Reversing Unknown Quantum Processes via Virtual Combs for Channels with Limited Information

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The inherent irreversibility of quantum dynamics for open systems poses a significant barrier to the inversion of unknown quantum processes. To tackle this challenge, we propose the framework of virtual combs that exploits the unknown process iteratively with additional classical postprocessing to simulate the process inverse. Notably, we demonstrate that an n-slot virtual comb can exactly reverse a depolarizing channel with one unknown noise parameter out of $n+1$ potential candidates, and a 1-slot virtual comb can exactly reverse an arbitrary pair of quantum channels. We further explore the approximate inversion of an unknown channel within a given channel set. A worst-case error decay of $\mathcal{O}(n^{-1})$ is unveiled for depolarizing channels within a specified noise region. Moreover, we show that virtual combs can universally reverse unitary operations and investigate the trade-off between the slot number and the sampling overhead.

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Introduction—Suppose a physical apparatus is provided that is guaranteed to perform some unknown process $\mathcal{N}$. It can be regarded as a black box with no more prior information. Is it possible to simulate the inverse of this process by employing this black box multiple times? For such a task of executing a desired transformation based on the given operations, the most comprehensive method entails using a quantum network [1]. Formally, the problem here is to construct a feasible quantum network $\mathcal{C}$, connected to the black box $\mathcal{N}$ for n times, to perform its inverse satisfying $\mathcal{C}(\mathcal{N}^{\otimes n})\circ\mathcal{N}$ be the identity channel, where such a quantum network is generally an n-slot quantum comb [1,2]. The significance of this task lies in revealing fundamental capabilities and properties of quantum operations [3,4], insights to quantum algorithm design [5–7], and applications to quantum error cancellation [8]. Understanding the power of quantum channels can shed further light on theoretical and applied quantum physics [9,10].

A simple strategy is to apply process tomography to obtain the full matrix representation, which is usually resourceful [11,12]. If the unknown process is restricted to unitary operations, numerous works have been carried out to explore efficient methods that can implement the inverse of any unknown unitary (see, e.g., [13–21]). Recently, deterministic and exact protocols for reversing any unknown unitary have been discovered for qubit case [22] and arbitrary dimensions [23], indicating full knowledge of the process through tomography is not necessary for this task. Nevertheless, how to extend such protocols to cases where the process is a general quantum channel remains an open question.

The challenge of reversing general unknown quantum processes is twofold. First, the inverse map of a quantum channel is generally not a physical process as it is not even positive. Such unphysical inverse maps fall under a broader scope of quantum operations, specifically Hermitian-preserving and trace-preserving linear maps. Second, even though we know that all such linear maps are simulatable via sampling quantum operations and postprocessing [24,25] or measurement-controlled postprocessing [26], implementing the inverse map via existing methods unavoidably requires the complete description of the quantum process.

In this Letter, to explore the full potential of reversing an unknown quantum process, we introduce the notion of *virtual combs* by lifting the positivity requirement on quantum combs. Physically, a virtual comb corresponds to sampling quantum combs with positive and negative coefficients and performing postprocessing. We find an affirmative answer that simulating the inverse of an unknown channel can be achieved with a virtual comb under certain conditions. Taking into account the unknown channel belonging to a given set without any prior information about its specific identity, we find that for two arbitrarily given quantum channels, the exact inverse could always be realized with a 1-slot virtual comb without knowing which specific channel is provided. For depolarizing channels, we unveil the remarkable capability of an n-slot virtual comb to exactly reverse a depolarizing channel with an unknown noise parameter among $n+1$ possible candidates (see Fig. 1). Intriguingly, we also establish a no-go theorem, elucidating the impossibility

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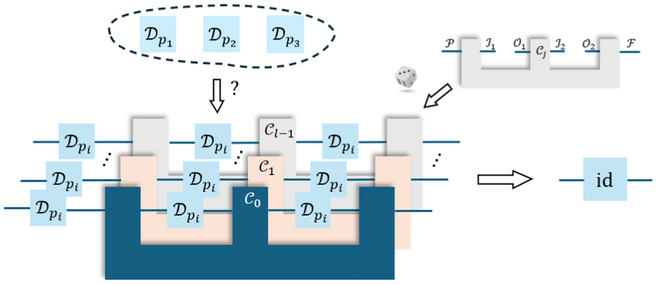

