Large Deviation Full Counting Statistics in Adiabatic Open Quantum Dynamics

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The state of an open quantum system undergoing an adiabatic process evolves by following the instantaneous stationary state of its time-dependent generator. This observation allows one to characterize, for a generic adiabatic evolution, the average dynamics of the open system. However, information about fluctuations of dynamical observables, such as the number of photons emitted or the time-integrated stochastic entropy production in single experimental runs, requires controlling the whole spectrum of the generator and not only the stationary state. Here, we show how such information can be obtained in adiabatic open quantum dynamics by exploiting tools from large deviation theory. We prove an adiabatic theorem for deformed generators, which allows us to encode, in a biased quantum state, the full counting statistics of generic time-integrated dynamical observables. We further compute the probability associated with an arbitrary "rare" time history of the observable and derive a dynamics which realizes it in its typical behavior. Our results provide a way to characterize and engineer adiabatic open quantum dynamics and to control their fluctuations.

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Introduction.—Systems evolving adiabatically, i.e., via slow driving protocols, find many applications in physics. In closed quantum systems, adiabatic dynamics are characterized by the decoupled evolution of the Hamiltonian eigenvectors [1–10], which is crucial for adiabatic quantum computation [11–15] and important experimental protocols such as stimulated Raman adiabatic passage [16]. The presence of decoherence and dissipation typically imposes a fundamental timescale in which this decoupled evolution can be observed [17–21], as explored in the context of optimal control [22–25] and of noisy quantum computation [26,27]. Nevertheless, genuine adiabatic dynamics in open quantum systems occur when the state of the system follows the instantaneous stationary state of its dynamical generator [28–33], as is the case for quasistatic thermodynamic processes [34–37].

Single realizations of open quantum dynamics, or quantum trajectories, are stochastic [38,39], which can manifest in the occurrence of quantum jumps, for instance related to photon emissions [40–45]. For Markovian dynamics, the full counting statistics of jump-related observables, such as entropy production currents [44,46,47], can be obtained using deformed dynamical generators, introduced within the framework of large deviation theory [48–61]. However, much less is known about the characterization of dynamical fluctuations in open quantum dynamics with time-dependent generators [62–64], including the case of adiabatic processes.

In this Letter, we show how to fully characterize the counting statistics of jump-related observables in adiabatic open quantum dynamics in which the system follows the instantaneous stationary state of the dynamical generator [cf. Fig. 1(a)]. We prove an adiabatic theorem for deformed dynamical generators [57,59], which allows us to demonstrate that, in these processes, the statistics of time-integrated observables assumes a large deviation form [48]. Furthermore, we show that adiabatic open quantum dynamics obey a so-called temporal additivity principle [65–68]. That is, the observables follow an instantaneous large deviation principle at all times [cf. Figs. 1(b) and 1(c)]. This fact opens up the possibility of deriving the probability of any time history of the observable, see sketch in Figs. 1(b) and 1(c). Such a probability provides a higher level of description of dynamical fluctuations in the adiabatic process than what can be obtained from the full counting statistics of time-integrated observables. The latter can indeed be obtained from the former through a contraction principle [48], [cf. Fig. 1(c)]. Finally, we construct an auxiliary dynamics [57,59,69] which can realize, as typical realization, any rare realization of the observable time history in the original adiabatic process. Our findings (see Refs. [65–67,70–79] for related results in classical dynamics) shed new insights on open quantum adiabatic processes and provide a powerful approach to control, even as a function of time, their fluctuating properties. Our methods can be used for studying fluctuations in adiabatic quantum machines [43,80,81], both in or out of equilibrium, or for dissipative quantum computation [82–85].

