Catalytic Transformation from Computationally Universal to Strictly Universal Measurement-Based Quantum Computation

Yuki Takeuchi*

NTT Communication Science Laboratories, NTT Corporation, 3-1 Morinosato Wakamiya, Atsugi, Kanagawa 243-0198, Japan and NTT Research Center for Theoretical Quantum Information, NTT Corporation, 3-1 Morinosato Wakamiya, Atsugi, Kanagawa 243-0198, Japan

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There are two types of universality in measurement-based quantum computation (MBQC): *strict* and *computational*. It is well known that the former is stronger than the latter. We present a method of transforming from a certain type of computationally universal MBQC to a strictly universal one. Our method simply replaces a single qubit in a resource state with a Pauli-Y eigenstate. We applied our method to show that hypergraph states can be made strictly universal with only Pauli measurements, while only computationally universal hypergraph states were known.

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Quantum computers solve several problems faster than classical computers with the best known classical algorithms [1–3]. Driven by this advantage, tremendous effort has been devoted to developing quantum computers, and several quantum-computing models were proposed such as quantum circuit model [4], measurement-based quantum computation (MBQC) [5,6], and adiabatic quantum computation [7]. Although these models have unique features, they are the same in terms of computational capability (i.e., what problems can be solved in polynomial time). More concretely, these models can execute "any" quantum computation, hence are called universal quantum-computing models.

There are two types of universality in quantum computation [8]. One is *strict* universality, which is the strongest notion of universality. It means that any unitary operator can be implemented with an arbitrary high accuracy; hence, any quantum state can also be generated. However, a restricted class of unitary operators is sufficient to generate the output probability distribution of any quantum circuit with an arbitrary high accuracy [8,9]. Therefore, we can define a weaker notion of universality called *computational* universality. To clarify the difference between these two notions, let us consider the quantum circuit model with n initialized input qubits $|0^n\rangle$. In this model, the universality is determined by gate sets; $\{H, T, \Lambda(Z)\}$ and $\{H, \Lambda(S)\}$ are examples of strictly universal gate sets. Here, $H \equiv |+\rangle\langle 0| + |-\rangle\langle 1|$, where $|\pm\rangle \equiv (|0\rangle \pm |1\rangle)/\sqrt{2}$, is the Hadamard gate, $T \equiv |0\rangle\langle 0| + e^{i\pi/4}|1\rangle\langle 1|$ is the T gate, $\Lambda(U) \equiv |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U$ is the controlled-U gate for any single-qubit unitary operator U, I is the two-dimensional identity gate, $Z \equiv T^4$ is the Pauli-Z gate, and $S \equiv T^2$ is the S gate. Real unitary operators, however, are sufficient to construct the computationally universal gate set $\{H, CCZ\}$ [10], where $CCZ \equiv I^{\otimes 3} - 2|111\rangle\langle 111|$ is the controlled-controlled-Z (CCZ) gate. By definition, it is trivial that strictly universal gate sets are also computationally universal, but the opposite does not hold. The computationally universal gate set $\{H, CCZ\}$ is insufficient to generate complex quantum states, the amplitudes of which include imaginary numbers. For example, the n-qubit quantum state $|\psi_t\rangle = (|0^n\rangle + i|1^n\rangle)/\sqrt{2}$ cannot be generated with fidelity larger than $1/2$. This is because any quantum state generated by applying H and CCZ gates to $|0^n\rangle$ is written as a real quantum state $|\phi_r\rangle = \sum_{z\in\{0,1\}^n} c_z|z\rangle$ with real numbers $\{c_z\}_{z\in\{0,1\}^n}$ satisfying $\sum_{z\in\{0,1\}^n} c_z^2 = 1$; hence, $|\langle\psi_t|\phi_r\rangle|^2 = (c_{0^n}^2 + c_{1^n}^2)/2 \leq 1/2$.

The difference between complex and real quantum states becomes more apparent when we focus on multiparty quantum information processing. There exists a task conducted by three parties that can be achieved by using complex quantum states but cannot by using real ones [11], and their difference was already experimentally observed by using photons [12] and superconducting qubits [13]. Given the importance of complex quantum states, the resource theory of imaginarity has also been developed [14–17]. These results would indicate the necessity of strict universality.

