Primordial Black Holes from Null Energy Condition Violation during Inflation

Yong Cai,[1,*] Mian Zhu,[2,†] and Yun-Song Piao[3,4,5,6,‡]

[1]*Institute for Astrophysics, School of Physics, Zhengzhou University, Zhengzhou 450001, China*
[2]*Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, 30-348 Krakow, Poland*
[3]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*
[4]*International Centre for Theoretical Physics Asia-Pacific, University of Chinese Academy of Sciences, 100190 Beijing, China*
[5]*School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*
[6]*Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China*

(Received 24 May 2023; revised 29 April 2024; accepted 5 June 2024; published 8 July 2024)

Primordial black holes (PBHs) and the violation of the null energy condition (NEC) have significant implications for our understanding of the very early Universe. We present a novel approach to generate PBHs via the NEC violation in a single-field inflationary scenario. In our scenario, the Universe transitions from a first slow-roll inflation stage with a Hubble parameter $H = H_{\text{inf}\,1}$ to a second slow-roll inflation stage with $H = H_{\text{inf}\,2} \gg H_{\text{inf}\,1}$, passing through an intermediate stage of NEC violation. The NEC violation naturally enhances the primordial scalar power spectrum at a certain wavelength, leading to the production of PBHs with masses and abundances of observational interest. We also investigate the phenomenological signatures of scalar-induced gravitational waves resulting from the enhanced density perturbations. Our work highlights the potential of utilizing a combination of PBHs, scalar-induced gravitational waves, and primordial gravitational waves as a valuable probe for studying NEC violation during inflation, opening up new avenues for exploring the early Universe.

DOI: 10.1103/PhysRevLett.133.021001

Introduction.—Primordial black holes (PBHs) are powerful probes for studying the physics of the early Universe [1–3]. In contrast to the astrophysical black holes, which evolve from massive stars and contain masses larger than $5M_\odot$ [4], PBHs can have a wide mass range from tens of micrograms to millions of solar masses. In view of that, PBHs can be relevant to astrophysical and cosmological phenomena such as the origin of dark matter [5–8], and the seeds of the supermassive black holes [9,10]. Therefore, the formation of PBHs in the early Universe is widely studied, see, e.g., [11–44].

In the literature, PBHs are thought to arise from overdense regions collapsing due to self-gravity. Inflation, as the most widely accepted paradigm of the very early Universe, is capable of generating primordial scalar (or density) perturbations that are consistent with observations of the cosmic microwave background (CMB). Therefore, the key to PBH formation in inflationary cosmology is to obtain a significant extra enhancement of the amplitudes of primordial scalar perturbations on small scales (see, e.g., [45–57]), while simultaneously satisfying the observational constraints on the CMB scale.

In the single-field slow-roll inflation scenario, the power spectrum of primordial scalar perturbations is dependent on the Hubble parameter H, the slow-roll parameter $\epsilon \equiv -\dot{H}/H^2$ (or its generalized formulation), and the sound speed c_s of the scalar perturbation mode. In standard single-field slow-roll inflation, the scalar power spectrum is nearly scale-invariant on all scales. To produce sizable PBHs in single-field inflation models, various mechanisms have been investigated, including (but not limited to) ultra-slow-roll (USR) inflation [58–68], and modifications to the dispersion relation or the sound speed c_s of scalar perturbations [69–72] (see also [73–77] for the mechanism of parametric resonance). In addition to these models, many inflationary models exhibit sudden strong enhancement of the power spectrum on certain scales when slow-roll conditions are violated at some stages, such as the Starobinsky model when there is a nonsmooth potential [78,79], which not only has the capacity to generate a significant amount of PBHs but may also produce a large gravitational wave (GW) background; see, e.g., [80,81].

The violation of the null energy condition (NEC), or more precisely, the null congruence condition in modified gravity, is closely related to potential solutions for the singularity problem in the context of the big bang and inflationary cosmology [82]. It may play a crucial role in the very early Universe. Fully stable NEC violation can be achieved in "beyond Horndeski" theories [83–88]. In this Letter, we propose a new approach to generate PBHs in a single-field inflation scenario by enhancing the curvature perturbations through intermediate NEC violation.

In this scenario, the Universe transits from a first stage of slow-roll inflation with a Hubble parameter $H = H_{\text{inf}\,1}$, to a second stage of slow-roll inflation with $H = H_{\text{inf}2} \gg H_{\text{inf}1}$, through an intermediate NEC violation stage (see Fig. 1 for

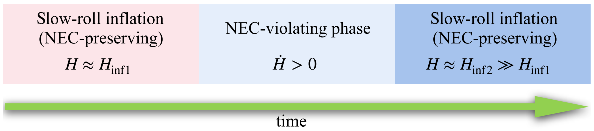

FIG. 1. In our scenario, the Universe begins with a period of slow-roll inflation, and then transitions into a second stage of slow-roll inflation with a higher energy scale, after passing through a phase of violating the null energy condition.

an illustration). The NEC violation is able to naturally boost the Hubble parameter H and consequently the power spectrum. We have constructed the background evolution of such a scenario in [89] and investigated the resulted enhanced power spectrum of the primordial GWs in [90,91]. Since the current bound of primordial GWs at the CMB band indicates a tensor-to-scalar ratio $r_{0.002} \leq 0.035$ at 95% confidence level [92], the rich phenomenology of our scenario occurs mainly on smaller scales, including the observational windows of pulsar timing array (PTA) and space-borne GW detectors.

In this Letter, we demonstrate that the NEC violation can significantly amplify the abundance of PBHs in single-field inflation through an intermediate violation of the NEC, offering valuable insights into the NEC violation during inflation. Moreover, we investigate the signals of scalar-induced gravitational waves (SIGWs) arising from the amplified density perturbations and show that our results are consistent with current observational constraints. Our findings present a compelling case for the study of PBHs, SIGWs, and the primordial GWs as crucial probes for understanding the NEC violation in the very early Universe.