FIG. 1. Schematic diagram of reversing depolarizing channels with unknown parameters via a 2-slot virtual comb. A virtual comb is represented as a quasiprobabilistic mixture of quantum combs $\tilde{\mathcal{C}} = \sum_{j=0}^{l-1} \eta_j \mathcal{C}_j$ where $\mathcal{C}_j$ is a quantum comb. The systems of a quantum comb $\mathcal{C}_j$ are labeled as $\mathcal{P}$, $\mathcal{I}_i$, $\mathcal{O}_i$, $\mathcal{F}$. Given a depolarizing channel $\mathcal{D}_{p_i}$ with an unknown parameter p_i out of three distinct choices, $\tilde{\mathcal{C}}$ can exactly reverse $\mathcal{D}_{p_i}$ by $\tilde{\mathcal{C}}(\mathcal{D}_{p_i}^{\otimes 2}) \circ \mathcal{D}_{p_i} = \text{id}$ for $i = 1, 2, 3$.

of a virtual comb to universally reverse an arbitrary quantum channel with finite uses of the channel.

Beyond exact inversion, our investigation extends to approximately reversing unknown quantum channels through virtual combs. For depolarizing channels within an arbitrary noise region, we find a protocol with worst-case error decay of $\mathcal{O}(n^{-1})$ using n calls of the channel. Notably, it shows the potential application of virtual combs in error cancellation, where our protocol works for mitigating depolarizing noises without requiring prior knowledge of noise parameters. Furthermore, virtual combs are applied to reverse unknown unitary operations. We show that a 1-slot virtual comb suffices to reverse any d-dimensional unitary operation and explore its relationship with the previous unitary inversion problem. Our findings offer fresh perspectives on the interplay between information reversibility and irreversibility in quantum dynamics and provide new avenues for higher-order quantum transformations.

Exact channel inversion—Since the inverse of a quantum channel is not necessarily completely positive and a legitimate quantum comb must adhere to be completely positive [27], to explore the inversion task and fully explore the power of supermaps, we introduce the *virtual comb* as follows.

Definition 1: Virtual comb—Let $\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{l-1}$ be quantum combs. An affine combination of them $\tilde{\mathcal{C}} = \sum_{i=0}^{l-1} \eta_i \mathcal{C}_i$ is called a virtual comb where $\sum_{i=0}^{l-1} \eta_i = 1$, $\eta_i \in \mathbb{R}$, $\forall\ i$.

We remark that the name "virtual comb" is bestowed for two reasons: first, its functionality extends beyond that of a conventional comb; second, its virtual nature considering negative values of η_i is manifested through its feasibility, achieved by sampling its quasiprobability decomposition and subsequent postprocessing.

Now we first focus on a scenario where the quantum process is guaranteed to be within a set of depolarizing channels characterized by varying degrees of noise. We show that the exact inversion of all channels in this set can be achieved with explicit construction of the virtual comb. Furthermore, we establish a no-go theorem for this task, which highlights the limitations in process reversibility imposed by quantum mechanics.

Specifically, the unknown channel belongs to a family of d-dimensional depolarizing channels with m elements $\{\mathcal{D}_{p_1}, \mathcal{D}_{p_2}, \ldots \mathcal{D}_{p_m}\}$, where $\mathcal{D}_p(\cdot) = (1-p)(\cdot) + pI_d/d$ is a depolarizing channel with a noise parameter p. The task is to implement the inverse of an arbitrary channel $\mathcal{D}_{p_i}$ by querying the unknown channel n times. Based on this setting, we present our main result as follows.

Theorem 1: Depolarizing channel inversion—For any $n \geq 1$, let $\mathcal{D}_{p_1}, \ldots, \mathcal{D}_{p_{n+1}}$ be $n+1$ d-dimensional depolarizing channels with distinct noise parameters $p_1, \ldots, p_{n+1} \in [0, 1)$. There exists an n-slot virtual comb $\tilde{\mathcal{C}}$ satisfying

$$\tilde{\mathcal{C}}(\mathcal{D}_{p_i}^{\otimes n}) = \mathcal{D}_{p_i}^{-1}, \quad \forall\ i = 1, \ldots, n+1. \tag{1}$$

The main idea is to utilize the symmetry condition, whereby $\tilde{\mathcal{C}}(\mathcal{D}_{p_i}^{\otimes n})$ can be decomposed into a combination of the identity channel and the depolarizing channel. Based on this, we can formulate Eq. (1) into a linear system, and derive a solvability condition and the corresponding construction for the virtual comb. Detailed proofs of the theorems in this manuscript are deferred to [28]. Theorem 1 unveils an intrinsic application of the virtual comb framework, enabling the exact inversion of a family of depolarizing channels. Remarkably, the protocol applies to a set of noises, and the number of distinct channels within the set that it can exactly reverse increases with the number of slots.