As mentioned above, MBQC is a universal quantum-computing model [5]. It proceeds by adaptively measuring qubits of a resource state one by one. Its universality is determined by a given resource state (and available measurement bases). For both types of universality, several resource states were proposed. Cluster states [18], Affleck-Kennedy-Lieb-Tasaki (AKLT) states [19], and parity-phase graph states [20], which are weighted graph states [21,22], are common examples of strictly universal resource states. For computational universality, several

*Contact author: yuki.takeuchi@ntt.com

hypergraph states were found, which require only Pauli measurements [23–27]. As with the quantum circuit model, strictly universal resource states are trivially computationally universal, but the opposite is not true. Despite the importance of understanding the hardness of developing strictly universal quantum computers, the gap between these two types of universality has been less explored in MBQC.

In this Letter, we present a method of transforming to the strictly universal MBQC from any computationally universal one that can precisely implement H and CCZ. Our method is quite simple in that it simply replaces a qubit in a resource state of a computationally universal MBQC with a Pauli-Y eigenstate $|+i\rangle \equiv (|0\rangle + i|1\rangle)/\sqrt{2}$. Since the added $|+i\rangle$ works like a catalyst in chemistry, we call the transformation with our method catalytic transformation. In fact, $|+i\rangle$ achieves a strictly universal MBQC, while it is invariant during MBQC [see Fig. 3(b)]. An advantage of our method is that the required measurement bases are the same before and after the transformation. To devise our method, we first show that $|1\rangle$ can be deterministically prepared by applying H and CCZ gates to $|000\rangle$. We then show that S is deterministically applicable to any quantum state $|\psi\rangle$ by applying H and CCZ gates to $|1\rangle|+i\rangle|\psi\rangle$. As an important point, $|+i\rangle$ is not consumed when we implement the S gate; hence, it can be repeatedly used to apply multiple S gates. This approach can be considered as an example of the catalytic embeddings introduced in Ref. [28], i.e., a catalytic embedding of S over $\{H, CCZ\}$. Thus, our results would exhibit the usefulness of the catalytic embeddings for MBQC. Another example was given in Ref. [29], which generalizes the representation of complex numbers used in Ref. [8]. Our approach is also related to conversions with catalyst states [30]. In summary, by using our method, we achieve strictly universal MBQC that can execute any quantum computation composed of $\{H, S, CCZ\}$.

A weakness of our method is that the S gates cannot be applied in parallel because each S requires a single $|+i\rangle$, but the transformed resource state includes only a single $|+i\rangle$. Toward relaxing this weakness, we also propose a technique of duplicating $|+i\rangle$ in the Supplemental Material [31]. Its construction is inspired by the $|T\rangle$-catalyzed $|CCZ\rangle \to 2|T\rangle$ factory (C2T factory) [33], which is a technique in quantum error correction. Here, $|T\rangle \equiv T|+\rangle$ and $|CCZ\rangle \equiv CCZ(|+\rangle^{\otimes 3})$ are magic states [34] for the T and CCZ gates, respectively. The C2T factory outputs $|T\rangle^{\otimes 3}$ by applying Clifford gates to $|CCZ\rangle|T\rangle$. In keeping with the terminologies in previous studies [35,36], the third output qubit is called a catalyst. As an interesting point, the third output qubit can be used as an input of the next C2T factory; hence, we can reinterpret it as a method of generating $|T\rangle^{\otimes 2}$ from $|CCZ\rangle$. We propose a similar technique for the magic state $|+i\rangle$ of the S gate. We generate $|+i\rangle^{\otimes 2}$ by applying H and CCZ gates to $|1\rangle|0\rangle|+i\rangle$ and can similarly use the second output qubit as an input of the next duplication.

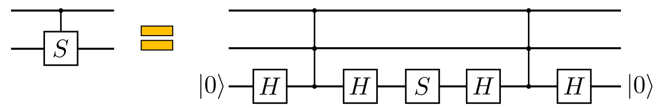


FIG. 1. Decomposition of $\Lambda(S)$ in terms of H, S, and CCZ gates. Single ancillary qubit $|0\rangle$ input into the right quantum circuit returns to $|0\rangle$ at the end of the quantum circuit.

