Conjectures about the Chiral Phase Transition in QCD from Anomalous Multi-Instanton Interactions

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To date numerical simulations of lattice QCD have not found a chiral phase transition of first order that is expected to occur for sufficiently light pions. We show how the restoration of an exact global chiral symmetry can strongly decrease the breaking of the approximate, anomalous $U_A(1)$ symmetry. This is testable on the lattice through simulations for one through four flavors. In QCD a small breaking of the $U_A(1)$ symmetry in the chirally symmetric phase generates novel experimental signals.

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One of the most beautiful phenomena in quantum field theory is the axial anomaly of Adler, Bell, and Jackiw [1–4]. In four spacetime dimensions massless fermions are chiral, whose spin is either opposite or along the direction of motion, and so respectively left or right handed. For chirally symmetric interactions, as with a gauge field, the current for the total number of fermions, left plus right, is always conserved. In contrast, the axial current, equal to the difference of the left and right handed currents, is conserved classically but not quantum mechanically. Instead, the divergence of the axial current is proportional to the density of the topological charge for the gauge field.

In the vacuum of quantum chromodynamics (QCD), large fluctuations in the topological charge explain why the flavor singlet meson, the η', is not a Goldstone boson [5–7]. It also affects other phenomena, albeit more indirectly [8–16]. The axial anomaly also appears in condensed matter systems [17,18].

The relationship between the divergence of the axial vector current and topological charge density, computed at one loop order, is exact [1,3,4]. Even so, this does not tell one how *large* the topologically nontrivial fluctuations are [19,20]. At zero temperature they must be large in order to make the η' heavy. In contrast, at high temperature instantons are the dominant topologically nontrivial fluctuations [21,22]. In this limit the density of instantons can be computed semiclassically, which implies that the magnitude of the topological charge susceptibility vanishes as a high power of the temperature T, as $T \to \infty$. Even though it vanishes at infinite T, it is natural to expect that the density of topologically nontrivial fluctuations is nonzero for any finite T.

This leaves the relationship between the restoration of the exact chiral symmetry, and the approximate, anomalous $U_A(1)$ symmetry, obscure. Based upon extensive results from numerical simulations in lattice QCD, in this Letter we outline how the restoration of an exact chiral symmetry strongly affects the approximate restoration of the anomalous $U_A(1)$ symmetry. This can be tested in lattice QCD with different numbers of flavors, especially for a single flavor. The approximate restoration of the anomalous $U_A(1)$ symmetry has dramatic implications for the collisions of heavy ions, and surely implications for condensed matter systems as well.

Effective Lagrangians.—We consider QCD-like theories, with a $SU(N_c)$ gauge field coupled to N_f flavors of massless quarks in the fundamental representation. As massless fields, the Lagrangian is invariant under the global chiral rotations $q_{L,R} \to e^{i(\theta_V \mp \theta_A)/2} U_{L,R} q_{L,R}$, where q_L and q_R are left and right handed quarks, and U_L and U_R elements of the global symmetry groups $SU_L(N_f)$ and $SU_R(N_f)$, respectively. There are two $U(1)$ groups, one for quark number, θ_V, and one for axial quark number, θ_A.

We assume that, in vacuum, the exact global chiral symmetry is characterized by an expectation value for a color singlet, spin-zero field Φ, $\Phi = \bar{q}_L q_R$, where Φ transforms under the fundamental representation of the global symmetry group of $\mathcal{G}_{\rm cl} = SU_L(N_f) \times SU_R(N_f) \times U_A(1)$ as $\Phi \to e^{i\theta_A} U_L^\dagger \Phi U_R$. As Φ is invariant under $U_V(1)$, this symmetry can be ignored.

Through the axial anomaly, the $U_A(1)$ symmetry is violated quantum mechanically by topologically nontrivial fluctuations such as instantons. The exact chiral symmetry that remains is just $\mathcal{G}_{\rm qu} = SU_L(N_f) \times SU_R(N_f)$.

In vacuum, it is expected that chiral symmetry breaks to the maximal diagonal subgroup of $SU_V(N_f)$, $\langle \Phi^{ab} \rangle = \phi_0 \delta^{ab}$, where $a, b = 1 \ldots N_f$ are the indices for the $SU_L(N_f)$ and $SU_R(N_f)$ groups. Phenomenologically, this pattern certainly occurs in QCD, where $N_c = 3$ and $N_f = 2$ or 3. Coleman and Witten proved that it arises in the limit of large N_c and small N_f [23].

