g Theorem from Strong Subadditivity

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We show that strong subadditivity provides a simple derivation of the g theorem for the boundary renormalization group flow in two-dimensional conformal field theories. We work out its holographic interpretation and also give a derivation of the g theorem for the case of an interface in two-dimensional conformal field theories. We also geometrically confirm strong subadditivity for holographic duals of conformal field theories on manifolds with boundaries.

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Introduction—Strong subadditivity (SSA) [1,2]

$$S_{AB} + S_{BC} - S_{ABC} - S_B \geq 0, \qquad (1)$$

is a fundamental property which explains the nature of quantum information in the form of certain monotonicity relation, analogous to the second law of thermodynamics. For example, SSA shows that the conditional mutual information is non-negative. Here, we write the entanglement entropy for the subsystem A as S_A. To define S_A, we introduce the reduced density matrix ρ_A by tracing the density matrix for the whole system over the complement of the region A and then consider its von Neumann entropy $S_A = -\mathrm{Tr}\rho_A \log \rho_A$.

SSA also plays an important role in quantum field theories (QFTs) as it offers a universal property for the degrees of freedom under the renormalization group (RG) flow. Indeed we can derive the c theorem [3] in two-dimensional (2D) QFTs and the F theorem [4] in 3D QFTs from the SSA relation (1). The a theorem in 4D QFTs was shown via a more elaborate method in [5].

Let us briefly recount the entropic c theorem in the 2D case [3]. Consider the entanglement entropy S_A for an interval A. We write its Lorentz invariant length as $|A| = l$, and then the entropy becomes a function of l, which is expressed as $S_A(l)$. It is also useful to rewrite SSA (1) as

$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}, \qquad (2)$$

where we regard AB and BC in (1) as A and B, respectively. By taking advantage of the relativistic invariance of 2D QFT, we can choose the subsystems $A, B, A\cap B$, and $A \cup B$ as in Fig. 1. If we set $|A\cap B| = l_1$ and $|A\cup B| = l_2$, then we find $|A| = |B| = \sqrt{l_1 l_2}$. Thus, SSA (2) leads to the inequality $2S_A(\sqrt{l_1 l_2}) \geq S_A(l_1) + S_A(l_2)$, which implies that S_A is concave as a function of $\log l$:

$$\frac{d}{dl}\left[l \frac{dS_A(l)}{dl}\right] \leq 0. \qquad (3)$$

The entanglement entropy for 2D CFT vacua is known to take the form $S_A = (c/3)\log(l/\epsilon)$, where c is the central charge and ϵ is the UV cutoff [6]. Therefore, we can regard $C(l) = 3l[dS_A(l)/dl]$ as an effective central charge at the length scale l. In this way, the inequality (3) shows the c theorem, which states that the degrees of freedom monotonically decrease under the RG flow.

Even though the c theorem was originally derived using the more traditional field-theoretic method [7], the above SSA argument provides us with a much simpler derivation and shows that at its essence lies the monotonicity of quantum information.

The purpose of this Letter is to extend this beautiful and geometrical derivation of the important monotonicity of QFTs, using the entanglement entropy, to cases with

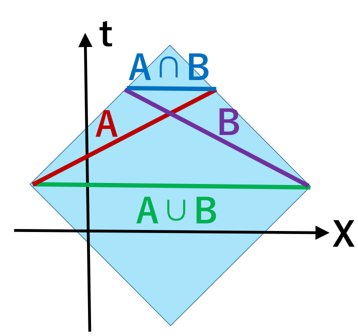

FIG. 1. The setup of deriving entropic c theorem.

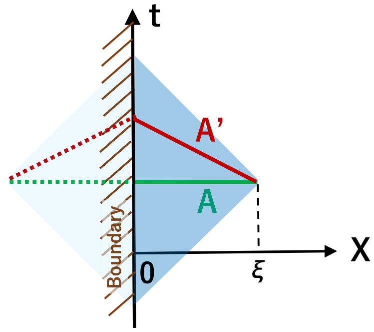

FIG. 2. Sketches of subsystem A and A' entanglement entropy in a 2D BCFT on a half plane $x > 0$. Since they have the same domain of dependence (blue region), we find $S_A = S_{A'}$.

boundaries or defects when their bulk theories are conformally invariant.

Entropic derivation of the g theorem for BCFTs—Consider a 2D CFT on a 2D Lorentzian flat spacetime, whose coordinates are denoted by (x, t) and put a timelike boundary at $x = 0$ by limiting the spacetime to the right half plane $x \geq 0$. When the boundary condition at $x = 0$ preserves a half of the bulk conformal invariance, this theory is called a boundary conformal field theory (BCFT) [8].