To concretely reveal the usefulness of our transformation method, we apply it to the MBQC with the hypergraph state in Ref. [26]. The MBQC in Ref. [26] achieves computationally universal quantum computation with only Pauli-X and -Z basis measurements. In any MBQC with hypergraph states, $|+i\rangle$ can be prepared by a measurement in the Pauli-Y basis. Therefore, our transformation shows that there are strictly universal hypergraph states with measurements in the Pauli-X, -Y, and -Z bases. To the best of our knowledge, only computationally universal hypergraph states have been known for Pauli measurements.

Strictly universal quantum computation with $\{H, S, CCZ\}$—As a preliminary to our main results, we show that the gate set $\{H, S, CCZ\}$ is sufficient for strictly universal quantum computation. A set $\mathcal{G}$ of quantum gates is called strictly universal if there is a positive constant n_0 such that the subgroup of unitary operators generated by $\mathcal{G}$ is dense in the special unitary group $\mathrm{SU}(2^n)$ for any natural number $n \geq n_0$ [8]. Simply speaking, by combining quantum gates in a strictly universal gate set, any unitary operator can be constructed with an arbitrarily high accuracy. Kitaev showed $\{H, \Lambda(S)\}$ to be a strictly universal gate set with $n_0 = 2$ [37]. With this fact in mind, it is sufficient for our purpose to give a decomposition of $\Lambda(S)$ in terms of H, S, and CCZ gates. We give the decomposition in Fig. 1 (for the proof, see the Supplemental Material [31]).

Main results—Resource states for a MBQC consist of three sections: input section $\mathcal{C}_I$, body $\mathcal{C}_M$, and output section $\mathcal{C}_O$. This division was introduced for graph states [6], but we do not assume that the resource states are graph states. In fact, our argument holds even for undiscovered computationally universal states that may not be graph states. For any natural number n, let V_n be any n-qubit unitary operator composed of $\{H, CCZ\}$. In this Letter, we call the MBQC computationally universal if and only if for any $n \geq n_0$, V_n can be applied on $\mathcal{C}_O$ by measuring all qubits in $\mathcal{C}_I \cup \mathcal{C}_M$ one by one in appropriate bases.

Let $|\Psi_n\rangle$ be a computationally universal resource state with n input qubits $|\psi_{\mathrm{in}}(n)\rangle \equiv (\otimes_{i=1}^{n} U_{\mathrm{in}}^{(i)})|0^n\rangle$, where the single-qubit unitary operator $U_{\mathrm{in}}^{(i)}$ is I or H for each $1 \leq i \leq n$. Our purpose is to transform $|\Psi_n\rangle$ to a strictly universal resource state that deterministically implements any unitary operator composed of $\{H, S, CCZ\}$ (up to a byproduct). To this end, we first expand the size from n to $N = n + 2$ regardless of the number of the S gates to be applied. We then replace a single qubit in $\mathcal{C}_I$ of $|\Psi_N\rangle$ with $|+i\rangle$, as shown in Fig. 2 [38]. By using this transformed