Our mechanism.—The "no-go" theorems [93,94] indicate that an NEC violation will generically lead to ghost or gradient instabilities in cosmology constructed by the Horndeski theory. In view of that, we should realize our scenario with theories beyond Horndeski [83–88]. For simplicity, we will work with the effective field theory (EFT) action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}\mathsf{R} + P(\phi, X) + L_{\delta g^{00}\mathsf{R}^{(3)}}\right], \quad (1)$$

where $X = \nabla_\mu\phi\nabla^\mu\phi$, the EFT operator $L_{\delta g^{00}\mathsf{R}^{(3)}} = [f(\phi)/2]\delta g^{00}\mathsf{R}^{(3)}$ is adopted to thoroughly eliminate the instabilities, δg^{00} is the perturbation of the 00th component of the metric, $\mathsf{R}^{(3)}$ is the three-dimensional Ricci scalar on the spacelike hypersurface; see, e.g., [86] for details.

The operator $L_{\delta g^{00}\mathsf{R}^{(3)}}$ is irrelevant to the background dynamics [83]. Therefore, the background evolution is determined by the k-essence action

$$P(\phi, X) = -\frac{g_1(\phi)}{2}M_{\rm P}^2X + \frac{g_2(\phi)}{4}X^2 - M_{\rm P}^4V(\phi), \quad (2)$$

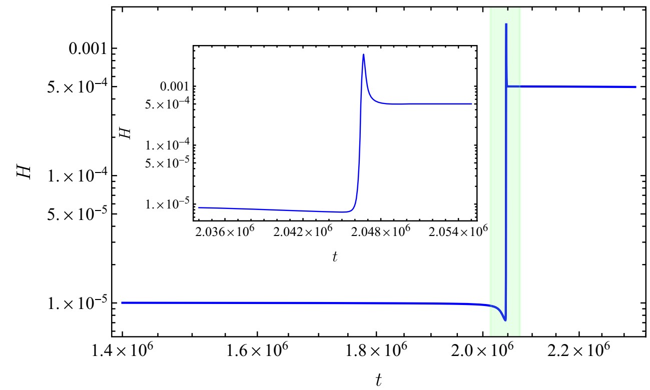

FIG. 2. A numerical solution of the Hubble parameter H with respect to time t in our model, which results in the blue curve of P_ζ in Fig. 3. We have set the Planck scale $M_{\rm P} = 1$.

where the details are presented in the Supplemental Material [95]. The background dynamics of our scenario is illustrated in Fig. 1. The evolution of the Hubble parameter H is displayed in Fig. 2 by setting a set of parameters. The NEC violating phase can be defined by $\dot{H} > 0$.

The quadratic action of scalar perturbation for the action (1) can be written as

$$S_\zeta^{(2)} = \int d^4xa^3Q_s\left[\dot{\zeta}^2 - c_s^2\frac{(\partial\zeta)^2}{a^2}\right], \quad (3)$$

where

$$Q_s = \frac{2\dot{\phi}^4P_{XX} - M_{\rm P}^2\dot{H}}{H^2}, \qquad c_s^2 = \frac{M_{\rm P}^2}{Q_s}\left(\frac{\dot{c}_3}{a} - 1\right) \quad (4)$$

and $c_3 = a[1 + (2f/M_{\rm P}^2)]/H$; see, e.g., [83]. Obviously, the ghost instability (i.e., $Q_s < 0$) and the gradient instability (i.e., $c_s^2 < 0$) can be easily cured with appropriate construction of $P(\phi, X)$ and the EFT operator $L_{\delta g^{00}\mathsf{R}^{(3)}}$.

The equation of motion for ζ can be written as

$$v_k'' + \left(c_s^2k^2 - \frac{z_s''}{z_s}\right)v_k = 0, \quad (5)$$

where $v_k = z_s\zeta$ and $z_s = \sqrt{2a^2Q_s}$, $' \equiv d/d\tau$, $d\tau = a^{-1}dt$. In the following, we will choose specific model parameters in $L_{\delta g^{00}\mathsf{R}^{(3)}}$ such that the sound speed is canonical, i.e., $c_s^2 \equiv 1$. The perturbation mode is in the vacuum state initially, i.e., $v_k \simeq (1/\sqrt{2k})e^{-ik\tau}$. The resulting spectrum of ζ at the radiation domination stage is $P_\zeta = (k^3/2\pi^2)|\zeta|^2$, which is evaluated after the perturbation modes exited their horizons, i.e., $aH/k \gg 1$.

Since $c_s^2 \equiv 1$, it can be inferred that the enhancement of the power spectrum is due to the variation of Q_s (primarily the growth of H) during the NEC violation, which is

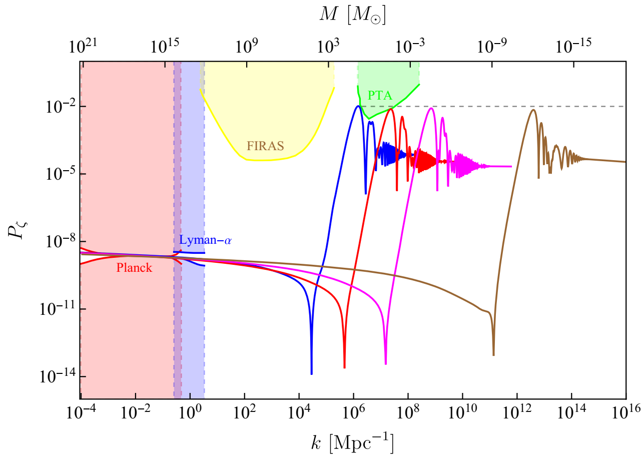

FIG. 3. The numerical results of the scalar power spectra, P_ζ, are presented for four different parameter sets. The constraints on P_ζ from Planck, Lyman-α, FIRAS, and PTA are depicted as the shadowed regions [60]. The gray dashed line represents the constraint $P_\zeta \simeq 10^{-2}$, which ensures a sufficient abundance of PBHs.

intrinsically different from many other mechanisms (including the USR inflation; see the Supplemental Material [95] for details). The scalar power spectrum is illustrated in Fig. 3 by numerically solving the background evolution and Eq. (5) with four different sets of parameters. It should be noted that, for illustrative purposes, the blue and red curves in Fig. 3 are chosen to narrowly satisfy the PTA constraint.