It is known that the entanglement entropy for an interval A which stretches from the boundary $x = 0$ to a point $x = \xi$ at any time $t = t_0$, takes the form [9]

$$S_A = \frac{c}{6}\log\frac{2\xi}{\epsilon} + \log g, \tag{4}$$

where ϵ is the UV cutoff and $\log g$ is called the boundary entropy.

Even if we deform the subsystem (called A') such that it ends on (ξ, t_0) and a boundary point $x = 0$ at a time $t_0 - \xi < t < t_0 + \xi$, which is within the domain of dependence of A (and its mirror), the entanglement entropy does not change, i.e., $S_A = S_{A'}$. See Fig. 2 for a sketch. This is true for any relativistic field theory with a boundary and is due to the complete reflection at the boundary.

Now, we break the conformal invariance at the boundary by a relevant boundary perturbation $\int dt\, O(t, x = 0)$. The basic property that the degrees of freedom at the boundary monotonically decrease under the boundary RG flow is known as the g theorem [10]. The g theorem argues that the boundary entropy $\log g$ in (4) as a function of length scale, so-called the g function, is monotonically decreasing under the boundary RG flow. This theorem was proved by examining the boundary RG flow in [11], by using a symmetry argument in [12], and by calculating the relative entropy in [13,14]. For higher dimensional versions of g functions, refer to [12,15–25].

Below, we would like to present another simpler derivation of the g theorem directly from SSA. Consider the Lorentzian setup of Fig. 3 and the implication of SSA:

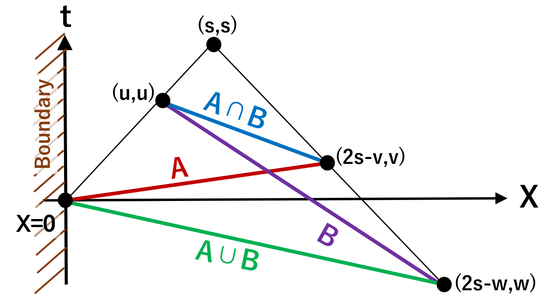

FIG. 3. The Lorentzian setup for the SSA in a 2D BCFT.

$$\Delta S \coloneqq S_A + S_B - S_{A\cup B} - S_{A\cap B} \geq 0. \tag{5}$$

We write the entanglement entropy S_A for an interval A as $S(x_1, t_1; x_2, t_2)$, whose end points are set to be $P_1\colon (x_1, t_1)$ and $P_2\colon (x_2, t_2)$. When P_1 is situated at the boundary $x_1 = 0$, then the entanglement entropy S_A only depends on x_2 as we already explained in Fig. 2, and we write this as $S_{\rm dis}(x)$.

Now we choose the subsystems such that each of the spacelike intervals $A, B, A\cup B$, and $A\cap B$ connects two points on the two null lays which intersect at the point (s, s) and such that they satisfy $|A||B| = |A\cup B||A\cap B|$, as illustrated in Fig. 3. Then, their entanglement entropies are described by

$$\begin{aligned} S_{A\cup B} &= S_{\rm dis}(2s - w), \qquad S_{A\cap B} = S(u, u; 2s - v, v),\\ S_A &= S_{\rm dis}(2s - v), \qquad S_B = S(u, u; 2s - w, w), \end{aligned} \tag{6}$$

where we assume $w < v < s$ and $s > 0$. Below, we appropriately choose the values of s, u, v, and w to obtain the tightest bound from SSA.

First, we take the limit $u \to s$, where B and $A\cap B$ become lightlike, which is equivalent to the zero width or equally the UV limit. We can understand this by regarding the two-point function of twist operators, which computes the entanglement entropy as a four-point function via the mirror method, which is factoring into a square of two-point functions of null-separated twist operators. Moreover, this claim is also obvious in the holographic dual of BCFTs [26–28], where the extremal surface dual to S_A is localized near the boundary.