The appropriate effective Lagrangian for chiral symmetry breaking is well known [5–7,11,12,19,24–45]. There are two types of terms that enter. The first type are terms invariant under $\mathcal{G}_{\rm cl}$. Up to terms of sixth order in Φ, these are

$$\begin{aligned} \mathcal{L}_{\rm cl} = & \operatorname{tr}(|\partial_\mu \Phi|^2) + m^2 \operatorname{tr}(\Phi^\dagger \Phi) \\ & + \lambda_1 (\operatorname{tr}(\Phi^\dagger \Phi))^2 + \lambda_2 \operatorname{tr}(\Phi^\dagger \Phi)^2 \\ & + \kappa_1 (\operatorname{tr}(\Phi^\dagger \Phi))^3 + \kappa_2 \operatorname{tr}(\Phi^\dagger \Phi) \operatorname{tr}(\Phi^\dagger \Phi)^2 \\ & + \kappa_3 \operatorname{tr}(\Phi^\dagger \Phi)^3. \end{aligned} \tag{1}$$

Our trace is normalized, so $\operatorname{tr}\mathbf{1} = 1$. For a gauge theory in $3+1$ dimensions, a phase transition at a nonzero temperature T is characterized by an effective theory in three dimensions. Couplings to sixth order then represent the relevant operators: the mass squared, m^2; two quartic coupling constants, λ_1 and λ_2, with dimensions of mass; and the six point couplings: κ_1, κ_2, and κ_3, with dimensionless coupling constants. Terms of eighth and higher order are irrelevant operators, whose coupling constants have negative mass dimension.

The second class of terms are invariant under $\mathcal{G}_{\rm qu}$ but not $U_A(1)$, and so are generated by topologically nontrivial fluctuations [5–7,11,12,19,30,34,36–38]:

$$\begin{aligned} \mathcal{V}_{\rm qu} = & -\xi_1 (\det \Phi + \det \Phi^\dagger) \\ & - \xi_1^1 (\operatorname{tr} \Phi^\dagger \Phi)(\det \Phi + \det \Phi^\dagger) \\ & - \xi_2 [(\det \Phi)^2 + (\det \Phi^\dagger)^2]. \end{aligned} \tag{2}$$

For three flavors, these are terms to third, fifth, and sixth order in Φ.

The Atiyah-Singer index theorem relates the change in the axial fermion number to the topological charge as $n_L - n_R = N_f Q$. An instanton with topological charge $Q > 0$ has $N_f Q$ left handed zero modes, q_L, while for an anti-instanton with $Q < 0$, the quark zero modes are right handed [46]. Thus, the first two terms, $\sim \det \Phi$, arise from instantons with charge 1, [5–7,24], while the last term, $\sim (\det \Phi)^2$, is due to instantons with charge 2 [11,12].

We comment that instead of the term $\sim \xi_1$, for three flavors, Refs. [30,36–38] use $\xi_1 = \xi_1^1 = 0$, and just a single anomalous coupling, $\sim \xi_2' [(\det \Phi)^2 - (\det \Phi^\dagger)^2]$. The operators $\sim \xi_2$ and $\sim \xi_2'$ differ by a term $\sim \det \Phi^\dagger \det \Phi = \det(\Phi^\dagger \Phi)$. This operator is invariant under $U_A(1)$, and so for three flavors, the coupling $\sim \xi_2'$ is equivalent to that $\sim \xi_2$, plus a modification of the $U_A(1)$ invariant couplings of sixth order in Eq. (1). A similar relation applies for any number of flavors ≥ 2. We prefer to use the coupling $\sim \xi_2$, as that is uniquely generated by instantons with charge 2.

The anomalous couplings in Eq. (2) are the first terms in an infinite series

$$\mathcal{V}_{\rm qu} = \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} \xi_i^j f_i^j (\Phi^\dagger \Phi)((\det \Phi)^i + (\det \Phi^\dagger)^i), \tag{3}$$

where $f_i^j(\Phi^\dagger \Phi)$ involves all independent terms of order $(\Phi^\dagger \Phi)^j$ [47]. Terms with couplings $\sim \xi_i^j$ are generated by fluctuations with topological charge $|Q| = i$. For ease of notation we denote $\xi_i^0 \equiv \xi_i$.