Primordial black holes.—In this Letter, we adopt the standard paradigm of PBH formation, where PBHs originate from the gravitational collapse of overdense regions in the early Universe. We define the density contrast as $\delta \equiv \delta\rho/\bar{\rho}$, where $\bar{\rho}$ is the energy density at the background level, and $\delta\rho \equiv \rho - \bar{\rho}$ is the density fluctuation. Moreover, we assume that the comoving curvature perturbation ζ and the density contrast δ follow a Gaussian distribution. In Fourier space, we have

$$\delta_k = \frac{2}{3}\left(\frac{k}{aH}\right)^2 \Phi_k \simeq \frac{4}{9}\left(\frac{k}{aH}\right)^2 \zeta_k, \tag{6}$$

where Φ is the Bardeen potential in the Newtonian gauge, $\Phi \simeq \frac{2}{3}\zeta$ on superhorizon scales. Therefore, the power spectrum of density contrast is

$$P_\delta(k) = \frac{16}{81}\left(\frac{k}{aH}\right)^4 P_\zeta(k). \tag{7}$$

In the standard Press-Schechter formalism, the mass fraction function $\beta(M)$, defined as the fraction of PBHs compared to the total energy of the Universe at the formation time t_i, is given by

$$\beta(R) \simeq \frac{\sigma_R}{\sqrt{2\pi}\delta_c} e^{-\frac{\delta_c^2}{2\sigma_R^2}}, \tag{8}$$

where we assume a Gaussian distribution function for density fluctuations, and σ_R represents the corresponding variance. The suggested threshold for PBH formation is $0.4 \le \delta_c \le 0.7$ [101]. In our case, we will take $\delta_c = 0.5$.

The smoothed density field δ_R is defined as $\delta_R(\vec{x}) \equiv \int d^3y W(\vec{x} - \vec{y}; R)\delta(\vec{y})$, where W is a window function associated with a characteristic length scale $R \equiv k^{-1}$. We choose the spherically symmetric real-space top-hat window function, i.e., $W(k;R) \equiv 3[\sin(kR) - kR\cos(kR)](kR)^{-3}$, since it requires the smallest amplitude of density perturbations for a fixed PBH abundance compared to alternative choices [102].

We have $\sigma_R^2 \equiv \langle\delta_R^2\rangle$, where $\langle\delta_R^2\rangle$ is suggested to be [103,104]

$$\langle\delta_R^2\rangle = \int_0^\infty \frac{dk}{k} W^2 \frac{16}{81}(kR)^4 T^2(k, \tau = R) P_\zeta(k), \tag{9}$$

the scalar transfer function at the radiation dominated era can be given by

$$T(k,\tau) \equiv \frac{9\sqrt{3}}{(k\tau)^3}\left[\sin\left(\frac{k\tau}{\sqrt{3}}\right) - \frac{k\tau}{\sqrt{3}}\cos\left(\frac{k\tau}{\sqrt{3}}\right)\right]. \tag{10}$$

The mass of the PBH is related to the wave number k by [105]

$$\frac{M}{M_\odot} \simeq \left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{10.75}\right)^{-\frac{1}{6}}\left(\frac{k}{1.9\times 10^6\ \mathrm{Mpc}^{-1}}\right)^{-2}, \tag{11}$$

where $M_\odot$ is the solar mass, γ represents the collapsing efficiency, and g_* denotes the effective number of degrees of freedom for the energy density at PBH formation. In this Letter, we take $\gamma = 0.2$ and $g_* = 106.75$ [3]. Accordingly, the current energy fraction, $f_{\mathrm{PBH}}(M)$, is given by [105]

$$f_{\mathrm{PBH}}(M) = \frac{\beta(M)}{2.70\times 10^{-8}}\left(\frac{k}{1.9\times 10^6\ \mathrm{Mpc}^{-1}}\right)^{-2}. \tag{12}$$

We present the plot of f_{PBH} as a function of PBH mass in Fig. 4. The production of PBHs is efficient across various mass scales, as indicated by the brown, magenta, and blue curves. Notably, our model can successfully account for the OGLE ultrashort-timescale microlensing events with specific parameter choices. Additionally, within the mass range between the red and blue curves (approximately $4\times 10^{-2}M_\odot \sim 0.8M_\odot$) in Fig. 4, the PBH abundance predicted by our model is constrained to be $f_{\mathrm{PBH}} < 1\%$ due to the PTA constraint on P_ζ, as illustrated by the red and blue curves in Fig. 3.

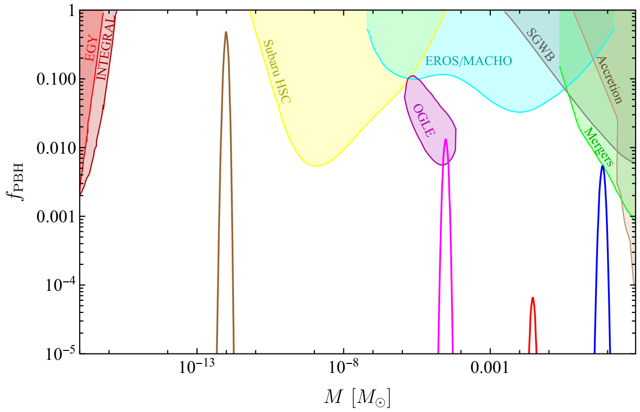

FIG. 4. The current energy fractions of PBHs, $f_{\rm PBH}$, are shown for different parameter sets. The curves of $f_{\rm PBH}$ correspond to those of P_ζ with the same color as P_ζ in Fig. 3. The current constraints on PBH abundance are adopted from [106], and the purple shaded region represents the PBH abundance inferred by the OGLE result [107].