Therefore, in this limit, we can approximate S_B and $S_{A\cap B}$ by their values in the CFT vacuum ignoring the presence of the boundary at $x = 0$:

$$\begin{aligned} S_B &\simeq \frac{c}{3}\log\frac{|B|}{\epsilon} = \frac{c}{6}\log\left[4(s-u)(s-w)/\epsilon^2\right],\\ S_{A\cap B} &\simeq \frac{c}{3}\log\frac{|A\cap B|}{\epsilon} = \frac{c}{6}\log\left[4(s-u)(s-v)/\epsilon^2\right]. \end{aligned} \tag{7}$$

Thus, ΔS defined in (5) is evaluated to be

$$\Delta S = S_{\rm dis}(2s - v) - S_{\rm dis}(2s - w) + \frac{c}{6}\log\frac{s-w}{s-v}. \quad (8)$$

Next, we take the value of v very close to w by setting $v = w + \delta$, where δ is an infinitesimally small and positive constant. Then (8) can be rewritten as

$$\Delta S = \delta\left(-\frac{dS_{\rm dis}(\xi)}{d\xi}\bigg|_{\xi=2s-w} + \frac{c}{6}\frac{1}{s-w}\right). \quad (9)$$

Finally, by assuming $w < 0$ and taking s to be very small such that $s \ll |w|$, we find that the SSA $\Delta S \geq 0$ gives the tightest bound:

$$\xi\frac{dS_{\rm dis}(\xi)}{d\xi} \leq \frac{c}{6}, \quad (10)$$

where $\xi \simeq -w > 0$ takes an arbitrary positive value.

Now we define the entropic g function $g(\xi)$ at the length scale x by

$$\log g(\xi) \coloneqq S_{\rm dis}(\xi) - \frac{c}{6}\log\frac{2\xi}{\epsilon}, \quad (11)$$

such that it gives the boundary entropy at each fixed point following the formula (4). Then SSA (10) leads to the inequality:

$$\frac{d}{d\xi}\log g(\xi) \leq 0. \quad (12)$$

This completes the derivation of the entropic g theorem.

Our proof has the crucial advantage that we only use the SSA property of entanglement entropy, which reveals the essential reason why the g theorem holds. The earlier quantum information theoretic proof [13,14] requires calculations of relative entropy between the IR and UV. In our approach, we only need the ground state wave function and its boosts. Therefore, it has tractable counterparts in lattice theories or quantum many-body systems. One more benefit is that since our formulation of the g function involves only entanglement entropy, it is directly connected to holography as we will see later.

Entropic g theorem for interface CFTs—Next, we extend our previous derivation of the g theorem to interfaces in 2D CFTs. Consider a 2D CFT with central charge c on the (x, t) plane and place an interface along $x = 0$ as depicted in Fig. 4. If the interface preserves half of the bulk conformal symmetry, a so-called interface CFT [29–31], then the entanglement entropy $S_{\rm int}(\xi)$ for the interval $-\xi \leq x \leq \xi$ at any time t_0 takes the form

$$S_{\rm int}(\xi) = \frac{c}{3}\log\frac{2\xi}{\epsilon} + \log g_I, \quad (13)$$

where the constant $\log g_I$ is the interface entropy. When we consider a relevant perturbation localized on the interface,

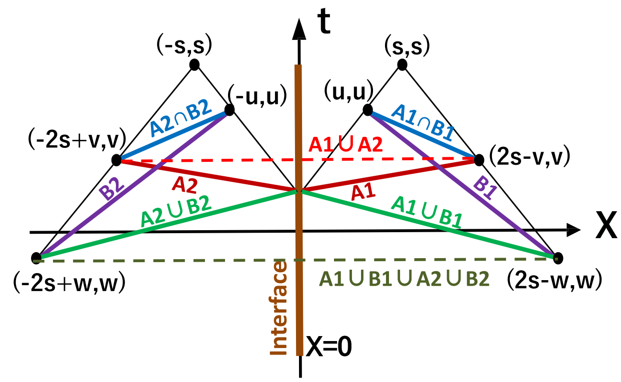

FIG. 4. The Lorentzian setup for the SSA in a 2D interface conformal field theory. The horizontal dotted intervals provide the g function.

the entropic g theorem for interface CFTs claims that the entropic g function

$$\log g_I(\xi) = S_{\rm int}(\xi) - \frac{c}{3}\log\frac{2\xi}{\epsilon}, \quad (14)$$

is monotonically decreasing as a function of ξ. Refer to [32] for an earlier attempt toward an entropic g theorem. The main improvement in this article from [32] is to apply SSA to a setup which is symmetric about the interface which allows us to give a complete derivation of g theorem. This is because the g function in the interface CFT becomes ambiguous if we allow asymmetric choices of the interval. Refer also to [33] for an interesting implication from SSA when one of the end points of the interval A is chosen to sit at the interface.