A conjecture about anomalous couplings.—At the outset we recognize that especially in vacuum, the topologically nontrivial configurations are surely truly quantum objects and far from any semiclassical approximation [20]. For ease of discussion, we refer to the dominant configurations in vacuum as quantum instantons, and those that dominate when $T \to \infty$ as semiclassical instantons. The contribution of a single semiclassical instanton to the partition function is $\sim \exp[-8\pi^2/g^2(T)]$, so by asymptotic freedom this falls off as a high power of the temperature [48]. Numerical simulations of lattice QCD indicate that the topological susceptibility falls off close to this power down to temperatures of $T_{\rm qu} \sim 300$ MeV [49]. While astonishingly low, this is still about twice the temperature for the chiral crossover in QCD, at $T_\chi \sim 156$ MeV [50–54]. Thus, we can take $T_{\rm qu}$ as an estimate of the change from quantum to semiclassical instantons [55].

The essential question is what is the relative magnitude of the anomalous coupling constants in vacuum and as the temperature increases. The standard assumption with effective Lagrangians is that the couplings with the highest mass dimension dominate. For the $U_A(1)$ symmetric Lagrangian of Eq. (1), that is the mass squared, followed by the quartic couplings, etc. In the standard Wilsonian paradigm, this is inescapable, because the only way of differentiating these different operators is through their mass dimension.

Of course some operators have a larger symmetry than others: $m^2 \operatorname{tr}(\Phi^\dagger \Phi)$ and $\lambda_1 [\operatorname{tr}(\Phi^\dagger \Phi)]^2$ are invariant under $O(2N_f^2)$, while the coupling $\lambda_2 \operatorname{tr}(\Phi^\dagger \Phi)^2$ is only invariant under $\mathcal{G}_{\rm cl}$. But this is standard, and does not affect the renormalization group flow [56]. The only time that couplings of sixth order need to be included is at isolated points where both λ_1 and λ_2 vanish; then there is a tricritical point, controlled by the evolution of the six-point coupling constants, κ_1, κ_2, and κ_3.

For the anomalous coupling constants, the operator with the lowest mass dimension is $\xi_1 \det \Phi$. Thus, naively one expects that this operator dominates the infrared behavior near the chiral phase transition [19].

However, there is something special about the anomalous couplings that is *not* true in standard effective theories. Terms $\sim \det \Phi$ are due, uniquely, to the zero modes of an instanton with charge 1; those $\sim(\det \Phi)^2$, to the zero modes of an instanton with charge 2, etc. [5–7,11,12].

In vacuum, when chiral symmetry breaking occurs the effective coupling for the *first* anomalous coupling, $\sim \det \Phi$, is a sum of an infinite number of terms

$$\xi_1^{\rm eff}(T) = \sum_{i=1}^{\infty}\sum_{j=0}^{\infty} \xi_i^j(T) f_i^j(\phi_0(T)^2) i \phi_0(T)^{(i-1)N_f}. \quad (4)$$

As indicated, all of the anomalous coupling constants, the ξ_i^j, and the expectation value of the scalar field, ϕ_0, are functions of temperature. At very high temperature, the anomalous coupling constants ξ_i can be computed semiclassically, and are all nonzero [5–7,11,12].

We conjecture the following. In vacuum, the contribution of $\xi_1(0)$ to the total coupling, $\xi_1^{\rm eff}(0)$, is small. Instead, terms nominally of higher order in Φ in the effective action are enhanced by corresponding powers of the chiral condensate, such as $\sim\phi_0(0)^{N_f}\xi_2(0)$, $\sim\phi_0(0)^{2N_f}\xi_3(0)$, etc. Our conjecture is that these terms dominate $\xi_1(0)$ numerically.

In contrast, in the chirally symmetric phase for $T \geq T_\chi$, the chiral condensate vanishes, $\phi_0(T) = 0$. For $T > T_{\rm qu}$, the $\xi_i(T) \sim [\xi_1(T)]^i$, and then $\xi_1(T)$ certainly dominates over $\xi_{i\geq 2}(T)$. This is just because in weak coupling, semiclassical instantons necessarily form a dilute gas [11,12].

Why should our conjecture be valid? Consider forming an effective Lagrangian for chiral symmetry breaking from the underlying gauge theory. We integrate out quarks and gluons to form an effective theory for Φ over some volume V_χ. The essential question is, then, what is the distribution of quantum instantons that contributes in V_χ.