Recently, it was claimed that the PBH formation from single-field inflation is ruled out because the enhanced density perturbations give too large one-loop correction to those on CMB scales in the USR inflationary scenario [34], while its validity is still in debate (e.g., [35] pointed out a problem in the computation of [34] and that the large one-loop correction disappears; see also [36]). Our scenario might be promising to give a loophole for the argument of this problem, due to the intrinsic differences between the NEC violation and the USR mechanism. In our scenario, the enhancement of P_ζ results primarily from the growth of H instead of the decrease of $\dot{H}$ (or equivalently a very tiny $\epsilon \ll 1$ as in the USR inflation). In fact, we have $|\epsilon| \gg 1$ during the NEC-violating phase. Additionally, in the USR scenario, the coefficient of the dominant term in the cubic action and ζ almost simultaneously reach their maximum values at the end of USR. In contrast, in our scenario, when the coefficient functions in the cubic action reach their maximum values, ζ or its derivatives are far from reaching their maxima. Consequently, the argument of [34] does not directly apply to our scenario. Using the approach outlined in [34], we computed the one-loop corrections to the CMB scale power spectrum for the spectra shown in Fig. 3, as detailed in the Supplemental Material [95]. The results indicate that our scenario may offer a potentially novel approach to yielding smaller one-loop corrections to the CMB scale power spectrum [95]. However, since we have made some simplifications in the calculations, obtaining conclusive proof that the result in [34] does not invalidate our scenario necessitates further investigation in the future.

Scalar-induced gravitational waves.—In order to generate a significant abundance of PBHs, it is necessary to enhance the primordial curvature perturbation. This enhancement can potentially lead to the production of large SIGW signals. Hence, it is crucial to examine the corresponding SIGWs within our model to ensure self-consistency. In our analysis, we adopt the standard approach, where the SIGWs are generated as the scalar perturbation modes re-enter the horizon during the radiation domination era.

The power spectrum for SIGW is [108]

$$P_h(\tau,k) = 576\int_0^\infty dt\int_{-1}^1 ds P_\zeta\left(k\frac{t+s+1}{2}\right) \times P_\zeta\left(k\frac{t-s+1}{2}\right)\frac{[-5+s^2+t(2+t)]^4}{(1-s+t)^6(1+s+t)^6} \times\left\{\left[\frac{s^2-(t+1)^2}{-5+s^2+t(2+t)}+\frac{1}{2}\ln\left|\frac{-2+t(2+t)}{3-s^2}\right|\right]^2 + \frac{\pi^2}{4}\Theta(t-\sqrt{3}+1)\right\}, \tag{13}$$

where Θ is the Heaviside function. It is related to the energy density parameter per logarithmic interval of k, $\Omega_{\rm GW}(\tau,k)$, as

$$\Omega_{\rm GW}(\tau_r,k) = \frac{\overline{P_h}(\tau,k)}{24}\left(\frac{k}{a(\tau_r)H(\tau_r)}\right)^2 = \frac{\overline{P_h}(\tau,k)}{24}, \tag{14}$$

where we evaluate the energy density at the horizon reentry ($k = aH$) with a conformal time τ_r. The energy density spectrum today for SIGW is

$$\Omega_{\rm GW}(k)h^2 = 0.83\left(\frac{g_*}{10.75}\right)^{-\frac{1}{3}}\Omega_{r,0}h^2\Omega_{\rm GW}(\tau_r,k), \tag{15}$$

where $\Omega_{r,0}h^2 \simeq 4.2\times10^{-5}$ is the current density parameter of radiation.

In Fig. 5, we present the energy density spectra of SIGWs for the same parameter sets as in Figs. 3 and 4. The

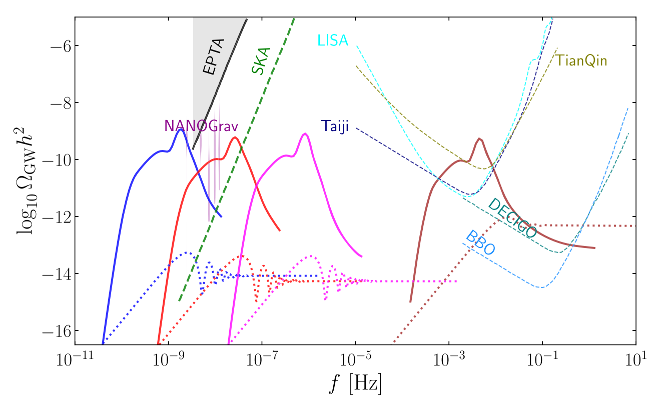

FIG. 5. The predicted current energy spectra of SIGWs (solid curves) and primordial GWs (dotted curves) are shown for different parameter sets. The curves of $\Omega_{\rm GW}h^2$ correspond to those of P_ζ with the same color as shown in Fig. 3. The shaded region represents the current constraint from EPTA. We also include the expected sensitivity curves of future GW observations as dashed curves, including SKA, LISA, Taiji, TianQin, DECIGO, and BBO. The magenta vertical violinlike bars correspond to the data of NANOGrav.

SIGW signals corresponding to our parameter sets are consistent with the constraints from the EPTA [109]. Notably, the solid blue, red, and brown curves exhibit detectable signatures that fall within the observational windows of future GW detectors, including those of PTA and space-borne GW detectors. Additionally, the red and blue curves may account for the evidence of a stochastic common-spectrum process reported by the NANOGrav Collaboration [110].

Distinctive features of our scenario.—Basically, the power spectrum of primordial GWs depends primarily on the Hubble parameter H, as long as the propagating speed of primordial GWs is $c_T \equiv 1$. In our scenario, H experiences significant growth due to the violation of the NEC, which is unique compared to other single-field PBH formation scenarios, e.g., the USR inflation. Consequently, the resulting primordial GWs spectrum will be significantly enhanced on certain scale and is nearly scale-invariant on smaller scale [89]. These distinctive features of primordial GWs are promising to be detected by future observations (e.g., BBO and DECIGO), allowing for distinguishing our NEC violation scenario from the other single-field scenarios of the PBH formation from an observational point of view.

For the four sets of parameter configurations used in Fig. 3, we have plotted the corresponding primordial GW signals in Fig. 5 as dotted curves. The blue, red, and magenta dotted curves predict $\Omega_{\mathrm{GW}}h^2 \sim 10^{-14}$ on small scales, which is narrowly beyond the sensitivity of the BBO. In contrast, the brown dotted curve predicts $\Omega_{\mathrm{GW}}h^2 \sim 10^{-12}$ within the observation windows of BBO and DECIGO. At this scale and smaller scales, it significantly exceeds the corresponding SIGW signal. Therefore, synchronized observations of PBHs, SIGWs, and primordial GWs may potentially probe NEC violation during inflation. Although some of the primordial GW background is borderline detectable in the example cases given, the principle is intriguing and the combination of signals is unique and characteristic for this class of models.