Our argument goes in parallel with our previous one in BCFTs. By doubling the setup of Fig. 2, we choose the subsystems depicted in Fig. 4. We have two copies of the boosted subsystems, each of which is identical to the ones $A, B, \ldots$ in Fig. 2, named as $A_1, B_1, \ldots$ and $A_2, B_2, \ldots$. Then we set $A = A_1 \cup A_2$ and $B = B_1 \cup B_2$ in the SSA relation (5). In the $u \to s$ limit, this inequality leads to

$$\Delta S = S_{\rm int}(2s - v) - S_{\rm int}(2s - w) + \frac{c}{3}\log\frac{s-w}{s-v}, \quad (15)$$

which is a straightforward extension of (8). As in the BCFT case, we further consider the limit $v \to w$ and $s \ll |w|$, and we finally obtain

$$\xi\frac{dS_{\rm int}(\xi)}{d\xi} \leq \frac{c}{3}, \quad (16)$$

which is equivalent to the g theorem $[d\log g_I(\xi)/d\xi] \leq 0$.

We would like to mention that the above analysis can be straightforwardly extended to the case where the CFT in the left and right half have different central charges c_1 and c_2. We just need to simply replace c with $(c_1 + c_2)/2$.

Holographic SSA and the null energy condition—The anti–de Sitter/conformal field theory (AdS/CFT) correspondence argues that gravity on a $d+1$ dimensional AdS spacetime is equivalent to a d dimensional CFT [34–36]. In AdS/CFT, we can calculate the entanglement entropy S_A in a geometrical way, known as the holographic entanglement entropy (HEE) [37–39]. It is computed from the area of an extremal surface Γ_A, denoted by $|\Gamma_A|$, which ends on the boundary of and is homologous to the subsystem A in AdS as

$$S_A = \frac{|\Gamma_A|}{4G_N}, \tag{17}$$

where G_N is the Newton constant in the AdS gravity. Interestingly, this HEE allows us to derive SSA in a more geometrical way [40,41], which essentially follows from the triangle inequality in classical geometry.

We can extend the AdS/CFT correspondence to the gravity dual of a CFT on a manifold with boundaries by introducing end-of-the-world (EOW) branes [26–28], called the AdS/BCFT correspondence. On the EOW brane, we impose the Neumann boundary condition

$$K_{ab} - Kh_{ab} = 8\pi G_N T^{(E)}_{ab}, \tag{18}$$

where h_{ab}, K_{ab}, and $T^{(E)}_{ab}$ are the induced metric, extrinsic curvature, and matter energy stress tensor on the EOW brane. The HEE in AdS/BCFT is again given by the formula (17) with an important addition that the extremal surface Γ_A can end on an EOW brane [26,27]. This can be viewed as a change in the homology constraint such that Γ_A is homologous to A *relative* to the EOW brane.

For a 2D CFT defined on a space with a boundary, its gravity dual is given by a region of 3D AdS (AdS_3) surrounded by an EOW brane. Assuming the pure gravity theory in the bulk, we can always choose the metric to be that of the pure AdS_3

$$ds^2 = \frac{dz^2 - dt^2 + dx^2}{z^2}. \tag{19}$$

We specify the profile of EOW brane by $z = z(x)$ such that $z(0) = 0$ as in Fig. 5, assuming that it is static. The gravity dual is given by the region $z < z(x)$. The cutoff ϵ of the z coordinate is identified with the UV cutoff ϵ of the dual CFT. When $z(x) \propto x$, the boundary preserves the conformal invariance, i.e., becomes a 2D BCFT. In general, the nontrivial profile of $z(x)$ encodes the detailed information of the boundary RG flow (see, e.g., [42] for an example).

Let us calculate the HEE by using this 3D holographic setup and compare it with our previous arguments for the g theorem. When the subsystem A is given by an interval which stretches from the boundary $x = 0$ to a point $x = \xi$ at any time, the HEE S_A is given by

$$S_{\rm dis}(\xi) = \frac{c}{6}|\Gamma_P|. \tag{20}$$

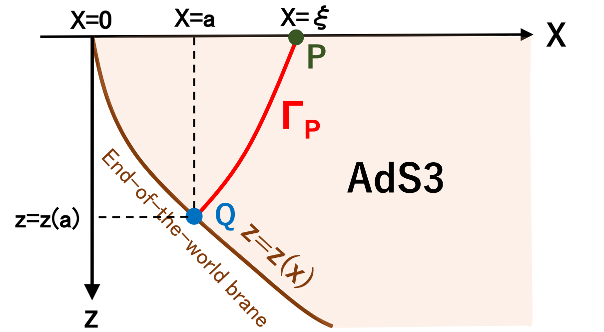

FIG. 5. The calculation of geodesic length in $AdS_3/BCFT_2$.