To highlight the unique power of reversing a family of depolarizing channels with unknown noises provided by virtual combs, we note that such an exact channel inversion task cannot be accomplished via a quantum comb, even probabilistically. We defer the detailed statement and proof to [28].

Significantly, we also obtain a no-go theorem that no n-slot virtual comb can be universally capable of exactly reversing every set of $n+2$ channels. This is indicated by the fact that the theoretical maximum for an n-slot virtual comb to reverse a collection of depolarizing channels exactly is limited to $n+1$.

Theorem 2—For any $n \geq 1$, let $\mathcal{D}_{p_1}, \ldots, \mathcal{D}_{p_{n+2}}$ be $n+2$ d-dimensional depolarizing channels with distinct noise parameters $p_1, \ldots, p_{n+2} \in [0, 1)$. There is no n-slot virtual comb $\tilde{\mathcal{C}}$ such that $\tilde{\mathcal{C}}(\mathcal{D}_{p_i}^{\otimes n}) = \mathcal{D}_{p_i}^{-1}$, $\forall\ i = 1, \ldots, n+2$.

Theorem 2 exposes the inherent limit in reversing an unknown channel, affirming that virtual combs with finite slots cannot achieve the inversion of arbitrary unknown quantum channels.

Approximate channel inversion—Although Theorem 2 imposes restrictions on achieving the exact inversion for arbitrary quantum channels with a determined virtual comb, the approximate inversion is not prohibited. In approximate inversion, given a set of quantum channels $\Theta = \{(p_i, \mathcal{N}_i)\}_i$ where p_i is the prior probability for $\mathcal{N}_i$, we want to find an n-slot virtual comb that can make $\tilde{\mathcal{C}}(\mathcal{N}_i^{\otimes n})\circ\mathcal{N}_i$ as close to the identity channel "id" as possible. With an n-slot virtual comb $\tilde{\mathcal{C}}$, the average error for reversing the channel set Θ can be expressed as

$$e_{\mathrm{ave}}^n(\tilde{\mathcal{C}}, \Theta) = \frac{1}{2}\sum_{i=1}^{m} p_i \left\| \tilde{\mathcal{C}}(\mathcal{N}_i^{\otimes n})\circ\mathcal{N}_i - \mathrm{id} \right\|_\diamond,$$

where $\|\mathcal{F}\|_\diamond \coloneqq \sup_{k\in\mathbb{N}}\sup_{\|X\|_1\le 1}\|(\mathcal{F}\otimes \mathrm{id}_k)(X)\|_1$ denotes the diamond norm of a linear operator $\mathcal{F}$. The worst-case error is defined as

$$e_{\mathrm{wc}}^n(\tilde{\mathcal{C}}, \Theta) = \max\left\{ \frac{1}{2}\left\| \tilde{\mathcal{C}}(\mathcal{N}_i^{\otimes n})\circ\mathcal{N}_i - \mathrm{id} \right\|_\diamond : \mathcal{N}_i \in \Theta \right\}.$$

Note that for any two Hermitian-preserving and trace-preserving maps $\mathcal{N}_1$, $\mathcal{N}_2$ from system A to B, the completely bounded trace distance can be evaluated by semidefinite programming (SDP), which is a powerful tool in quantum information [29–31]. Then the optimal average error for approximately reversing quantum channels within the set Θ is determined via an SDP, the details of which are provided in [28].