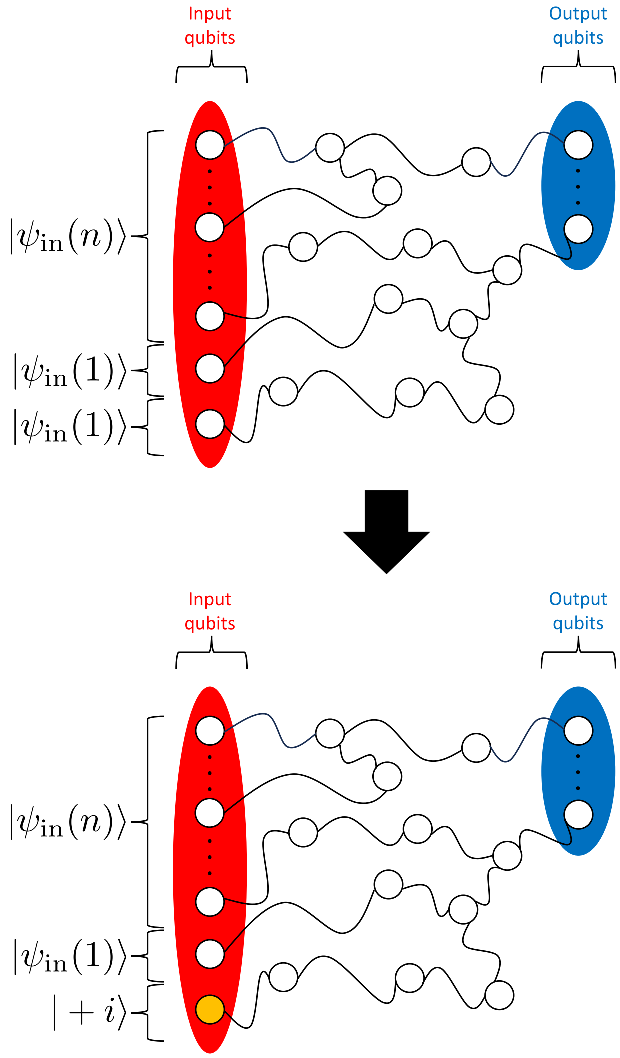
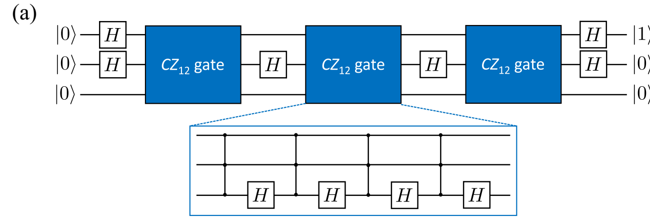


FIG. 2. Schematic of our transformation. Each circle represents a qubit. Red and blue ellipses represent $\mathcal{C}_I$ and $\mathcal{C}_O$, respectively. Other qubits (not covered by an ellipse) correspond to $\mathcal{C}_M$. By replacing the input qubit (i.e., $|\psi_{\rm in}(1)\rangle$) with $|+i\rangle$ in a given computationally universal resource state $|\Psi_N\rangle$, it becomes strictly universal. Another $|\psi_{\rm in}(1)\rangle$ will be used to prepare $|1\rangle$. Note that this transformation is applicable to any resource state that can precisely implement H and CCZ.



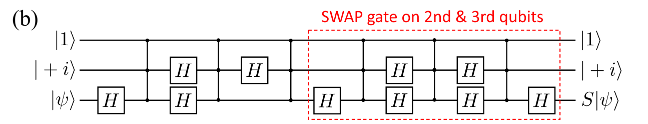


FIG. 3. Quantum circuits used to implement the S gate. It is important that necessary quantum gates are only H and CCZ gates. (a) Bit flip on $|0\rangle$ can be implemented using only H and CCZ gates. "CZ_{12} gate" means that the quantum circuit enclosed by a blue line is equivalent to $\Lambda(Z) \otimes I$. (b) S can be applied to any single-qubit state $|\psi\rangle$ by using $|1\rangle|+i\rangle$ as a catalyst. $|+i\rangle$ is given as an input qubit due to our transformation, and $|1\rangle$ is prepared with a quantum circuit in (a).

resource state, strictly universal quantum computation is executed on the first n input qubits $|\psi_{\rm in}(n)\rangle$ with the aid of the two ancillary qubits $|\psi_{\rm in}(1)\rangle|+i\rangle$.

