tachikawa, Central charges of $N=2$ superconformal field theories in four dimensions, J. High Energy Phys. 09 (2008) 109.

[29] The work [27] and its predecessors make their own assumptions, of course, one of which (free generation) was discussed earlier. Nevertheless, their results are expected to be very robust, sufficient to perform a quite general classification of $\mathcal{N}=2$ SCFTs. In indirect support of this, the rationality of central charges was confirmed by [30] using different, Higgs branch-based methods and employing independent assumptions.

[30] L. Rastelli and B. C. Rayhaun, Rationality in four dimensions, Phys. Rev. D **109**, 105018 (2024).

[31] D. M. Hofman and J. Maldacena, Conformal collider physics: Energy and charge correlations, J. High Energy Phys. 05 (2008) 012.

[32] P. C. Argyres and M. Martone, Scaling dimensions of Coulomb branch operators of 4d $N=2$ superconformal field theories, arXiv:1801.06554.

[33] M. Caorsi and S. Cecotti, Geometric classification of 4d $\mathcal{N}=2$ SCFTs, J. High Energy Phys. 07 (2018) 138.

[34] Throughout the Letter, we use $\sim$ to indicate asymptotic scaling up to numerical prefactors and $\approx$ to indicate asymptotic equality including numerical prefactors, with additive corrections left implicit in all cases.

[35] Let us register a complementary (but more restricted) argument that $a \approx c \sim r$ theories cannot be dual to semiclassical Einstein gravity specifically: With this scaling, the species scale $\Lambda_s = (c/n_L)^{1/3}$, where n_L is the number of bulk fields with energies below Λ_s, would become $\mathcal{O}(1)$, invalidating Einstein gravity as a good semiclassical EFT.

[36] The works [37,38] study putative 4D $\mathcal{N}=1$ SCFTs, labeled "dense" theories, which have $a \sim c \sim N$, where N is the rank of a UV gauge theory from which the IR SCFT flows. These theories have bands of operators within finite $\mathcal{O}(1)$ windows with $1/N$-suppressed splittings, which, therefore, become continuous in the large-N limit, violating the finiteness criterion (8).

[37] P. Agarwal and J. Song, Large N gauge theories with a dense spectrum and the weak gravity conjecture, J. High Energy Phys. 05 (2021) 124.

[38] P. Agarwal, K.-H. Lee, and J. Song, Classification of large N superconformal gauge theories with a dense spectrum, J. High Energy Phys. 10 (2021) 049.

[39] Equivalently, $\mathbb{N}(\Delta_-, \Delta_{\max}) \sim o(r)$ for any $\Delta_- \gg \Delta_*(z)$: There can be states with $\Delta \gg \Delta_*(z)$, but not too many of them, lest the scaling (18) be violated.

[40] For example, at higher energies $\Delta \gg E_{\rm BH}$, black hole states obey $\mathbb{N}(\Delta, \Delta+\delta) \sim e^{S(\Delta)}$ for $\delta \sim \mathcal{O}(1)$.

[41] J. B. Gutowski and H. S. Reall, Supersymmetric AdS_5 black holes, J. High Energy Phys. 02 (2004) 006.

[42] V. Balasubramanian, M. Berkooz, A. Naqvi, and M. J. Strassler, Giant gravitons in conformal field theory, J. High Energy Phys. 04 (2002) 034.

[43] P. Basu, J. Bhattacharya, S. Bhattacharyya, R. Loganayagam, S. Minwalla, and V. Umesh, Small hairy black holes in global AdS spacetime, J. High Energy Phys. 10 (2010) 045.

[44] S. Bhattacharyya, S. Minwalla, and K. Papadodimas, Small hairy black holes in $\mathrm{AdS}_5 x S^5$, J. High Energy Phys. 11 (2011) 035.

[45] J. Markeviciute and J. E. Santos, Hairy black holes in $\mathrm{AdS}_5\mathrm{S}^5$, J. High Energy Phys. 06 (2016) 096.

[46] S. Choi, S. Kim, E. Lee, S. Lee, and J. Park, Towards quantum black hole microstates, J. High Energy Phys. 11 (2023) 175.

[47] I. Bena and N. P. Warner, Resolving the structure of black holes: Philosophizing with a hammer, arXiv:1311.4538.

[48] I. Bah and P. Heidmann, Topological stars and black holes, Phys. Rev. Lett. **126**, 151101 (2021).

[49] S. Kim, S. Kundu, E. Lee, J. Lee, S. Minwalla, and C. Patel, Grey Galaxies' as an endpoint of the Kerr-AdS superradiant instability, J. High Energy Phys. 11 (2023) 024.

[50] I. Heemskerk, J. Penedones, J. Polchinski, and J. Sully, Holography from conformal field theory, J. High Energy Phys. 10 (2009) 079.

[51] D. Anselmi, D. Z. Freedman, M. T. Grisaru, and A. A. Johansen, Nonperturbative formulas for central functions of supersymmetric gauge theories, Nucl. Phys. **B526**, 543 (1998).

[52] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.061601, which includes Refs. [53–56], for a brief examination of the meaning of "pure (super)gravity" in higher dimensions.

[53] H. Maxfield and G. J. Turiaci, The path integral of 3D gravity near extremality; or, JT gravity with defects as a matrix integral, J. High Energy Phys. 01 (2021) 118.

[54] P. Kraus and F. Larsen, Microscopic black hole entropy in theories with higher derivatives, J. High Energy Phys. 09 (2005) 034.

[55] L. Eberhardt, Off-shell partition functions in 3d gravity, Commun. Math. Phys. **405**, 76 (2024).

[56] S. Collier, L. Eberhardt, and M. Zhang, Solving 3d gravity with Virasoro TQFT, SciPost Phys. **15**, 151 (2023).

[57] The threshold $c \sim r^{3/2}$ is also notable from a different perspective: It is the scaling for which $\delta c_r \coloneqq c_{r+1} - c_r \sim M_p$. The possible significance of this comment will be motivated elsewhere [58].

[58] E. Perlmutter, KITP seminar: What is N? On the Large Scale Structure of CFT Space (to be published).

[59] S. Hellerman, A universal inequality for CFT and quantum gravity, J. High Energy Phys. 08 (2011) 130.

[60] N. Afkhami-Jeddi, T. Hartman, and A. Tajdini, Fast conformal bootstrap and constraints on 3d gravity, J. High Energy Phys. 05 (2019) 087.

[61] T. Hartman, C. A. Keller, and B. Stoica, Universal spectrum of 2d conformal field theory in the large c limit, J. High Energy Phys. 09 (2014) 118.

[62] E. Perlmutter and B. Smith (to be published).

[63] We expect even more potential for progress if we combine Coulomb and Higgs branch physics. See Ref. [64] for an approach to classification of $\mathcal{N}=2$ SCFTs that does so to impressive effect. As noted there, as $r \to \infty$, the degree d of the modular differential equation governing the Higgs branch vertex operator algebra seems to scale at least linearly with the rank. Finding an emergent "effective" approach at large r and large d poses a difficult but interesting challenge.

[64] J. Kaidi, M. Martone, L. Rastelli, and M. Weaver, Needles in a haystack. An algorithmic approach to the classification of 4d $\mathcal{N}=2$ SCFTs, J. High Energy Phys. 03 (2022) 210.

[65] Two possibly fruitful avenues may be the study of protected sectors of 3D $\mathcal{N}=4$ SCFTs or of 4D $\mathcal{N}=1$ SCFTs obtained by RG flow from $\mathcal{N}=2$ SCFTs.