Based on the numerical calculations of the SDP, we present intriguing results for reversing general quantum channels. The numerical calculations are implemented in MATLAB [32] with the interpreters CVX [33,34] and QETLAB [35]. In each experiment, we generate m random qubit-to-qubit quantum channels by the proposed measures in [36], e.g., generating random Choi operators, and calculate the average error of approximately reversing them (with equal prior probability) via 1-slot virtual combs by SDP. Then we apply the *computer-assisted proofs* given in Ref. [37] to construct a feasible solution for the virtual combs. It is worth noting that when $m \le 13$, we observe that the average errors across 1000 experimental iterations remain consistently below a tolerance of 1×10^{-5} whenever the channels are invertible. However, intriguingly, when $m \ge 14$, the average errors increase up to the first decimal place, indicating that no virtual comb could achieve near-exact inversion for all these channels. Therefore, we conjecture an upper limit of 13 elements in the channel set for reversing quantum channels using virtual combs. Notably, we present a theorem demonstrating that the exact inversion is always achievable for any pair of quantum channels.

Theorem 3: General channel inversion—For any two invertible quantum channels $\mathcal{N}_1$, $\mathcal{N}_2$, there exists a 1-slot virtual comb $\tilde{\mathcal{C}}$ satisfying $\tilde{\mathcal{C}}(\mathcal{N}_i) = \mathcal{N}_i^{-1},\ \forall\ i = 1, 2$.

Theorem 3 highlights the remarkable capability of a 1-slot virtual comb to reverse an *arbitrary* given pair of quantum channels, even when the input and output systems of the channels have different dimensions.

For depolarizing channels, we now consider that the noise levels are not a few fixed values but fall within a specified range $[p_1, p_2]$. As an n-slot virtual comb $\tilde{\mathcal{C}}$ could exactly reverse $n+1$ distinct noise level, using the unknown channel more times is surely to enhance performance. Here, we show that as the number of slots in the virtual comb (or the calls for the channel) increases, the worst-case error in channel inversion diminishes at least at a rate of $\mathcal{O}(n^{-1})$.

Theorem 4—Let $0 \le p_1 < p_2 \le 1$, the minimum worst-case error of approximately reversing a depolarizing channel $\mathcal{D}_p$ with $p \in [p_1, p_2]$ using an n-slot virtual comb is at most $\mathcal{O}(n^{-1})$.

This result can be simply understood as follows: by Theorem 1, we can exactly reverse a depolarizing channel whose noise parameter is from $\{p_1 + (p_2 - p_1)k/n\}_{k=0}^n$ via an n-slot virtual comb. Then, in a continuous case, we demonstrate that the worst-case error is at most $\mathcal{O}(n^{-1})$ within each interval. Based on this scheme, we present the upper bounds of the minimum worst-case error for the cases p_1, p_2, which are (0, 0.2), (0, 0.4), and (0, 0.6) in Fig. 2. A detailed explanation can be found in [28]. As the number of calls to the unknown channel increases, performance rapidly converges to exact for all these channels.

Application to error cancellation of unknown depolarizing noises—The task of reversing an unknown quantum channel is interlinked with quantum error cancellation. In quantum information processing, estimating the expectation value $\mathrm{Tr}[O\rho]$ for a given observable O and a quantum state ρ plays an essential role [38]. In practice, a state ρ is inevitably affected by noise that is modeled by

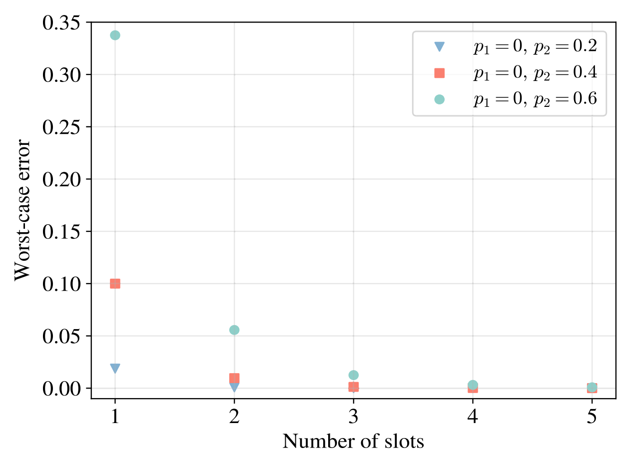

FIG. 2. Upper bounds on the minimum worst-case error for reversing a depolarizing channel with an unknown noise parameter $p \in [p_1, p_2]$. The x axis represents the number of slots in a virtual comb.

a quantum channel $\mathcal{N}$. Consequently, many methods have been proposed to recover $\mathrm{Tr}[O\rho]$ against noises rather than obtaining $\mathrm{Tr}[O\mathcal{N}(\rho)]$ [39–41].