If in V_χ quantum instantons with net charge 1 dominate, then so will the operator $\sim\xi_1 \det \Phi$. If instead V_χ predominately contains quantum instantons with net charge 2, then the operator $\sim\xi_2(\det \Phi)^2$ will be more important. We suggest, then, that in vacuum quantum instantons with charge 2 and greater dominate V_χ. Of course in all, the topological charge of the vacuum vanishes. But it need not within a finite volume V_χ [57].

We now discuss the implications of our conjecture, beginning with the case of three flavors that motivated it.

Three flavors.—In QCD there is no true phase transition, only a crossover (albeit with a large increase in the pressure). If $\xi_1(T_\chi) \neq 0$, however, for three flavors the operator $\sim \det \Phi$ is a cubic operator. The presence of a cubic operator implies that the standard effective Lagrangian for a second-order phase transition, with only terms quartic and quadratic in the fields, cannot be reached, and so the transition is of first order. Hence, a chiral phase transition of first order *must* emerge for sufficiently light pions, $m_\pi < m_\pi^{\rm crit}$ [19]. For simplicity we discuss the case of three degenerate quark flavors.

How large $m_\pi^{\rm crit}$ is depends upon the magnitude of $\xi_1(T_\chi)$. We suggest that in vacuum the η' is heavy *not* because ξ_1 is large, but because the higher order terms, such as ξ_2, ξ_3, etc., contribute and overwhelm ξ_1. At the chiral phase transition, however, $\phi_0 = 0$, and one is left with just $\xi_1^{\rm eff}(T_\chi) = \xi_1(T_\chi)$. If $\xi_1(T_\chi)$ is small, then so is $m_\pi^{\rm crit}$.

In mean field theory, it is customary to assume that $\xi_1(T)$ is independent of temperature. Since the η' is so heavy at zero temperature, in vacuum $\xi_1(0)$ must be large, and $m_\pi^{\rm crit}$ should also be large. In a quark meson model, one finds $m_\pi^{\rm crit} \approx 150$ MeV if the vacuum fluctuations of quarks are ignored [58], and $m_\pi^{\rm crit} \approx 86$ MeV if they are included [59]. Similarly, using mean field theory in a chiral matrix model yields $m_\pi^{\rm crit} \approx 110$ MeV [60].

Going beyond mean field theory, mesonic fluctuations can be included by using the functional renormalization group. This gives rise to a critical mass that is dramatically smaller but still nonzero, $m_\pi^{\rm crit} \approx 17$ MeV [59]. Presumably this occurs because the functional renormalization group is including, at least in part, higher-order anomalous contributions as in Eq. (4) [61].

In contrast, *no* simulation of lattice QCD has ever found evidence of a first order transition, with $m_\pi^{\rm crit} < 50$ MeV from Ref. [63] and $m_\pi^{\rm crit} < 100$ MeV in Ref. [64]. By considering the position of the tricritical point as a function of N_f, Ref. [65] finds that even for three flavors, the chiral transition is of second order in the chiral limit. This is also consistent with Ref. [66], by extrapolation from $m_\pi^{\rm crit} \sim 80$ MeV.

We note that a small value of $\xi_1(0)$ is perfectly consistent with hadronic phenomenology at zero temperature, for both hadronic masses and decay widths. In fact, these quantities can be reproduced successfully in low-energy models even with ξ_2 as the *only* anomalous coupling [30,31,36–38]. This will be analyzed in greater detail in future work [67].

While we assume that $\xi_1(T_\chi) \neq 0$, we stress that we *cannot* exclude the possibility that $\xi_1(T_\chi) = 0$. From the viewpoint of effective Lagrangians, this is most unnatural, as then two parameters—$m^2(T)$ and $\xi_1(T)$—vanish as one thermodynamic parameter, the temperature, is varied.

If the result of Ref. [65] holds and the chiral transition is of second order, then we speculate that not just $\xi_1(T_\chi)$, but *all* of the anomalous couplings vanish at the critical temperature

$$\xi_i^j(T_\chi) = 0. \quad (5)$$

This implies that the anomalous $U_A(1)$ symmetry is restored at T_χ. This can only happen precisely at T_χ, since as $T \to \infty$ semiclassical instantons are present and give $\xi_i^j(T) \neq 0$. It certainly is not a generic property of effective models, as seen by different approaches [40,44,68,69]. Thus, it *must* be specific to its origin from the properties of fermion zero modes at T_χ, which must then decouple from the critical modes. The relation of Eq. (5) to the 't Hooft anomaly condition is also of interest [70,71].