Conclusion and outlook.—We report a novel mechanism capable of generating sizable PBHs in the context of single-field inflationary cosmology, by introducing an intermediate stage of NEC violation during inflation, which offers a unique avenue to enhance the Hubble parameter H and consequently the primordial power spectrum. Our scenario can be realized in the EFT framework of inflation. The primordial curvature perturbation is significantly enhanced in a narrow band of comoving wavelengths corresponding to the NEC violation stage. Consequently, the primordial density perturbation is nearly scale-invariant in both large and small scales connected by a sharp peak. The sharp peak leads to the generation of a sizable amount of PBHs as well as signals of SIGW, which can be probed and tested in future cosmological surveys. Furthermore, the distinctive features in the power spectrum of primordial GWs (see Refs. [89–91]) will enable our scenario to be distinguished from other single-field PBH formation scenarios, including USR inflation, from an observational perspective.

In this Letter, we adopted an EFT approach in the context of single-field inflationary cosmology, where we assumed a sound speed $c_s \equiv 1$ for the scalar perturbations by introducing the EFT operator $L_{\delta g^{00}\mathrm{R}^{(3)}}$. As a result, our scenario effectively avoids issues related to large entropy fluctuations and superluminality. This choice allows for independent parametrization of scalar and tensor perturbations, as the EFT operator does not contribute to the background dynamics or tensor perturbations at quadratic order. However, in realistic cosmological scenarios derived from covariant actions, scalar and tensor perturbations are interconnected. For instance, when implementing the EFT operator using theories beyond Horndeski, the sound speeds of scalar and tensor perturbations can be modified. Therefore, to comprehensively investigate the contributions of our scenario to the GW background and confront them with observations, it is necessary to specify the covariant actions and conduct a detailed analysis in future studies.

Our work highlighted the potential of utilizing a combination of PBHs, SIGW signals, and primordial GWs as a valuable probe for exploring the NEC violation during inflation, particularly in the era of multimessenger and multiband observations.

We thank Chao Chen, Jie-Wen Chen, Chunshan Lin, Taotao Qiu, and Shengfeng Yan for valuable discussions. Y. C. is supported in part by the National Natural Science Foundation of China (Grant No. 11905224), the China Postdoctoral Science Foundation (Grant No. 2021M692942) and Zhengzhou University (Grant No. 32340282). M. Z. is supported by Grant No. UMO 2021/42/E/ST9/00260 from the National Science Centre, Poland. Y.-S. P. is supported by the National Natural Science Foundation of China under Grant No. 12075246 and the Fundamental Research Funds for the Central Universities. We acknowledge the use of the computing server *Arena317*@ZZU.

[*]Contact author: yongcai_phy@outlook.com, caiyong@zzu.edu.cn
[†]Contact author: mian.zhu@uj.edu.pl
[‡]Contact author: yspiao@ucas.ac.cn

[1] S. Hawking, Gravitationally collapsed objects of very low mass, Mon. Not. R. Astron. Soc. **152**, 75 (1971).
[2] B. J. Carr and S. W. Hawking, Black holes in the early Universe, Mon. Not. R. Astron. Soc. **168**, 399 (1974).
[3] B. J. Carr, The primordial black hole mass spectrum, Astrophys. J. **201**, 1 (1975).
[4] F. Ozel, D. Psaltis, R. Narayan, and J. E. McClintock, The black hole mass distribution in the galaxy, Astrophys. J. **725**, 1918 (2010).