For this, let us calculate the length of the geodesic Γ_P, which connects between a given point P on the AdS boundary $(z,x) = (\epsilon, \xi)$ and a point Q on the EOW brane, described by $(z,x) = (z(a), a)$. The value of a is fixed by minimizing Γ_P as a function of a. Notice that since the EOW brane is static, Γ_P is on a constant time slice, leading to $S_A = S_{A'}$ in Fig. 2.

Since the geodesic Γ_P is orthogonal to the EOW brane at Q and is given by a part of a circle in (x,z) plane, we find the relation between ξ and a:

$$\xi = a - \frac{z(a)}{\dot{z}(a)} + z(a)\sqrt{1 + \frac{1}{\dot{z}(a)^2}}, \tag{21}$$

and the length of the geodesic is computed as

$$|\Gamma_P| = \log\left[\frac{2z(a)\sqrt{1+\dot{z}(a)^2}}{\epsilon(\sqrt{1+\dot{z}(a)^2}+1)}\right]. \tag{22}$$

For example, if we set $z(x) = \lambda x$, then we find

$$S_{\rm dis} = \frac{c}{6}|\Gamma_P| = \frac{c}{6}\log\frac{2\xi}{\epsilon} - \frac{c}{6}\log\left[\frac{1}{\lambda} + \sqrt{1+\frac{1}{\lambda^2}}\right], \tag{23}$$

which leads to the standard form of the entanglement entropy (4) in 2D BCFT.

For a generic profile $z = z(x)$, we obtain

$$\frac{6}{c}\xi\frac{\partial S_{\rm dis}(\xi)}{\partial \xi} - 1 = \frac{az'(a) - z(a)}{z(a)\sqrt{1+z'(a)^2}}. \tag{24}$$

The non-negativity of this quantity is equivalent to the SSA condition (10). Indeed, we can find that (24) is non-negative if we assume the null energy condition, i.e., $T^{(E)}_{ab}n^a n^b \geq 0$ for any null vector n^a in AdS_3. The null energy condition on the EOW brane leads to the condition $z''(x) \leq 0$ as shown in [26,27], where a holographic g theorem was derived. This allows us to guarantee $az'(a) - z(a) \leq 0$. This is found as follows: first, in the UV limit $a \to 0$, we expect the boundary to become conformal, which means $z(a) \propto a$, leading to

$az'(a) - z(a) = 0$ at $a = 0$. Moreover, the derivative $[az'(a) - z(a)]' = z''(a)$ is nonpositive due to the null energy condition. Thus, these manifestly show $az'(a) - z(a) \leq 0$. In this way, SSA in the setup of Fig. 1 precisely requires that the classical gravity satisfies the null energy condition in the gravity dual.

Holographic SSA in static backgrounds—In the above calculations of SSA, it was crucial that we considered the Lorentzian setup taking advantage of boost operations in relativistic QFTs. On the other hand, if we assume all subsystems $(A, \ldots)$ and the dual extremal surfaces $(\Gamma_A, \ldots)$ are on the same time slice $t = t_0$, then we can show that the HEE always satisfies SSA for any profile of the EOW branes at $t = t_0$ as we show below (see also [43,44] for earlier confirmation of SSA in particular examples). More generally, this claim can also be applied to a setup with a time reversal symmetry $(t - t_0) \to -(t - t_0)$. Note that this does not contradict the null energy condition because we can compensate for the arbitrary shape of the EOW brane at the specific time $t = t_0$ by choosing an appropriate time evolution of the EOW brane profile such that the null energy condition is maintained.