A necessary condition for a second order transition is the existence of an infrared stable fixed point in the underlying critical theory. So far, no stable fixed point in the $\mathcal{G}_{\rm qu}$ universality class has been found [19,45,72]. Regarding $\mathcal{G}_{\rm cl}$ universality, analyses with the ϵ expansion in $4-\epsilon$ dimensions [19,73,74], perturbation theory in three dimensions [75], and Monte Carlo simulations [76,77] found no fixed point, indicating a first order transition. In contrast, recent results from the functional renormalization group [45] and the conformal bootstrap [78–80] find a stable fixed point. Hence, from critical physics a second order transition seems more likely if $U(1)_A$ is restored at T_χ. While unsettled, we therefore assume that if $\xi_1(T_\chi)=0$, three massless flavors could have a chiral transition of second order in the universality class of $\mathcal{G}_{\rm cl}$.

This is in accord with recent results using Dyson-Schwinger equations [81], where a second-order chiral transition is found in the chiral limit. In this case scaling analysis shows that the universal physics is described by mean field behavior without further external input. A second order transition then arises if $\xi_1(T_\chi)=0$, e.g., Refs. [59,67], providing strong indications that this is also true in Ref. [81].

Two and four flavors.—For two flavors, the term $\sim\xi_1^{\rm eff}$ is a mass term that splits the η meson from the pions. The couplings $\sim\xi_1^1$ and $\sim\xi_2$ are of quartic order. Thus, in the chiral limit, $\xi_1\neq 0$ implies that the η meson is massive at T_χ, and the universality class is that of $\mathcal{G}_{\rm qu}=SU_L(2)\times SU_R(2)\equiv O(4)$. Numerical simulations using Wilson fermions by Brandt *et al.* [82] find that the mass of the η meson is much smaller near T_χ than at $T=0$, in accord with our conjecture. If the speculation of Eq. (5) holds, then the η meson is massless at T_χ, and the universality class is then $O(4)\times O(2)$. Interestingly, recent analyses show that stable fixed points exist not only for $O(4)$, but also for $O(4)\times O(2)$ universality [34,78,80].

For four flavors, the coupling $\sim\xi_1\det\Phi$ is of quartic order, and a relevant quartic coupling, of the same mass dimension as the couplings $\sim\lambda_1$ and $\sim\lambda_2$. The critical behavior of $\mathcal{G}_{\rm qu}$ for $N_f=4$ is unknown. Preliminary studies indicate that very light quarks are necessary to see a first order transition [83].

One flavor.—An interesting test of our conjecture is for a single, massless flavor [25,28,29]. Taking $\Phi=\phi+i\eta$, where $\Phi^\dagger\Phi=\phi^2+\eta^2$, and $(\det\Phi+\det\Phi^\dagger)=2\phi$. Including all couplings to quartic order, the effective Lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm qu}=&(\partial_i\phi)^2+(\partial_i\eta)^2+\xi_1\phi\\&+m^2(\phi^2+\eta^2)+\xi_2(\phi^2-\eta^2)\\&+\xi_3\phi(\phi^2-\eta^2)+\xi_1^1\phi(\phi^2+\eta^2)\\&+\lambda(\phi^2+\eta^2)^2\\&+\xi_4(\phi^4-6\phi^2\eta^2+\eta^4)+\xi_2^1(\phi^4-\eta^4).\end{aligned}\tag{6}$$

If $\xi_1\neq 0$, instantons directly induce a vacuum expectation value for ϕ.

If our conjecture is correct, then while there may be no true chiral phase transition, there could well be a sharp crossover from a low temperature phase, dominated by quantum instantons with large $\xi_1(T)$ and $\phi_0(T)$, to a phase dominated by semiclassical instantons, with small $\xi_1(T)$ and $\phi_0(T)$. As $T\to\infty$, $\xi_1(T)$ and $\phi_0(T)\to 0$.

If the speculation of Eq. (5) is true, only $\lambda(T_\chi)\neq 0$, with $m^2(T_\chi)$ and *all* $\xi_i^j(T_\chi)=0$. There is then a chiral phase transition of second order for an emergent $U_A(1)$ symmetry at T_χ. This would be most dramatic.

Implications for QCD.—We have worked exclusively in the chiral limit. What are the implications for QCD, where numerical simulations on the lattice find no true phase transition, but cross over [84–86]?

If QCD is close to the chiral limit for three massless flavors, then the restoration of the axial $U_A(1)$ symmetry at T_χ surely implies that the *approximate* restoration of the axial $U_A(1)$ symmetry is closely tied to the crossover temperature.