[5] P. Ivanov, P. Naselsky, and I. Novikov, Inflation and primordial black holes as dark matter, Phys. Rev. D **50**, 7173 (1994).
[6] K. M. Belotsky, A. D. Dmitriev, E. A. Esipova, V. A. Gani, A. V. Grobov, M. Y. Khlopov, A. A. Kirillov, S. G. Rubin, and I. V. Svadkovsky, Signatures of primordial black hole dark matter, Mod. Phys. Lett. A **29**, 1440005 (2014).
[7] B. Carr, F. Kuhnel, and M. Sandstad, Primordial black holes as dark matter, Phys. Rev. D **94**, 083504 (2016).
[8] B. Carr and F. Kuhnel, Primordial black holes as dark matter: Recent developments, Annu. Rev. Nucl. Part. Sci. **70**, 355 (2020).
[9] R. Bean and J. Magueijo, Could supermassive black holes be quintessential primordial black holes?, Phys. Rev. D **66**, 063505 (2002).
[10] M. Kawasaki, A. Kusenko, and T. T. Yanagida, Primordial seeds of supermassive black holes, Phys. Lett. B **711**, 1 (2012).
[11] M. Kawasaki, N. Kitajima, and T. T. Yanagida, Primordial black hole formation from an axionlike curvaton model, Phys. Rev. D **87**, 063519 (2013).
[12] J.-W. Chen, J. Liu, H.-L. Xu, and Y.-F. Cai, Tracing primordial black holes in nonsingular bouncing cosmology, Phys. Lett. B **769**, 561 (2017).
[13] J. Quintin and R. H. Brandenberger, Black hole formation in a contracting universe, J. Cosmol. Astropart. Phys. 11 (2016) 029.
[14] S. Wang, Y.-F. Wang, Q.-G. Huang, and T. G. F. Li, Constraints on the primordial black hole abundance from the first Advanced LIGO observation run using the stochastic gravitational-wave background, Phys. Rev. Lett. **120**, 191102 (2018).
[15] S. Wang, T. Terada, and K. Kohri, Prospective constraints on the primordial black hole abundance from the stochastic gravitational-wave backgrounds produced by coalescing events and curvature perturbations, Phys. Rev. D **99**, 103531 (2019).
[16] J. Garriga, A. Vilenkin, and J. Zhang, Black holes and the multiverse, J. Cosmol. Astropart. Phys. 02 (2016) 064.
[17] S. Pi, Y.-l. Zhang, Q.-G. Huang, and M. Sasaki, Scalaron from R^2-gravity as a heavy field, J. Cosmol. Astropart. Phys. 05 (2018) 042.
[18] T. Nakama and Y. Wang, Do we need fine-tuning to create primordial black holes?, Phys. Rev. D **99**, 023504 (2019).
[19] Q. Ding, T. Nakama, J. Silk, and Y. Wang, Detectability of gravitational waves from the coalescence of massive primordial black holes with initial clustering, Phys. Rev. D **100**, 103003 (2019).
[20] Y. Luo, C. Chen, M. Kusakabe, and T. Kajino, Impacts of Hawking radiation from primordial black holes in critical collapse model on the light element abundances, J. Cosmol. Astropart. Phys. 05 (2021) 042.
[21] G. Domènech, C. Lin, and M. Sasaki, Gravitational wave constraints on the primordial black hole dominated early universe, J. Cosmol. Astropart. Phys. 04 (2021) 062; 11 (2021) E01.
[22] A. Boudon, B. Bose, H. Huang, and L. Lombriser, Baryogenesis through asymmetric Hawking radiation from primordial black holes as dark matter, Phys. Rev. D **103**, 083504 (2021).
[23] K. Kawana and K.-P. Xie, Primordial black holes from a cosmic phase transition: The collapse of Fermi-balls, Phys. Lett. B **824**, 136791 (2022).
[24] D.-S. Meng, C. Yuan, and Q.-g. Huang, One-loop correction to the enhanced curvature perturbation with local-type non-Gaussianity for the formation of primordial black holes, Phys. Rev. D **106**, 063508 (2022).
[25] J. Lin, S. Gao, Y. Gong, Y. Lu, Z. Wang, and F. Zhang, Primordial black holes and scalar induced gravitational waves from Higgs inflation with noncanonical kinetic term, Phys. Rev. D **107**, 043517 (2023).
[26] L. Wu, Y. Gong, and T. Li, Primordial black holes and secondary gravitational waves from string inspired general no-scale supergravity, Phys. Rev. D **104**, 123544 (2021).
[27] X.-H. Tan, Y.-J. Yan, T. Qiu, and J.-Q. Xia, Searching for the signal of a primordial black hole from CMB lensing and γ-ray emissions, Astrophys. J. Lett. **939**, L15 (2022).
[28] S. Pi and M. Sasaki, Primordial black hole formation in non-minimal curvaton scenario, Phys. Rev. D **108**, L101301 (2023).
[29] G. Domènech, V. Takhistov, and M. Sasaki, Exploring evaporating primordial black holes with gravitational waves, Phys. Lett. B **823**, 136722 (2021).
[30] J. Liu, L. Bian, R.-G. Cai, Z.-K. Guo, and S.-J. Wang, Primordial black hole production during first-order phase transitions, Phys. Rev. D **105**, L021303 (2022).
[31] X. Wang, Y.-l. Zhang, R. Kimura, and M. Yamaguchi, Reconstruction of power spectrum of primordial curvature perturbations on small scales from primordial black hole binaries scenario of LIGO/VIRGO detection, Sci. China Phys. Mech. Astron. **66**, 260462 (2023).
[32] S. He, L. Li, Z. Li, and S.-J. Wang, Gravitational waves and primordial black hole productions from gluodynamics, Sci. China Phys. Mech. Astron. **67**, 240411 (2024).
[33] T. Papanikolaou, C. Tzerefos, S. Basilakos, and E. N. Saridakis, No constraints for f(T) gravity from gravitational waves induced from primordial black hole fluctuations, Eur. Phys. J. C **83**, 31 (2023).
[34] J. Kristiano and J. Yokoyama, Constraining primordial black hole formation from single-field inflation, Phys. Rev. Lett. **132**, 221003 (2024).
[35] J. Fumagalli, Absence of one-loop effects on large scales from small scales in non-slow-roll dynamics, arXiv:2305.19263.
[36] A. Riotto, The primordial black hole formation from single-field inflation is still not ruled out, arXiv:2303.01727.
[37] V. De Luca, G. Franciolini, A. Riotto, and H. Veermäe, Ruling out initially clustered primordial black holes as dark matter, Phys. Rev. Lett. **129**, 191302 (2022).
[38] V. De Luca, G. Franciolini, and A. Riotto, Heavy primordial black holes from strongly clustered light black holes, Phys. Rev. Lett. **130**, 171401 (2023).
[39] R. D'Agostino, R. Giambò, and O. Luongo, Constraining primordial black holes as a fraction of dark matter through accretion disk luminosity, Phys. Rev. D **107**, 043032 (2023).
[40] M. Lewicki, P. Toczek, and V. Vaskonen, Primordial black holes from strong first-order phase transitions, J. High Energy Phys. 09 (2023) 092.

[41] S. Choudhury, S. Panda, and M. Sami, Quantum loop effects on the power spectrum and constraints on primordial black holes, arXiv:2303.06066.

[42] S. Choudhury, S. Panda, and M. Sami, Galileon inflation evades the no-go for PBH formation in the single-field framework, J. Cosmol. Astropart. Phys. 08 (2023) 078.

[43] J. He, H. Deng, Y.-S. Piao, and J. Zhang, Implications of GWTC-3 on primordial black holes from vacuum bubbles, Phys. Rev. D **109**, 044035 (2024).

[44] C. Zhang and X. Zhang, Gravitational capture of magnetic monopoles by primordial black holes in the early universe, J. High Energy Phys. 10 (2023) 037.

[45] M. Drees and E. Erfani, Running spectral index and formation of primordial black hole in single field inflation models, J. Cosmol. Astropart. Phys. 01 (2012) 035.

[46] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida, Inflationary primordial black holes as all dark matter, Phys. Rev. D **96**, 043504 (2017).

[47] K. Kannike, L. Marzola, M. Raidal, and H. Veermäe, Single field double inflation and primordial black holes, J. Cosmol. Astropart. Phys. 09 (2017) 020.