One of the primary techniques employed is the *probabilistic error cancellation* [25,39] wherein the key idea is to represent the inverse map $\mathcal{N}^{-1}$ of the noisy channel as a quasiprobabilistic mixture of quantum channels. A crucial assumption in this protocol is that the noise is given *a priori*; otherwise, a high-precision tomography of the noise channel is required. In contrast, the scheme presented in Theorem 4 shows the potential to achieve high-precision error cancellation for certain unknown channels, e.g., depolarizing channels with unknown parameters within a given range.

In general, any n-slot quantum comb $\mathcal{C}$ can be equivalently realized by a sequence of quantum channels $\{\mathcal{E}_j\}_{j=1}^{n+1}$ with an ancillary system [1]. Thus, we can obtain a set of channels $\{\mathcal{E}_{ij}\}_j$ for each comb $\mathcal{C}_i$ in a decomposition of a virtual comb $\tilde{\mathcal{C}} = \sum_i \eta_i \mathcal{C}_i$. Given an unknown quantum channel oracle $\mathcal{N}$ and a noisy state $\mathcal{N}(\rho)$, if $\tilde{\mathcal{C}}(\mathcal{N}^{\otimes n}) = \mathcal{N}^{-1}$, then we can obtain $\mathrm{Tr}[O\rho] = \mathrm{Tr}[O\,\tilde{\mathcal{C}}(\mathcal{N}^{\otimes n})\circ\mathcal{N}(\rho)]$ by querying $\mathcal{N}$, sampling quantum channels for each $\mathcal{C}_i$ and applying classical postprocessing [25,41].

In each round out of S times sampling, we sample a sequence of quantum channels $\{\mathcal{E}_{sj}\}_{j=1}^{n+1}$ from $\{\mathcal{E}_{ij}\}_{ij}$ with probability $|\eta_s|/\gamma$, where $\gamma = \sum_i |\eta_i|$. Apply $\mathcal{E}_{s1}, \mathcal{N}, \mathcal{E}_{s2}, \ldots, \mathcal{N}, \mathcal{E}_{s,n+1}$ to the target state sequentially to obtain $\mathcal{E}_{s,n+1}\circ\mathcal{N}\circ\cdots\circ\mathcal{E}_{s2}\circ\mathcal{N}\circ\mathcal{E}_{s1}\circ\mathcal{N}(\rho) = \tilde{\mathcal{C}}_s(\mathcal{N}^{\otimes n})\circ\mathcal{N}(\rho)$, and then measure each qubit on a computational basis. We then denote λ_s as the measurement outcome and obtain a random variable $X^{(s)} = \gamma\, \mathrm{sgn}(\eta_s)\lambda_s \in [-\gamma, \gamma]$. After S rounds sampling, we calculate the empirical mean value $\zeta \coloneqq (1/S)\sum_{s=1}^{S} X^{(s)}$ as an estimation for the expectation value $\mathrm{Tr}[O\,\tilde{\mathcal{C}}(\mathcal{N}^{\otimes n})\circ\mathcal{N}(\rho)]$. By Hoeffding's inequality [42], to estimate the expectation value within error ϵ with probability no less than $1-\delta$, the number of samples required to be $S \geq 2\gamma^2 \log(2/\delta)/\epsilon^2$. Hence, γ is known as the sampling overhead. We note that the optimal sampling overhead can be calculated by SDP as given in [28].

In particular, if we aim to cancel the effect of an unknown depolarizing noise $\mathcal{D}_p$ from a set of distinct noise parameters as described in Theorem 1, we have a detailed protocol provided in [28], where we do not need to implement quantum combs and instead relies on three types of simple operations: (i) do nothing to the received state, (ii) replace the received state with a maximally mixed state, and (iii) apply the black box to the received state iteratively for i times.