In numerical simulations of lattice QCD, it is common to measure the violation of the anomalous $U_A(1)$ symmetry by computing the difference in the two point functions of pions and a_0 mesons [87–92]. This is useful for two light flavors, but since for three flavors a term $\sim\det\Phi$ is cubic in Φ, when $\phi_0=0$ anomalous terms do not affect mesonic two point functions [93–96]. In lattice QCD, at present the situation is unsettled [50,97]: Refs. [87,89,92,98–102] find that the anomalous symmetry is not even approximately restored by T_χ, while Refs. [65,66,82,88,90,91,103] find that it is.

Our analysis also applies to nonzero quark chemical potential, μ. For a theory at $T\neq 0$, the effective theory is three dimensional. If $T\ll\mu$, though, the relevant effective theory is then in four dimensions. Assuming that confinement gaps the quarks and gluons, the effective theory is again that of Eqs. (1) and (2). While the mass dimensions of the coupling constants change, the conclusion remains that if $\xi_1(T_\chi)\neq 0$, the chiral phase transition is of first order in the chiral limit.

Our analysis predicts that the breaking of the anomalous $U_A(1)$ symmetry is uniformly *small* in a chirally symmetric regime. The η' meson, which is heavy is vacuum, must become light.

There is an interesting possibility that arises. Like the $U_A(1)$ invariant coupling constants, the anomalous coupling constants are all functions of *both* temperature and chemical potential, $\xi_i^j(T,\mu)$. Analogous to the critical endpoint, where for two light flavors the $O(4)$ invariant quartic coupling constant vanishes, $\lambda(T^{\rm cr},\mu^{\rm cr})=0$ [104–107], since we have two thermodynamic parameters to vary, it is possible that there is a *single* point in the phase diagram where $\xi_1(T^A,\mu^A)=0$. About this point, instead of

$SU_V(3)$ flavor eigenstates, the π^0, η, and η' are eigenstates of flavor, and there is a large violation of isospin [19]. It is very intriguing that such a violation has been reported by the NA61/SHINE Collaboration recently [108,109].

If $\xi_1(T,\mu)$ vanishes at a point in the plane of T and μ, then perhaps there is a region where $\xi_1(T,\mu)$ is of *opposite* sign to that in the vacuum. If chiral symmetry is broken, then instead of the σ meson condensing, the η' does. This implies that CP symmetry is spontaneously broken by an η' condensate.

Other signals that have been suggested include Hanbury-Brown-Twiss correlations [110–112], possibly confirmed by the PHENIX experiment [113], and an excess of soft dileptons [114]. The HADES experiment finds that the η meson is about twice as abundant as expected from a statistical distribution [115,116]. Certainly when the η' meson becomes light, so does the η meson [117].

Besides the other implications of our results, it is also natural to wonder how the suppression of topologically nontrivial fluctuations in a chirally symmetric phase affects baryogenesis in the early Universe [119].

Note added.—After this Letter was submitted for publication, Ref. [120] showed in the local potential approximation of the functional renormalization group that if the $U_A(1)$ symmetry is restored at T_χ, then the chiral transition can be of second order for all $N_f \geq 2$, contrary to the ϵ expansion.