[48] S.-L. Cheng, W. Lee, and K.-W. Ng, Primordial black holes and associated gravitational waves in axion monodromy inflation, J. Cosmol. Astropart. Phys. 07 (2018) 001.

[49] Y.-F. Cai, C. Chen, X. Tong, D.-G. Wang, and S.-F. Yan, When primordial black holes from sound speed resonance meet a stochastic background of gravitational waves, Phys. Rev. D **100**, 043518 (2019).

[50] J. Lin, Q. Gao, Y. Gong, Y. Lu, C. Zhang, and F. Zhang, Primordial black holes and secondary gravitational waves from k and G inflation, Phys. Rev. D **101**, 103515 (2020).

[51] A. Ashoorioon, A. Rostami, and J. T. Firouzjaee, Examining the end of inflation with primordial black holes mass distribution and gravitational waves, Phys. Rev. D **103**, 123512 (2021).

[52] O. Özsoy and Z. Lalak, Primordial black holes as dark matter and gravitational waves from bumpy axion inflation, J. Cosmol. Astropart. Phys. 01 (2021) 040.

[53] S. Kawai and J. Kim, Primordial black holes from Gauss-Bonnet-corrected single field inflation, Phys. Rev. D **104**, 083545 (2021).

[54] A. Karam, N. Koivunen, E. Tomberg, V. Vaskonen, and H. Veermäe, Anatomy of single-field inflationary models for primordial black holes, J. Cosmol. Astropart. Phys. 03 (2023) 013.

[55] T. Papanikolaou, A. Lymperis, S. Lola, and E. N. Saridakis, Primordial black holes and gravitational waves from non-canonical inflation, J. Cosmol. Astropart. Phys. 03 (2023) 003.

[56] C. Fu and C. Chen, Sudden braking and turning with a two-field potential bump: Primordial black hole formation, J. Cosmol. Astropart. Phys. 05 (2023) 005.

[57] S. Choudhury, M. R. Gangopadhyay, and M. Sami, No-go for the formation of heavy mass primordial black holes in single field inflation, arXiv:2301.10000.

[58] J. Garcia-Bellido and E. Ruiz Morales, Primordial black holes from single field models of inflation, Phys. Dark Universe **18**, 47 (2017).

[59] C. Germani and T. Prokopec, On primordial black holes from an inflection point, Phys. Dark Universe **18**, 6 (2017).

[60] C. T. Byrnes, P. S. Cole, and S. P. Patil, Steepest growth of the power spectrum and primordial black holes, J. Cosmol. Astropart. Phys. 06 (2019) 028.

[61] C. Fu, P. Wu, and H. Yu, Primordial black holes from inflation with nonminimal derivative coupling, Phys. Rev. D **100**, 063532 (2019).

[62] C. Fu, P. Wu, and H. Yu, Scalar induced gravitational waves in inflation with gravitationally enhanced friction, Phys. Rev. D **101**, 023529 (2020).

[63] C. Fu, P. Wu, and H. Yu, Primordial black holes and oscillating gravitational waves in slow-roll and slow-climb inflation with an intermediate noninflationary phase, Phys. Rev. D **102**, 043527 (2020).

[64] H. V. Ragavendra, P. Saha, L. Sriramkumar, and J. Silk, Primordial black holes and secondary gravitational waves from ultraslow roll and punctuated inflation, Phys. Rev. D **103**, 083510 (2021).

[65] H. Di and Y. Gong, Primordial black holes and second order gravitational waves from ultra-slow-roll inflation, J. Cosmol. Astropart. Phys. 07 (2018) 007.

[66] H. Motohashi and W. Hu, Primordial black holes and slow-roll violation, Phys. Rev. D **96**, 063503 (2017).

[67] I. Dalianis, A. Kehagias, and G. Tringas, Primordial black holes from α-attractors, J. Cosmol. Astropart. Phys. 01 (2019) 037.

[68] Z. Yi, Q. Gao, Y. Gong, and Z.-h. Zhu, Primordial black holes and scalar-induced secondary gravitational waves from inflationary models with a noncanonical kinetic term, Phys. Rev. D **103**, 063534 (2021).

[69] G. Ballesteros, J. Beltran Jimenez, and M. Pieroni, Black hole formation from a general quadratic action for inflationary primordial fluctuations, J. Cosmol. Astropart. Phys. 06 (2019) 016.

[70] A. Y. Kamenshchik, A. Tronconi, T. Vardanyan, and G. Venturi, Non-canonical inflation and primordial black holes production, Phys. Lett. B **791**, 201 (2019).

[71] A. Ashoorioon, A. Rostami, and J. T. Firouzjaee, EFT compatible PBHs: Effective spawning of the seeds for primordial black holes during inflation, J. High Energy Phys. 07 (2021) 087.

[72] T. Qiu, W. Wang, and R. Zheng, Generation of primordial black holes from an inflation model with modified dispersion relation, Phys. Rev. D **107**, 083018 (2023).

[73] Y.-F. Cai, X. Tong, D.-G. Wang, and S.-F. Yan, Primordial black holes from sound speed resonance during inflation, Phys. Rev. Lett. **121**, 081306 (2018).

[74] R.-G. Cai, Z.-K. Guo, J. Liu, L. Liu, and X.-Y. Yang, Primordial black holes and gravitational waves from parametric amplification of curvature perturbations, J. Cosmol. Astropart. Phys. 06 (2020) 013.

[75] Y.-F. Cai, C. Lin, B. Wang, and S.-F. Yan, Sound speed resonance of the stochastic gravitational wave background, Phys. Rev. Lett. **126**, 071303 (2021).

[76] Z. Zhou, J. Jiang, Y.-F. Cai, M. Sasaki, and S. Pi, Primordial black holes and gravitational waves from resonant amplification during inflation, Phys. Rev. D **102**, 103527 (2020).

[77] Z.-Z. Peng, C. Fu, J. Liu, Z.-K. Guo, and R.-G. Cai, Gravitational waves from resonant amplification of curvature perturbations during inflation, J. Cosmol. Astropart. Phys. 10 (2021) 050.

[78] A. A. Starobinsky, Spectrum of adiabatic perturbations in the universe when there are singularities in the inflation potential, JETP Lett. **55**, 489 (1992), https://inspirehep.net/literature/147727.