Open quantum dynamics.—We consider quantum systems whose dynamics is described by the master equation $\dot{\rho}(t) = \mathcal{L}(t)[\rho(t)]$, with time-dependent generator

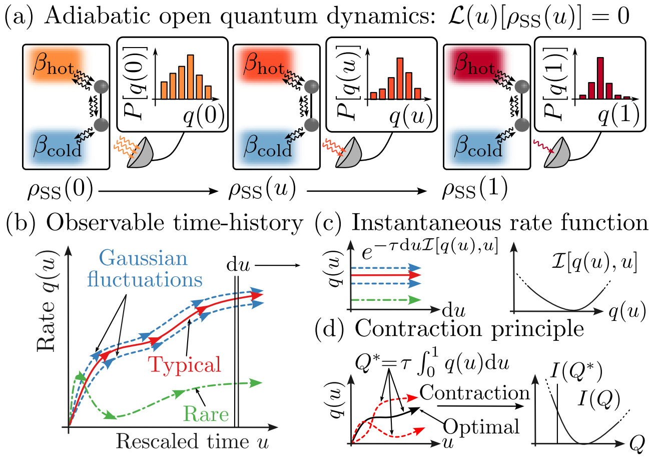

FIG. 1. Adiabatic open quantum dynamics and large deviations. (a) During an adiabatic dynamics (total evolution time τ), the system is described by the instantaneous steady state $\rho_{SS}(u)$ of the dynamical generator $\mathcal{L}(u)$, in the slow rescaled timescale $u = t/\tau$. (b) Sketch of different time histories of the instantaneous "rate" $q(u)$ of an observable of interest, e.g., the instantaneous photonemission rate. We highlight the *typical* time history, two with small Gaussian fluctuations and a rare one displaying a *large deviation* from the typical value. (c) Because of the slow dynamics, the system spends a large amount of time in each rescaled time interval du. This implies that the probability function for each instantaneous rate $q(u)$ [see illustration in panel (a)] obeys a large deviation principle. Combining them provides the probability functional for time histories of the observable. (d) The full counting statistics of the time-integrated observable, $Q(\tau) = \tau\int_0^1 du\, q(u)$, can be obtained from the optimal, i.e., most likely, trajectories providing the values of $Q(\tau) = Q^*$.

$$\mathcal{L}(t)[\rho] = -i[\tilde{H}(t)\rho - \rho\tilde{H}^\dagger(t)] + \sum_j \mathcal{J}_j(t)[\rho]. \quad (1)$$

Here, $\tilde{H}(t) = H(t) - (i/2)\sum_j J_j^\dagger(t)J_j(t)$ is the effective Hamiltonian [39], $\mathcal{J}_j(t)[\rho] = J_j(t)\rho J_j^\dagger(t)$, with $J_j(t)$ being the jump operators. The above equation generates the evolution of the system state $\rho(t)$ averaged over all possible realizations of the system-environment interaction [17,38]. Single dynamical realizations are instead described by quantum jump trajectories [57,59,86], generated by the stochastic process

$$d\psi(t) = \mathcal{B}(t)[\psi(t)]dt + \sum_j \left(\frac{\mathcal{J}_j(t)[\psi(t)]}{\mathrm{Tr}(\mathcal{J}_j(t)[\psi(t)])} - \psi(t)\right)dn_j(t), \quad (2)$$

which evolves pure quantum states $\psi = |\psi\rangle\langle\psi|$. Here, $d\psi(t)$ is the state increment while $dn_j(t)$ are Poisson increments, which can only take the value 0 or 1 with average value $\mathbb{E}_{\psi(t)=\psi}[dn_j(t)] = dt\mathrm{Tr}(\mathcal{J}_j(t)[\psi])$ [17,38], where $\mathbb{E}_{\psi(t)=\psi}$ denotes the expectation over the process conditional to the system being in ψ at time t. When a Poisson increment is equal to 1, the state undergoes a jump associated with the corresponding $\mathcal{J}_j(t)$. When all increments are zero, the system evolves continuously through the map

$$\mathcal{B}(t)[\psi] = -i\tilde{H}(t)\psi + i\psi\tilde{H}^\dagger(t) - \psi\mathrm{Tr}[-i\tilde{H}(t)\psi + i\psi\tilde{H}^\dagger(t)].$$