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[1] S. L. Adler, Axial vector vertex in spinor electrodynamics, Phys. Rev. **177**, 2426 (1969).
[2] J. S. Bell and R. Jackiw, A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ model, Nuovo Cimento A **60**, 47 (1969).
[3] S. L. Adler and W. A. Bardeen, Absence of higher order corrections in the anomalous axial vector divergence equation, Phys. Rev. **182**, 1517 (1969).
[4] W. A. Bardeen, Anomalous Ward identities in spinor field theories, Phys. Rev. **184**, 1848 (1969).
[5] G. 't Hooft, Symmetry breaking through Bell-Jackiw anomalies, Phys. Rev. Lett. **37**, 8 (1976).
[6] G. 't Hooft, Computation of the quantum effects due to a four-dimensional pseudoparticle, Phys. Rev. D **14**, 3432 (1976); **18**, 2199(E) (1978).
[7] G. 't Hooft, How instantons solve the U(1) problem, Phys. Rep. **142**, 357 (1986).
[8] G. Veneziano, Is there a QCD spin crisis?, Mod. Phys. Lett. A **04**, 1605 (1989).
[9] G. M. Shore, The $U_A(1)$ anomaly and QCD phenomenology, Lect. Notes Phys. **737**, 235 (2008).
[10] D. E. Kharzeev, J. Liao, S. A. Voloshin, and G. Wang, Chiral magnetic and vortical effects in high-energy nuclear collisions—A status report, Prog. Part. Nucl. Phys. **88**, 1 (2016).
[11] R. D. Pisarski and F. Rennecke, Multi-instanton contributions to anomalous quark interactions, Phys. Rev. D **101**, 114019 (2020).
[12] F. Rennecke, Higher topological charge and the QCD vacuum, Phys. Rev. Res. **2**, 033359 (2020).
[13] A. Tarasov and R. Venugopalan, Role of the chiral anomaly in polarized deeply inelastic scattering: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics, Phys. Rev. D **102**, 114022 (2020).
[14] A. Tarasov and R. Venugopalan, Role of the chiral anomaly in polarized deeply inelastic scattering. II. Topological screening and transitions from emergent axionlike dynamics, Phys. Rev. D **105**, 014020 (2022).
[15] F. Giacosa, A. Koenigstein, and R. D. Pisarski, How the axial anomaly controls flavor mixing among mesons, Phys. Rev. D **97**, 091901(R) (2018).
[16] F. Giacosa, S. Jafarzade, and R. Pisarski, Anomalous interactions for heterochiral mesons with $J^{PC} = 1^{+-}$ and 2^{-+}, Phys. Rev. D **109**, L071502 (2024).
[17] C. Rylands, A. Parhizkar, A. A. Burkov, and V. Galitski, Chiral anomaly in interacting condensed matter systems, Phys. Rev. Lett. **126**, 185303 (2021).
[18] E. Fradkin, Field theoretic aspects of condensed matter physics: An overview, arXiv:2301.13234.
[19] R. D. Pisarski and F. Wilczek, Remarks on the chiral phase transition in chromodynamics, Phys. Rev. D **29**, 338 (1984).
[20] V. P. Nair and R. D. Pisarski, Fractional topological charge in SU(N) gauge theories without dynamical quarks, Phys. Rev. D **108**, 074007 (2023).
[21] D. J. Gross, R. D. Pisarski, and L. G. Yaffe, QCD and instantons at finite temperature, Rev. Mod. Phys. **53**, 43 (1981).
[22] A. Boccaletti and D. Nogradi, The semi-classical approximation at high temperature revisited, J. High Energy Phys. 03 (2020) 045.
[23] S. R. Coleman and E. Witten, Chiral symmetry breakdown in large N chromodynamics, Phys. Rev. Lett. **45**, 100 (1980).
[24] G. 't Hooft, Confinement and topology in nonabelian gauge theories, Acta Phys. Austriaca Suppl. **22**, 531 (1980).
[25] H. Leutwyler and A. V. Smilga, Spectrum of Dirac operator and role of winding number in QCD, Phys. Rev. D **46**, 5607 (1992).
[26] S. P. Klevansky, The Nambu-Jona-Lasinio model of quantum chromodynamics, Rev. Mod. Phys. **64**, 649 (1992).

[27] D. U. Jungnickel and C. Wetterich, Effective linear meson model, Eur. Phys. J. C **1**, 669 (1998).

[28] M. Creutz, One flavor QCD, Ann. Phys. (Amsterdam) **322**, 1518 (2007).

[29] M. Creutz, Anomalies and chiral symmetry in QCD, Ann. Phys. (Amsterdam) **324**, 1573 (2009).

[30] D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, and D. H. Rischke, Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons, Phys. Rev. D **87**, 014011 (2013).

[31] P. Kovács and G. Wolf, Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons: Investigation of the $U(1)_A$ anomaly term, Acta Phys. Pol. B Proc. Suppl. **6**, 853 (2013).

[32] M. Grahl and D. H. Rischke, Functional renormalization group study of the two-flavor linear sigma model in the presence of the axial anomaly, Phys. Rev. D **88**, 056014 (2013).

[33] M. Mitter and B.-J. Schaefer, Fluctuations and the axial anomaly with three quark flavors, Phys. Rev. D **89**, 054027 (2014).

[34] A. Pelissetto and E. Vicari, Relevance of the axial anomaly at the finite-temperature chiral transition in QCD, Phys. Rev. D **88**, 105018 (2013).

[35] M. Grahl, $U(2)_A \times U(2)_V$-symmetric fixed point from the functional renormalization group, Phys. Rev. D **90**, 117904 (2014).