Application to universal unitary inversion—Now we investigate a particular scenario where the quantum process is known to be a unitary operation. Previously, several works have studied the problem of reversing unknown unitary operations, including deterministic nonexact protocol [17,18] and probabilistic exact protocols [19–21]. Notably, it is proved that the inverse operation U^{-1} cannot be implemented deterministically and exactly with a single use of U [17]. Recently, deterministic and exact protocols were proposed, requiring four calls of U in a qubit case [22], and $\mathcal{O}(d^2)$ calls for a general d-dimensional U [23]. In a virtual setting, it is interesting to ask whether a 1-slot virtual comb is enough for reversing an arbitrary unknown d-dimensional unitary channel $\mathcal{U}_d(\cdot) = U_d(\cdot)U_d^\dagger$ or not. Here, we find the answer is positive as the following result.

Proposition 5—For any dimension d, there exists a 1-slot virtual comb $\tilde{\mathcal{C}}$ that transforms all qudit-unitary channels $\mathcal{U}_d$ into their inverses $\mathcal{U}_d^{-1}$, i.e., $\tilde{\mathcal{C}}(\mathcal{U}_d)(\cdot) = U_d^\dagger(\cdot)U_d$.

Proposition 5 reveals that with a virtual comb, a deterministic and exact protocol for any dimensions can be achieved with just one call of the unitary. This result gives an alternative way to simulate the inverse of unknown unitary in practice with shallower circuits for estimating expectation values. We point out that when there exists depolarizing noise, i.e., the given channel is $\mathcal{U}_d\circ\mathcal{D}_p$, the 1-slot virtual comb will result in an overall operation as $\mathcal{U}_d^{-1}\circ\mathcal{D}_p$ and the probabilistic error cancellation could be used to mitigate this error. For the deterministic protocol, the circuit is generally not transversal [22,23], thus the depolarizing noise will accumulate and become difficult to handle.

Furthermore, we analyze the query complexity of a virtual protocol, specifically the number of times U needs to be queried to obtain the expectation value $\mathrm{Tr}[O\mathcal{U}_d^{-1}(\rho)]$. The optimal sampling overhead for an n-slot virtual comb that can exactly reverse all d-dimensional unitaries can be characterized via the following SDP:

$$\nu(d,n) = \min 2\eta + 1, \tag{2a}$$

$$\text{s.t. } \mathrm{Tr}[\tilde{C}\Omega] = 1, \tag{2b}$$

$$\tilde{\mathcal{C}} = (1+\eta)\mathcal{C}_0 - \eta\mathcal{C}_1, \quad \eta \geq 0, \tag{2c}$$

$$\mathcal{C}_0, \mathcal{C}_1 \quad \text{are } n\text{-slot quantum combs}, \tag{2d}$$

where Eq. (2b) ensures that $\tilde{\mathcal{C}}$ is a desired map that can exactly reverse an arbitrary unitary operation and Ω is a $d^{2(n+1)} \times d^{2(n+1)}$ positive matrix called the performance operator [17,43]. The detailed formula and numerical results on the sampling overhead for small d and n are provided in [28].

Notably, we find that the optimal sampling overhead for the virtual comb that can exactly reverse unknown unitary operations has a dual relationship with the problem of finding the optimal average fidelity of reversing unknown unitary operations by a quantum comb [43] as the following theorem.

Theorem 6—The optimal sampling overhead for the n-slot virtual comb that can exactly reverse all d-dimensional unitary operations satisfies

$$\nu(d,n) = \frac{2}{F_{\mathrm{opt}}(d,n)} - 1, \quad \forall\ d \geq 2, \quad n \geq 1, \quad (3)$$

where $F_{\mathrm{opt}}(d,n)$ is the optimal average channel fidelity of reversing all d-dimensional unitary operations with an n-slot quantum comb.

Here, the optimal average channel fidelity is given by $F_{\mathrm{opt}}(d,n) = \max \mathrm{Tr}[C\Omega]$ where Ω is the performance operator as appeared in Eq. (2b) and the maximization ranges over all n-slot quantum combs with Choi operators C [43]. When $n=1$, it has been shown that $F_{\mathrm{opt}}(d,1) = 2/d^2$ [17], leading to $\nu(d,1) = d^2 - 1$. Although the sufficient querying number of the unitary is governed by $\nu^2(d,1)$, scaling as $\mathcal{O}(d^4)$, worse than $\mathcal{O}(d^2)$ required by the deterministic and exact protocol [23], it is worthwhile to note that when the state or observable is given, the query complexity could be significantly reduced. Specifically, we find that to estimate the expectation value $\mathrm{Tr}[Z\mathcal{U}_d^{-1}(|0\rangle\langle 0|)]$, the 1-slot virtual protocol has a better performance in both average simulation error and standard deviation under the same number of queries of the unknown unitary. The details of the numerical analysis to show this potential advantage are provided in [28].