A generic time-integrated observable associated with quantum-jump events can thus be defined as

$$Q(t) = \sum_j \int_0^t f_j(v)dn_j(v). \quad (3)$$

When $f_j(v) = 1\ \forall\ j$, $Q(t)$ equals the total number of jumps occurred during a trajectory. For other choices, it is instead related to, for instance, stochastic heat or entropy production in thermal machines [43,44,47,87,88]. To characterize the properties of this observable, it is convenient to work with its moment generating function, defined as $Z_s(t) = \mathbb{E}[e^{-sQ(t)}]$ through the field s, which is conjugate to the observable. As shown in Supplemental Material (SM) [89], the moment generating function can be computed as $Z_s(t) = \mathrm{Tr}[\rho_s(t)]$, where $\dot{\rho}_s(t) = \mathcal{L}_s(t)[\rho_s(t)]$ and with $\mathcal{L}_s(t)$ being the deformed dynamical generator [57,90]

$$\mathcal{L}_s(t)[\rho] = \mathcal{L}(t)[\rho] + \sum_j (e^{-sf_j(t)} - 1)\mathcal{J}_j(t)[\rho]. \quad (4)$$

For time-independent deformed generators and large evolution times τ, $Z_s(\tau)$ obeys a large deviation principle, $Z_s(\tau) \approx e^{\tau\theta_s}$ with θ_s being the scaled cumulant generating function of $Q(\tau)$. In such a time-independent framework, θ_s coincides with the dominant eigenvalue of $\mathcal{L}_s$ [48,57] and fully characterizes the probability $P[Q(\tau) = Q^*]$. This also takes a large deviation form $P[Q(\tau) = Q^*] \approx e^{-\tau I(Q^*/\tau)}$, with rate function given by the Legendre-Fenchel transform $I(x) = \sup_{s\in\mathbb{R}}\{-sx - \theta_s\}$ [48]. In what follows, we derive the behavior of $Z_s(\tau)$ for the case of adiabatic open quantum dynamics. To this end, we consider that $\dot{Z}_s(t) = \mathrm{Tr}\{\mathcal{L}_s(t)[\varrho_s(t)]\}Z_s(t)$, where $\varrho_s(t) = \rho_s(t)/\mathrm{Tr}[\rho_s(t)]$, which we can use to express the moment generating function as

$$Z_s(\tau) = e^{\int_0^\tau \mathrm{Tr}(\mathcal{L}_s(t)[\varrho_s(t)])dt}. \quad (5)$$

As we show below, this expression allows us to write θ_s in terms of the instantaneous dominant eigenvalues of $\mathcal{L}_s(t)$.

Adiabatic theorem for deformed generators.—We consider $\mathcal{L}_s(t)$ to vary on the slow timescale $u = t/\tau$, with τ being the total evolution time and we assume it to be diagonalizable with right and left eigenmatrices, $r_s^m(t)$ and $\ell_s^m(t)$. These are such that $\mathcal{L}_s(t)[r_s^m(t)] = \lambda_s^m(t)r_s^m(t)$ and $\mathcal{L}_s^*(t)[\ell_s^m(t)] = \lambda_s^m(t)\ell_s^m(t)$, where $\lambda_s^m(t)$ are the instantaneous eigenvalues of $\mathcal{L}_s(t)$ and $\mathcal{L}_s^*(t)$ is the dual generator acting on operators. We consider the dominant eigenvalue $\lambda_s^0(t)$ to be unique (and thus real), so that $\lambda_s^0(t) > \mathrm{Re}\{\lambda_s^m(t)\}$, for $m \geq 1$. With these definitions, our adiabatic condition reads (C1) $\|\dot{r}_s^m(t)\|, \|\dot{\ell}_s^m(t)\|, |\dot{\lambda}_s^m(t)| \sim 1/\tau$, encoding that the generator varies slowly for large τ. Our second assumption is related to the uniqueness of the dominant eigenvalue $\lambda_s^0(t)$ and is conveniently expressed as the existence of a finite gap Δ for all times: (C2) $\Delta \coloneqq \inf_{m>0,\,\forall\, t}\{|\lambda_s^0(t) - \mathrm{Re}\{\lambda_s^m(t)\}|\} > 0$.