The MBQC on the transformed resource state proceeds as follows. (1) Initialize $|\psi_{\rm in}(n)\rangle|\psi_{\rm in}(1)\rangle|+i\rangle$ to $|0^{n+1}\rangle|+i\rangle$. This can be accomplished without the added $|+i\rangle$ because the given resource state is computationally universal, and $|\psi_{\rm in}(n)\rangle|\psi_{\rm in}(1)\rangle$ is a tensor product of $|0\rangle$'s and/or $|+\rangle$'s. (2) Run the quantum circuit in Fig. 3(a) on the $(n+1)$th, nth, and $(n-1)$th input qubits $|000\rangle$. Therefore, the $(n+1)$th qubit becomes $|1\rangle$, which will be used to implement the S gate in step 3 (for the proof, see the Supplemental Material [31]). This step can also be implemented without the added $|+i\rangle$ because the given resource state is computationally universal. (3) Depending on which quantum gate we want to apply on the first n qubits, carry out one of the following procedures: (a) When H or CCZ is applied, conduct the corresponding measurements in the original computationally universal MBQC. (b) When S is applied, run the quantum circuit in Fig. 3(b) by using $(n+1)$th and $(n+2)$th input qubits $|1\rangle|+i\rangle$ (for the proof, see the Supplemental Material [31]). Note that $|1\rangle|+i\rangle$ is invariant in step 3; hence, the S gate can be applied at any time. In other words, $|1\rangle|+i\rangle$ can be used recursively. (4) Finally, a desired output state is generated on the first n qubits in $\mathcal{C}_O$ (up to a by-product). An advantage of our method is that the set of the required measurement bases does not change before and after our transformation. This is because we use only H and CCZ gates in the above procedures.

Application to MBQC with hypergraph states—Since the pattern of measurements depends on a resource state to which our catalytic transformation is applied, we cannot discuss its detail without specifying the resource state(, and hence we give our results by using quantum circuits in the previous section). In this section, to state our results in terms of MBQC rather than the quantum circuit model, we apply our transformation to a concrete hypergraph state. Hypergraph states are generalizations of graph states [40]. Let $G \equiv (V, E_2, E_3)$ be a triplet of the set V of m vertices, the set E_2 of edges connecting two vertices, and the set E_3 of hyperedges connecting three vertices. Note that hyperedges connecting more than three vertices are also generally allowed, but they are unnecessary in this section. An m-qubit hypergraph state $|G\rangle$ corresponding to the hypergraph G is defined as $(\prod_{(j,k,l)\in E_3} CCZ_{jkl})(\prod_{(j,k)\in E_2} \Lambda(Z)_{jk})|+\rangle^{\otimes m}$, where the subscript represents to which qubits the quantum gate is applied.

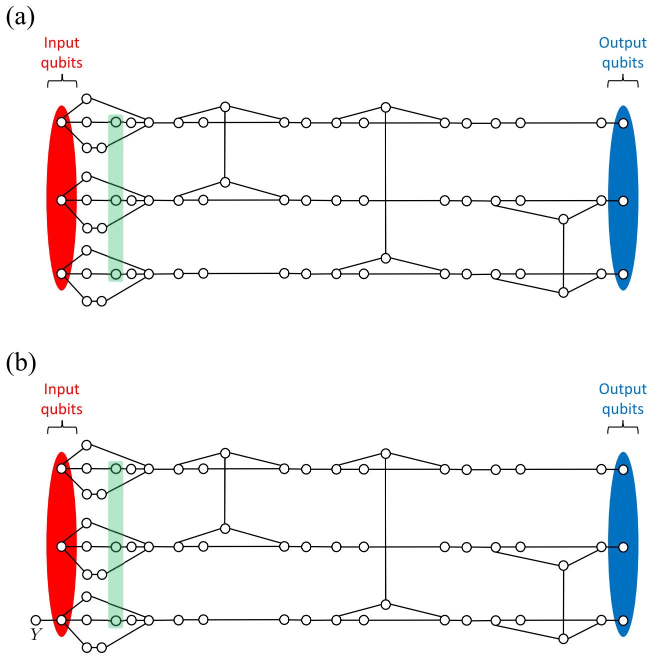


FIG. 4. Transformation of a computationally universal hypergraph state to a strictly universal one. Each vertex and edge represent $|+\rangle$ and the CZ gate, respectively. Each green rectangle represents CCZ. (a) The hypergraph state $|G_3^1\rangle$ in Ref. [26] with $n = 3$ input qubits and depth $d = 1$. Any 1-depth quantum circuit composed of $\{H, CCZ\}$ can be run on $|+\rangle^{\otimes 3}$ by measuring it in Pauli-X and -Z bases. By entangling a number of $|G_3^1\rangle$'s and $|+\rangle$'s using CZ gates, the computationally universal hypergraph state [26] is generated. (b) Transformed hypergraph state. Difference from (a) is that an additional single $|+\rangle$ is entangled with the third input qubit by using the CZ gate. By measuring it in the Pauli-Y basis, the third input qubit becomes $S|+\rangle = |+i\rangle$ (up to a by-product) due to gate teleportation.