[79] D. Polarski and A. A. Starobinsky, Spectra of perturbations produced by double inflation with an intermediate matter dominated stage, Nucl. Phys. **B385**, 623 (1992).

[80] J. Lesgourgues, D. Polarski, and A. A. Starobinsky, How large can be the primordial gravitational wave background in inflationary models?, Mon. Not. R. Astron. Soc. **308**, 281 (1999).

[81] G. Tasinato, An analytic approach to non-slow-roll inflation, Phys. Rev. D **103**, 023535 (2021).

[82] V. A. Rubakov, The null energy condition and its violation, Phys. Usp. **57**, 128 (2014).

[83] Y. Cai, Y. Wan, H.-G. Li, T. Qiu, and Y.-S. Piao, The effective field theory of nonsingular cosmology, J. High Energy Phys. 01 (2017) 090.

[84] P. Creminelli, D. Pirtskhalava, L. Santoni, and E. Trincherini, Stability of geodesically complete cosmologies, J. Cosmol. Astropart. Phys. 11 (2016) 047.

[85] Y. Cai, H.-G. Li, T. Qiu, and Y.-S. Piao, The effective field theory of nonsingular cosmology: II, Eur. Phys. J. C **77**, 369 (2017).

[86] Y. Cai and Y.-S. Piao, A covariant Lagrangian for stable nonsingular bounce, J. High Energy Phys. 09 (2017) 027.

[87] R. Kolevatov, S. Mironov, N. Sukhov, and V. Volkova, Cosmological bounce and Genesis beyond Horndeski, J. Cosmol. Astropart. Phys. 08 (2017) 038.

[88] A. Ilyas, M. Zhu, Y. Zheng, Y.-F. Cai, and E. N. Saridakis, DHOST bounce, J. Cosmol. Astropart. Phys. 09 (2020) 002.

[89] Y. Cai and Y.-S. Piao, Intermittent null energy condition violations during inflation and primordial gravitational waves, Phys. Rev. D **103**, 083521 (2021).

[90] Y. Cai and Y.-S. Piao, Generating enhanced primordial GWs during inflation with intermittent violation of NEC and diminishment of GW propagating speed, J. High Energy Phys. 06 (2022) 067.

[91] Y. Cai, Generating enhanced parity-violating gravitational waves during inflation with violation of the null energy condition, Phys. Rev. D **107**, 063512 (2023).

[92] P. A. R. Ade *et al.* (BICEP/Keck Collaboration), Improved constraints on primordial gravitational waves using Planck, WMAP, and BICEP/Keck observations through the 2018 observing season, Phys. Rev. Lett. **127**, 151301 (2021).

[93] M. Libanov, S. Mironov, and V. Rubakov, Generalized Galileons: Instabilities of bouncing and Genesis cosmologies and modified Genesis, J. Cosmol. Astropart. Phys. 08 (2016) 037.

[94] T. Kobayashi, Generic instabilities of nonsingular cosmologies in Horndeski theory: A no-go theorem, Phys. Rev. D **94**, 043511 (2016).

[95] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.021001, which includes Refs. [96–100], for additional information about the details of our model, the enhancement of curvature perturbations from the NEC violation, and a discussion of one-loop correction in our scenario.

[96] M. Zhu, A. Ilyas, Y. Zheng, Y.-F. Cai, and E. N. Saridakis, Scalar and tensor perturbations in DHOST bounce cosmology, J. Cosmol. Astropart. Phys. 11 (2021) 045.

[97] X. Gao and D. A. Steer, Inflation and primordial non-Gaussianities of 'generalized Galileons', J. Cosmol. Astropart. Phys. 12 (2011) 019.

[98] A. De Felice and S. Tsujikawa, Inflationary non-Gaussianities in the most general second-order scalar-tensor theories, Phys. Rev. D **84**, 083504 (2011).

[99] X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama, and S. Yokoyama, Full bispectra from primordial scalar and tensor perturbations in the most general single-field inflation model, Prog. Theor. Exp. Phys. **2013**, 053E03 (2013).

[100] S. Akama, S. Hirano, and T. Kobayashi, Primordial non-Gaussianities of scalar and tensor perturbations in general bounce cosmology: Evading the no-go theorem, Phys. Rev. D **101**, 043529 (2020).

[101] I. Musco, V. De Luca, G. Franciolini, and A. Riotto, Threshold for primordial black holes. II. A simple analytic prescription, Phys. Rev. D **103**, 063538 (2021).

[102] K. Ando, K. Inomata, and M. Kawasaki, Primordial black holes and uncertainties in the choice of the window function, Phys. Rev. D **97**, 103528 (2018).

[103] D. Blais, T. Bringmann, C. Kiefer, and D. Polarski, Accurate results for primordial black holes from spectra with a distinguished scale, Phys. Rev. D **67**, 024024 (2003).

[104] A. S. Josan, A. M. Green, and K. A. Malik, Generalised constraints on the curvature perturbation from primordial black holes, Phys. Rev. D **79**, 103520 (2009).

[105] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Primordial black holes—perspectives in gravitational wave astronomy, Classical Quantum Gravity **35**, 063001 (2018).

[106] B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, Constraints on primordial black holes, Rep. Prog. Phys. **84**, 116902 (2021).

[107] H. Niikura, M. Takada, S. Yokoyama, T. Sumi, and S. Masaki, Constraints on Earth-mass primordial black holes from OGLE 5-year microlensing events, Phys. Rev. D **99**, 083503 (2019).

[108] K. Kohri and T. Terada, Semianalytic calculation of gravitational wave spectrum nonlinearly induced from primordial curvature perturbations, Phys. Rev. D **97**, 123532 (2018).

[109] L. Lentati *et al.*, European Pulsar Timing Array limits on an isotropic stochastic gravitational-wave background, Mon. Not. R. Astron. Soc. **453**, 2576 (2015).

[110] Z. Arzoumanian *et al.* (NANOGrav Collaboration), The NANOGrav 12.5 yr data set: Search for an isotropic stochastic gravitational-wave background, Astrophys. J. Lett. **905**, L34 (2020).