Given the two assumptions above, we prove in the SM [89] that, within the rescaled slow timescale $u = t/\tau$,

$$\lim_{\tau\to\infty} \varrho_s(u) = r_s^0(u), \qquad 0 < u \leq 1. \tag{6}$$

Note that, with a slight abuse of notation we denote the dependence on the slow timescale u in the same way as that on the original timescale t. Equation (6) shows that under the evolution with the deformed dynamical generator the normalized state $\varrho_s(u)$ follows the path of the instantaneous dominant right eigenmatrix of $\mathcal{L}_s(t)$. This result thus extends the adiabatic theorem for open quantum systems [28–33] to deformed dynamical generators and includes, for $s = 0$, the case of completely generic open quantum dynamics satisfying conditions (C1)–(C2). Importantly, controlling the evolution of the state under the deformed dynamical generator, as in our result, does only provide information about the stationary state ($s = 0$ case) as in usual adiabatic theorems, but also encodes, in a nontrivial way, some information (for $s \neq 0$) about the spectrum of excitations of the generator of the adiabatic open quantum dynamics. Furthermore, our approach allows us to establish that Eq. (6) holds, for both $s = 0$ and $s \neq 0$, irrespectively of the initial state of the system.

As a consequence of Eq. (6), the moment generating function $Z_s(\tau)$ in Eq. (5) obeys a large deviation principle, in the limit $\tau \to \infty$, with scaled cumulant generating function given by

$$\theta_s^{\mathrm{ad}} = \int_0^1 \lambda_s^0(u)\mathrm{d}u. \tag{7}$$

As such, the statistics of $Q(\tau)$ also obeys a large deviation principle [48], characterized by the function $I(Q/\tau)$, obtained as the Legendre-Fenchel transform of θ_s^{ad}. Interestingly, Eq. (7) remains valid also in the case of degenerate dominant eigenvalues $\lambda_s^0(u)$ [89].

To benchmark these results, we consider a resonantly driven two-level atom, with excited state $|e\rangle$, ground state $|g\rangle$ and Hamiltonian $H(t) = \Omega(t/\tau)(\sigma_+ + \sigma_-)$, where we defined $\sigma_- = \sigma_+^\dagger = |g\rangle\langle e|$. We assume $\Omega(u) = \Omega_0 \cos(u\pi)$ for $u < 1/2$ and $\Omega_0 \sin(u\pi)$ for $u \geq 1/2$. The atom emits photons, which is described by the jump operator $J = \sqrt{\gamma}\sigma_-$, where γ is the emission rate. We focus on

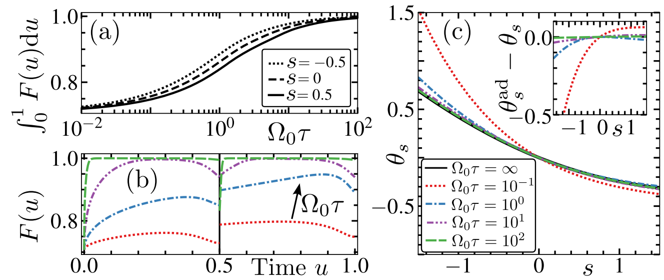

FIG. 2. Driven two-level atom. (a) Time-integrated fidelity $\int_0^1 F(u)\mathrm{d}u$, with $F(u) = \mathrm{Tr}(\sqrt{\varrho_s^{1/2}(u) r_s^0(u) \varrho_s^{1/2}(u)}$, for $s = -0.5$ (dotted line), $s = 0$ (dashed line), and $s = 0.5$ (solid line). (b) The instantaneous fidelity $F(u)$ for different values of $\Omega_0\tau$, see legend in panel (c). The parameters are $\Omega_0 = \gamma$ and $s = -0.5$. (c) Scaled cumulant generating function for the activity. The black solid line corresponds to θ_s^{ad}, while the other lines give the function θ_s for different values of $\Omega_0\tau$. The inset shows how θ_s approaches θ_s^{ad} as $\Omega_0\tau$ is increased.

the activity, i.e., the total number of quantum jumps, $A(\tau) = \int_0^\tau \mathrm{d}n(t)$ [57]. Figure 2(a) shows the time-averaged fidelity between $r_s^0(u)$ and $\varrho_s(u)$ as a function of the total time τ. The inset displays the fidelity as a function of the rescaled time u, for increasing τ. The results confirm our theorem as well as the convergence of the scaled cumulant generating function to the one in Eq. (7), as $\tau \to \infty$ [see Fig. 2(b)]. They further show that our findings remain valid in the case of piecewise-differentiable dynamical parameters.