The hypergraph state in Ref. [26] was shown to be computationally universal and prepared by applying the controlled-Z (CZ) gates to the $\Theta(n^4 d)$ $|+\rangle$'s and $\Theta(n^3 d)$ small hypergraph states shown in Fig. 4(a), where n and d are the number of input qubits and depth under the gate set $\{H, CCZ\}$, respectively. From our argument in the previous sections, the above hypergraph state [26] can be made strictly universal by replacing an input qubit with a single $|+i\rangle$. Such replacement can be accomplished by modifying one of the small hypergraph states such as that in Fig. 4(b). By measuring the added qubit in the Pauli-Y basis, the third input state becomes $|+i\rangle$ (up to a by-product) due to the gate teleportation. Since the original hypergraph state [26] requires only Pauli-X and -Z basis measurements for computational universality, the transformed hypergraph state achieves strict universality by using a single Pauli-Y basis measurement in addition to those Pauli measurements (see also the Supplemental Material [31]). Under the assumption that quantum computers are more powerful than classical computers, this Pauli universality cannot be obtained by using graph states because Pauli measurements on graph states can be efficiently simulated with a classical computer. Note that the combination of the hypergraph states in Ref. [26] and Pauli-X and -Z basis measurements cannot achieve the strict universality, i.e., cannot implement the S gate. This is because these states and measurements are real quantum states and operations, respectively. Therefore, our transformation surely makes a nonstrictly universal MBQC to a strictly universal one.

Our strictly universal hypergraph state should be useful for distributed quantum computation [41], which is quantum computation conducted on quantum internet [42]. It is a promising approach to realize a universal quantum computer because it only requires small or intermediate-scale quantum computers to build a large-scale quantum computer. To construct quantum internet, it would be important to convert from an entangled state to another one (e.g., from a graph state to the tensor product of Bell pairs [51]). Our hypergraph state can be used to prepare any entangled state (up to a by-product) by just performing Pauli measurements in each quantum computer. Furthermore, it is compatible with several quantum communication protocols such as quantum key distribution [52] and one-time programs [53] in the sense that they can also be implemented with Pauli measurements. In the Supplemental Material [31], to evaluate the practicality of this application, we discuss the required number of qubits.

Conclusion and discussion—We have presented a method of transforming from a computationally universal to strictly universal MBQC by simply replacing a single input qubit with a catalyst $|+i\rangle$ [54]. We believe our results will facilitate the discovery of novel strictly universal resource states. By applying our transformation to the hypergraph state [26], we have constructed a strictly universal hypergraph state. Our results in Fig. 4 should indicate that the gap between the computational and strict universalities is smaller than expected thus far. In fact, our constructed strictly universal hypergraph state has the same amount of magic (i.e., nonstabilizerness) with the computationally universal hypergraph state [26] when we quantify it by using the stabilizer rank [56]. It would be interesting to extensively explore the gap between computationally universal and strictly universal MBQCs with Pauli measurements from the viewpoint of magic (see, e.g., Ref. [57]).

In Ref. [20], the strict universality of weighted graph states with Pauli-X and -Z basis measurements was shown. Our hypergraph state also requires a Pauli-Y basis measurement to achieve strict universality. Although our results reduce the gap between them, weighted graph states are still slightly superior to hypergraph states. However, with respect to verifiability (i.e., how easily the fidelity between the ideal state and an actual state can be estimated), hypergraph states are conversely superior to weighted graph states. Although only Pauli-X and -Z basis measurements are sufficient for hypergraph states [58,59], non-Pauli measurements are required for weighted graph states [60]. It would be interesting to investigate differences between them, which are two different generalizations of graph states, more deeply. As another future work, it will be interesting to generalize our results to other nonstrictly universal gate sets.

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