Since the essence of this argument does not depend on the dimension, we will continue to focus on the specific example of $AdS_3/BCFT_2$. Let A be an interval, whose end points are given by (x_1, t_1) and (x_2, t_2), its HEE S_A is computed as the minimum of the area of two configurations of surfaces:

$$S_A = \mathrm{Min}[S_{\mathrm{con}}(x_1, t_1; x_2, t_2), S_{\mathrm{dis}}(x_1) + S_{\mathrm{dis}}(x_2)], \quad (25)$$

where $S_{\mathrm{dis}}(x)$ is the HEE for the disconnected geodesic (20) and S_{con} is the HEE for the connected geodesic, given by

$$S_{\mathrm{con}}(x_1, t_1; x_2, t_2) = \frac{c}{6} \log [(x_2 - x_1)^2/\epsilon^2 - (t_2 - t_1)^2/\epsilon^2].$$

We take three subsystems A, B, and C on Σ. Recalling that the entanglement wedge preserves the order of inclusion, we have

$$E_B \subset E_{AB}, \qquad E_{BC} \subset E_{ABC}, \quad (26)$$

where E_A denotes the homology region which is the entanglement wedge of A projected onto Σ. This leads to the implication that the intersection $E_{AB} \cap E_{BC}$ is nonempty. Therefore, the extremal surface Γ_{AB} of subsystem AB can be divided into three parts:

$$\Gamma_{AB} = \Gamma_{AB}^{(1)} \cup \Gamma_{AB}^{(2)} \cup \gamma. \quad (27)$$

The first one denotes the part containing E_{BC}, i.e., $\Gamma_{AB}^{(1)} \coloneqq \Gamma_{AB} \cap E_{BC}$ and the second one is the outer part. The last one is the common part $\gamma \coloneqq \Gamma_{AB} \cap \Gamma_{BC}$. $\Gamma_{BC}^{(1)}$ and $\Gamma_{BC}^{(2)}$ are defined similarly (see Fig. 6). Furthermore, for subsystem B, Q_B denotes the parts of the EOW brane which enclose E_B. Namely, by the homology condition on the homology region, we have

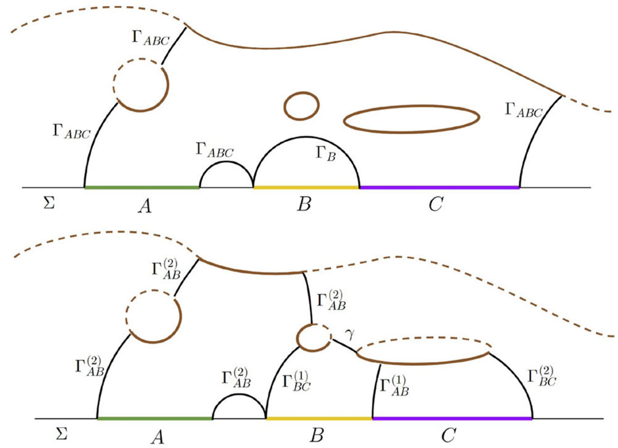

FIG. 6. An example of general EOW branes and extremal surfaces.

$$\partial E_B = B \cup \Gamma_B \cup Q_B. \quad (28)$$

We define E'_B as the intersection $E_{AB} \cap E_{BC}$, then E'_B satisfies the homology condition:

$$\partial E'_B = B \cup \left(\Gamma_{AB}^{(1)} \cup \Gamma_{BC}^{(1)} \cup \gamma\right) \cup (Q_{AB} \cap Q_{BC}). \quad (29)$$

Because of the extremality of Γ_B, we must have

$$|\Gamma_B| \leq |\Gamma_{AB}^{(1)}| + |\Gamma_{BC}^{(1)}| + |\gamma|. \quad (30)$$

Furthermore, $X'_{ABC} \coloneqq E_{AB} \cup E_{BC}$ also satisfies the homology condition

$$\partial E'_{ABC} = ABC \cup \left(\Gamma_{AB}^{(2)} \cup \Gamma_{BC}^{(2)} \cup \gamma\right) \cup (Q_{AB} \cup Q_{BC}), \quad (31)$$

thus we have

$$|\Gamma_{ABC}| \leq |\Gamma_{AB}^{(2)}| + |\Gamma_{BC}^{(2)}| + |\gamma|. \quad (32)$$

Finally, by adding the two inequalities (30) and (32), we have

$$|\Gamma_B| + |\Gamma_{ABC}| \leq \left(|\Gamma_{AB}^{(1)}| + |\Gamma_{AB}^{(2)}| + |\gamma|\right) \quad (33)$$

$$+ \left(|\Gamma_{BC}^{(1)}| + |\Gamma_{BC}^{(2)}| + |\gamma|\right) \quad (34)$$

$$= |\Gamma_{AB}| + |\Gamma_{BC}|. \quad (35)$$

Therefore, SSA on Σ holds.

It is also possible to show that the monogamy of mutual information [45] holds for the same setup of AdS/BCFT at a time slice as we explain in Supplemental Material [46].

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