Time history of the observable.—The scaled cumulant generating function in Eq. (7), together with its Legendre-Fenchel transform, characterizes the time-integrated observable $Q(\tau)$ during an adiabatic process. However, it is also relevant to characterize the probability of the different time histories of the observable [cf. Fig. 1(b)] realizing different values of $Q(\tau)$. To arrive at such a higher level of description of the process, we observe that, due to the adiabatic nature of the open quantum dynamics, the system spends an infinite amount of time in each of the infinitesimal (rescaled) time intervals du. For each du, it is possible to define a coarse-grained instantaneous rate $q(u)$, representing the time-averaged value of the observable at the rescaled time u (see Ref. [89] for details). We can thus write $Q(\tau) = \tau \int_0^1 q(u)\mathrm{d}u$, where $\{q(u)\}$ is a (stochastic) time history of the observable rate, as illustrated in Fig. 1(b). Discretizing time and considering that each $q(u)$ obeys an independent large deviation principle, we have that the probability over time histories is given by $P[\{q(u)\}] \approx \prod_u P[q(u)]$ and, in the continuous-time limit, with $\tau \to \infty$,

$$P[\{q(u)\}] \approx e^{-\tau\varphi[\{q(u)\}]}, \qquad \varphi[\{q(u)\}] = \int_0^1 \mathcal{I}[q(u), u]\mathrm{d}u, \tag{8}$$

where $\mathcal{I}[q(u), u]$ is the instantaneous large deviation function of $q(u)$, i.e., the Legendre-Fenchel transform of $\lambda_s^0(u)$

[91]. From a physical perspective, we expect time histories $\{q(u)\}$ to be sufficiently regular, e.g., piecewise analytic functions of time. Essentially, this shows that adiabatic open quantum dynamics obey the so-called temporal additivity principle introduced in Ref. [65]. (See SM for the formal proof [89]).

The functional in Eq. (8) contains the full information about time histories of the observable rate $\{q(u)\}$ and, thus, a complete description of fluctuations at the rescaled timescale u. The typical time history is the one minimizing the functional φ, that is, the one passing through the minima of the instantaneous rate functions $\mathcal{I}[q(u), u]$. The functional φ can further be used to derive the statistics of any observable constructed from the time history $\{q(u)\}$. An example is again the time-integrated observable $Q(\tau)$, whose functional I can be retrieved, via a contraction principle [48], as

$$I(x) = \inf_{\forall\ \{q(u)\}:\ x=\int_0^1 q(u)\mathrm{d}u} \varphi[\{q(u)\}]. \tag{9}$$

Physically, this means that the probability of observing $Q = Q^*$ is equal to the probability of the most likely time history $\{q^*(u)\}$ providing value of the time-integrated observable [cf. Fig. 1(d)].

While the general derivation of the contraction in Eq. (9) is provided in SM [89], we discuss it here using the example of the two-level atom, setting for convenience $\gamma(u) = 4\Omega(u)$ [57]. In this case, we find $\lambda_s^0(u) = 2\Omega(u)(e^{-s/3} - 1)$ and $\mathcal{I}[a(u), u] = 3\{a(u)\log[a(u)/a_0(u)] - [a(u) - a_0(u)]\}$, where $a_0(u) = (2/3)\Omega(u)$ is the typical time history of the activity rate. To compute the minimization in Eq. (9), we perform a functional derivative and set it to zero. This results in $a^*(u) = a_0(u)e^{-\mu/3}$ where μ is a Lagrange multiplier introduced to enforce the constraint in Eq. (9). Integrating $a^*(u)$ over time, we find $A^* = A_0 e^{-\mu/3}$ which fixes the Lagrange multiplier to $\mu^* = 3\log(A_0/A^*)$ with A_0 being the typical value of the time-integrated observable $A(\tau)$. Substituting this information into the functional φ [cf. Eq. (8)], we find $I(A^*/\tau) = 3[(A^*/\tau)\log A^*/A_0 - (A^* - A_0)/\tau]$, which is the same result one gets by calculating the Legendre transform of θ_s^{ad} given in Eq. (7) [57].