Concluding remarks—In this Letter, we addressed the problem of reversing an unknown quantum process by introducing the *virtual comb*. Our theoretical analysis demonstrated its ability and shows its potential to help us further understand the properties and capabilities of channels, combs, and virtual processes. One may already notice that a qubit channel can be determined by 12 parameters, which coincides with our numerical result that if the number of random qubit channels exceeds 13, no perfect 1-slot virtual inversion protocol could be found. In terms of applications, the examples we provided suggest that the virtual combs may potentially become an alternative solution in specific experimental settings. It might offer trade-offs in terms of query complexity, circuit depth, and the number of auxiliary qubits; therefore, it is intriguing to conduct further analysis and construct concrete circuits for specific experimental scenarios.

The virtual combs may also shed light on other research directions for unknown processes, particularly in quantum learning. By transmitting quantum states through an unknown process, we can infer its characteristics and replicate it or execute related tasks. Such studies have been done for learning unitary gates [19,44,45], measurements [46,47], and Pauli noises [48]. How to further extend this setting to learning and using unknown channels remains open. Moreover, virtual combs may also be useful in transforming Hamiltonian dynamics [49,50], shadow tomography [38], virtual resource manipulation [51], and randomized quantum algorithms [52,53] for its unique attributes regarding quantum memory effect, sampling, and classical postprocessing.

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[1] G. Chiribella, G. M. D'Ariano, and P. Perinotti, Europhys. Lett. **83**, 30004 (2008).
[2] G. Chiribella, G. M. D'Ariano, and P. Perinotti, Phys. Rev. Lett. **101**, 060401 (2008).
[3] F. Caruso, V. Giovannetti, C. Lupo, and S. Mancini, Rev. Mod. Phys. **86**, 1203 (2014).
[4] A. S. Holevo, *Quantum Systems, Channels, Information: A Mathematical Introduction* (Walter de Gruyter, Berlin, Boston, 2013), Vol. 16.
[5] A. Gilyén, Y. Su, G. H. Low, and N. Wiebe, in *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing* (ACM, New York, NY, USA, 2019), pp. 193–204.
[6] J. M. Martyn, Z. M. Rossi, A. K. Tan, and I. L. Chuang, PRX Quantum **2**, 040203 (2021).
[7] Y. Wang, L. Zhang, Z. Yu, and X. Wang, Phys. Rev. A **108**, 062413 (2023).
[8] Z. Cai, R. Babbush, S. C. Benjamin, S. Endo, W. J. Huggins, Y. Li, J. R. McClean, and T. E. O'Brien, Rev. Mod. Phys. **95**, 045005 (2023).
[9] M. M. Wilde, *Quantum Information Theory* (Cambridge University Press, Cambridge, England, 2017).
[10] M. Hayashi, *Quantum Information Theory*, Graduate Texts in Physics (Springer Berlin Heidelberg, Berlin, Heidelberg, 2017).
[11] I. L. Chuang and M. A. Nielsen, J. Mod. Opt. **44**, 2455 (1997).
[12] J. Haah, R. Kothari, R. O'Donnell, and E. Tang, arXiv: 2302.14066.
[13] D. Trillo, B. Dive, and M. Navascués, Quantum **4**, 374 (2020).
[14] M. Navascués, Phys. Rev. X **8**, 031008 (2018).
[15] D. Trillo, B. Dive, and M. Navascués, Phys. Rev. Lett. **130**, 110201 (2023).
[16] P. Schiansky, T. Strömberg, D. Trillo, V. Saggio, B. Dive, M. Navascués, and P. Walther, Optica **10**, 200 (2023).
[17] G. Chiribella and D. Ebler, New J. Phys. **18**, 093053 (2016).