The functional φ is formally derived as the Legendre-Fenchel transform of the scaled cumulant generating "functional" $\Theta[\{s(u)\}]$, associated with a time dependent field $s(u)$ [89]. The function $\Theta[\{s(u)\}]$ is obtained as in Eq. (7) from the eigenvalues of the deformed operator in Eq. (4) defined with a time-dependent $s(u)$. The knowledge of such a deformed operator gives us a handle to manipulate on-demand time histories of the observable: by choosing $s^*(u)$ such that $-\delta\Theta[\{s^*(u)\}]/\delta s^*(u) = q^*(u)$, we can indeed define a suitable open quantum dynamics which produces, as typical, the rare time history $\{q^*(u)\}$ of the original process [57,59,69]. Such an auxiliary dynamics is given by [89]

$$\begin{aligned} H^{\mathrm{A}}(u) &= \frac{1}{2}\left\{\left[\ell^0_{s^*(u)}(u)\right]^{1/2}\tilde{H}(u)\left[\ell^0_{s^*(u)}(u)\right]^{-1/2} + \mathrm{H.c.}\right\}, \\ J_j^{\mathrm{A}}(u) &= e^{-\frac{s^*(u)}{2}f(u)}\left[\ell^0_{s^*(u)}(u)\right]^{1/2} J_j(u)\left[\ell^0_{s^*(u)}(u)\right]^{-1/2}, \end{aligned} \tag{10}$$

where $\ell^0_{s^*(u)}(u)$ is the dominant left eigenmatrix of the deformed operator with time-dependent field $s^*(u)$.

Applications.—As a first application, we consider a system composed by two two-level atoms attached to different thermal baths. The system Hamiltonian is $H = \omega(\sigma_e^{\mathrm{hot}} + \sigma_e^{\mathrm{cold}}) + \Omega(\sigma_-^{\mathrm{hot}}\sigma_+^{\mathrm{cold}} + \mathrm{H.c.})$, with $\sigma_e = |e\rangle\langle e|$ and where the superscripts hot and cold indicate the atom in contact with the corresponding thermal reservoir. The dynamics is governed by a time-dependent Lindblad generator derived via a weak coupling of the system with the thermal baths [17,89]. The jump operators thus read $J_{ij}^b = \sqrt{\gamma}\sqrt{(\omega_{ji}/\omega)^3 N_b(\omega_{ji})}|\epsilon_i\rangle\langle\epsilon_j|$, where b indexes the baths, γ is a rate, $N_b(\omega) = 1/(e^{\beta_b\omega} - 1)$, and $\omega_{ij} = \epsilon_j - \epsilon_i$ is the difference between the energies of the eigenstates $|\epsilon_j\rangle$ and $|\epsilon_i\rangle$ of H [17]. They implement transitions between the eigenstates $|\epsilon_i\rangle$ and $|\epsilon_j\rangle$ of H. In addition, we include a phenomenological laser driving term, $H_{\mathrm{laser}} = g(\sigma_x^{\mathrm{hot}} + \sigma_x^{\mathrm{cold}})$ [92–94]. The entropy production associated with any quantum jump in the dynamics is defined as the energy exchanged with the corresponding thermal bath responsible for the transition, $\pm\Delta\sigma_{ij}^b = \pm\omega_{ij}\beta_b$ [43,44,47]. The time-integrated entropy flow from the hot bath to the cold one can thus be defined as $\Sigma(\tau) = \sum_{i,j,b}\int_0^\tau \beta_b(t)\omega_{ij}\mathrm{d}n_{ij}^b(t)$, where $\mathrm{d}n_{ij}^b$ are the increments associated with the different jump operators [47]. The inverse temperature of the hot bath follows the protocol $\beta_{\mathrm{hot}}(t) = \beta_{\mathrm{hot}}^0/[1 + (1/2)\sin^2(t\pi/\tau)]$.

Figure 3(a) shows the instantaneous rate function for the stochastic entropy current, which displays a symmetry [95,96] related to the existence of entropy fluctuation relations at all times t [see inset of Fig. 3(a)]. Figure 3(a), also provides numerical results from quantum trajectories for both the original dynamics and an auxiliary one displaying a rare realization of the observables. We note that this analysis can be extended to other quantities, such as stochastic heat and work [88,97,98].

We then consider a three-level system, with basis states $|0\rangle, |1\rangle, |2\rangle$, and time-dependent Hamiltonian $H(t) = \Omega_1(t)|0\rangle\langle 1| + \Omega_2|0\rangle\langle 2| + \mathrm{H.c.}$ The system is subject to decay from state $|1\rangle$ to $|0\rangle$, described by the jump operator $J = \sqrt{\gamma}|0\rangle\langle 1|$. Such a system can be interpreted as a nonequilibrium quantum machine [94]. It can be readily realized in experiments [99] and may find application as a microscopic engine [100,101].

For this three-level system, we consider the activity $A(\tau) = \int_0^\tau \mathrm{d}n(t)$, which is equivalent (up to the rate γ) to the heat dissipated into the environment. We further consider

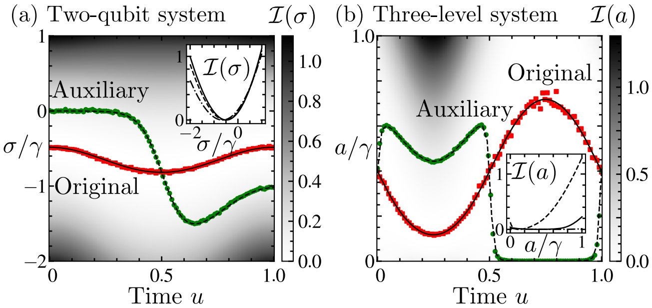

FIG. 3. Stochastic entropy production and heat fluctuations. (a), (b) Instantaneous rate function $\mathcal{I}$ as a function of time for the stochastic entropy production $[\sigma(u)]$ and the heat (proportional to the activity) rates $[a(u)]$ for the two-atom and the three-level systems, respectively. The black solid line shows the typical time history while the dashed line a rare one. Red squares are estimates obtained by running a quantum trajectory evaluated at $s = 0$, while the green bullets a quantum trajectory with the auxiliary system in Eq. (10), for $s(u) = (1/2)\tanh(10u - 5)$ in panel (a) and $s(u) = -\sin(2\pi u)$ in panel (b). The total evolution for (a) and (b) are, respectively, $\gamma\tau = 2 \times 10^6$ and $\gamma\tau = 5 \times 10^6$. Both insets show the instantaneous rate function for $u = 0$ (solid line), $u = 1/4$ (dashed line), and the dot-dashed line for (a) represents $u = 1/2$ while for (b) $u = 3/4$. The parameters for (a) are $g = 2\Omega = \omega = \gamma$ and the temperatures are $\beta^0_{\rm hot} = \beta_{\rm cold}/2 = 1$, in units of $1/\omega$. For panel (b), $10\Omega_2 = \gamma/4 = \Omega^0_1$.

$\Omega_1(t) = \Omega^0_1[1 - \sin(2\pi t/\tau)/2]$. Typical trajectories of the system feature coexistence between an active phase (frequent emissions) and an inactive one (no emissions) [57,99,102]. As such, the instantaneous rate function of the activity shows a broad minimum associated with very large fluctuations, which can also be observed in a single realization of the quantum dynamics [see red squares in Fig. 3(b)]. We then bias the system dynamics in a way that it is found in the active phase for $u < 1/2$ and in the inactive one for $u \geq 1/2$. In this case, the total dissipated heat is essentially determined by the emissions during the active phase. This simple example highlights the importance of investigating fluctuations of thermodynamic quantities in quantum machines as well as of controlling their full time history beyond the global time-integrated value.

Discussion.—We have derived a complete statistical characterization for open quantum systems in the adiabatic regime. Our analysis can be extended to different quantum stochastic processes, such as diffusive quantum trajectories associated with homodyne-detection experiments [103]. It would be interesting to explore whether the auxiliary quantum dynamics derived here can be exploited to control the performance of (adiabatic) quantum machines [104–106]. With regard to applications in adiabatic quantum computing [21,107], it would be important to generalize our analysis to characterize the full counting statistics of state-dependent observables [50], such as the fidelity. This would require the application of numerical schemes as in Ref. [108] or the derivation of a level 2.5 formalism for adiabatic open quantum dynamics [50,109]. Full counting statistics find applications in quantum sensing and interferometry [110–113]. Our results may thus allow for novel protocols for sensing critical values of dynamical parameters by detecting sudden changes, during an adiabatic evolution, of the statistics of emission related observables.

The codes used to produce the numerical results of this Letter are available on Zenodo [114].

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