[18] I. S. B. Sardharwalla, T. S. Cubitt, A. W. Harrow, and N. Linden, arXiv:1602.07963.
[19] M. Sedlák, A. Bisio, and M. Ziman, Phys. Rev. Lett. **122**, 170502 (2019).
[20] M. T. Quintino, Q. Dong, A. Shimbo, A. Soeda, and M. Murao, Phys. Rev. A **100**, 062339 (2019).
[21] M. T. Quintino, Q. Dong, A. Shimbo, A. Soeda, and M. Murao, Phys. Rev. Lett. **123**, 210502 (2019).
[22] S. Yoshida, A. Soeda, and M. Murao, Phys. Rev. Lett. **131**, 120602 (2023).
[23] Y.-A. Chen, Y. Mo, Y. Liu, L. Zhang, and X. Wang, arXiv: 2403.04704.
[24] F. Buscemi, M. Dall'Arno, M. Ozawa, and V. Vedral, arXiv: 1312.4240.
[25] J. Jiang, K. Wang, and X. Wang, Quantum **5**, 600 (2021).
[26] X. Zhao, L. Zhang, B. Zhao, and X. Wang, arXiv:2309.09963.
[27] M.-D. Choi, Linear Algebra Appl. **10**, 285 (1975).
[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.030801 for detail discussions.
[29] J. Watrous, arXiv:0901.4709.
[30] X. Wang, Ph.D. thesis, University of Technology Sydney, 2018.
[31] P. Skrzypczyk and D. Cavalcanti, arXiv:2306.11637.
[32] T. M. Inc., MATLAB version: 9.13.0 (r2022b) (2022).
[33] M. Grant and S. Boyd, CVX: MATLAB software for disciplined convex programming, version 2.1, http://cvxr.com/cvx (2014).
[34] M. Grant and S. Boyd, in Recent Advances in Learning and Control, *Lecture Notes in Control and Information Sciences*, edited by V. Blondel, S. Boyd, and H. Kimura (Springer-Verlag Limited, Berlin, 2008), pp. 95–110.
[35] N. Johnston, QETLAB: A MATLAB toolbox for quantum entanglement, version 0.9, https://qetlab.com (2016).
[36] R. Kukulski, I. Nechita, L. Pawela, Z. Puchala, and K. Zyczkowski, J. Math. Phys. (N.Y.) **62**, 062201 (2021).
[37] J. Bavaresco, M. Murao, and M. T. Quintino, Phys. Rev. Lett. **127**, 200504 (2021).
[38] S. Aaronson, in *Proceedings of the 50th Annual ACM SIGACT Symposium on Theory of Computing* (Association for Computing Machinery, New York, 2018), pp. 325–338.
[39] K. Temme, S. Bravyi, and J. M. Gambetta, Phys. Rev. Lett. **119**, 180509 (2017).
[40] S. Endo, S. C. Benjamin, and Y. Li, Phys. Rev. X **8**, 031027 (2018).
[41] R. Takagi, Phys. Rev. Res. **3**, 033178 (2021).
[42] W. Hoeffding, *The Collected Works of Wassily Hoeffding* (Springer Science & Business Media, Springer New York, 2012).
[43] M. T. Quintino and D. Ebler, Quantum **6**, 679 (2022).
[44] A. Bisio, G. Chiribella, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Rev. A **81**, 032324 (2010).
[45] Y. Mo and G. Chiribella, New J. Phys. **21**, 113003 (2019).
[46] M. Sedlák and M. Ziman, Phys. Rev. A **90**, 052312 (2014).
[47] H.-C. Cheng, M.-H. Hsieh, and P.-C. Yeh, Quantum Inf. Comput. **16**, 0615 (2016).
[48] Y. Chen, Z. Yu, C. Zhu, and X. Wang, arXiv:2305.04148.
[49] T. Odake, H. Kristjánsson, A. Soeda, and M. Murao, Phys. Rev. Res. **6**, L012063 (2024).
[50] T. Odake, H. Kristjánsson, P. Taranto, and M. Murao, arXiv:2312.08848.
[51] X. Yuan, B. Regula, R. Takagi, and M. Gu, Phys. Rev. Lett. **132**, 050203 (2024).
[52] H.-Y. Huang, R. Kueng, and J. Preskill, Nat. Phys. **16**, 1050 (2020).
[53] K. Wan, M. Berta, and E. T. Campbell, Phys. Rev. Lett. **129**, 030503 (2022).