[36] L. Olbrich, M. Zétényi, F. Giacosa, and D. H. Rischke, Three-flavor chiral effective model with four baryonic multiplets within the mirror assignment, Phys. Rev. D **93**, 034021 (2016).

[37] F. Divotgey, P. Kovacs, F. Giacosa, and D. H. Rischke, Low-energy limit of the extended linear sigma model, Eur. Phys. J. A **54**, 5 (2018).

[38] D. Parganlija and F. Giacosa, Excited scalar and pseudoscalar mesons in the extended linear sigma model, Eur. Phys. J. C **77**, 450 (2017).

[39] F. Rennecke and B.-J. Schaefer, Fluctuation-induced modifications of the phase structure in $(2+1)$-flavor QCD, Phys. Rev. D **96**, 016009 (2017).

[40] G. Fejos, Functional dependence of axial anomaly via mesonic fluctuations in the three flavor linear sigma model, Phys. Rev. D **92**, 036011 (2015).

[41] S. D. Bass and P. Moskal, η' and η mesons with connection to anomalous glue, Rev. Mod. Phys. **91**, 015003 (2019).

[42] D. Horvatić, D. Kekez, and D. Klabučar, η' and η mesons at high T when the $U_A(1)$ and chiral symmetry breaking are tied, Phys. Rev. D **99**, 014007 (2019).

[43] A. Gómez Nicola, J. Ruiz De Elvira, and A. Vioque-Rodríguez, The QCD topological charge and its thermal dependence: the role of the η', J. High Energy Phys. 11 (2019) 086.

[44] J. Braun, M. Leonhardt, J. M. Pawlowski, and D. Rosenblüh (QCD Collaboration), Chiral and effective $U(1)_{\mathrm{A}}$ symmetry restoration in QCD, arXiv:2012.06231.

[45] G. Fejos, Second-order chiral phase transition in three-flavor quantum chromodynamics?, Phys. Rev. D **105**, L071506 (2022).

[46] L. S. Brown, R. D. Carlitz, and C.-k. Lee, Massless excitations in instanton fields, Phys. Rev. D **16**, 417 (1977).

[47] That is, $f_i^j(\Phi^\dagger\Phi) = (\mathrm{tr}\Phi^\dagger\Phi)^j + \xi_i^{j,2}(\mathrm{tr}\Phi^\dagger\Phi)^{j-2}\mathrm{tr}(\Phi^\dagger\Phi)^2 + \ldots + \xi_i^{j,j}\mathrm{tr}(\Phi^\dagger\Phi)^j$.

[48] For a $SU(N_c)$ gauge theory coupled to N_f massless flavors of quarks, the topological susceptibility with massive quarks of mass m falls off as $\sim m^{N_f}/T^{(c-4)}$, $c = (11N_c - 4N_f C_f)/3$, $C_f = (N^2-1)/(2N)$. The factor of c is from the leading order coefficient of the β function, and the factor of -4 from integration over the instanton scale size. Note that we work in the chiral limit, where $m = 0$ and the topological susceptibility vanishes as m^{N_f}.

[49] The overall magnitude of the topological susceptibility from a computation of semiclassical instantons to one loop order is too small by about an order of magnitude, but surely the two loop computation is called for.

[50] A. Y. Kotov, M. P. Lombardo, and A. M. Trunin, Fate of the η' in the quark gluon plasma, Phys. Lett. B **794**, 83 (2019).

[51] P. T. Jahn, P. M. Junnarkar, G. D. Moore, and D. Robaina, Multicanonical reweighting for the QCD topological susceptibility, Phys. Rev. D **104**, 014502 (2021).

[52] S. Borsanyi and D. Sexty, Topological susceptibility of pure gauge theory using density of states, Phys. Lett. B **815**, 136148 (2021).

[53] Y.-C. Chen, T.-W. Chiu, and T.-H. Hsieh (TWQCD Collaboration), Topological susceptibility in finite temperature QCD with physical $(u/d, s, c)$ domain-wall quarks, Phys. Rev. D **106**, 074501 (2022).

[54] G. Aarts *et al.*, Phase transitions in particle physics: Results and perspectives from lattice quantum chromo-dynamics, Prog. Part. Nucl. Phys. **133**, 104070 (2023).

[55] It is notable that lattice QCD finds an intermediate region of $T_\chi < T < T_{\mathrm{qu}}$ [54], where the quantum instantons have a topological susceptibility different from $T > T_{\mathrm{qu}}$, but the presence of this intermediate regime does not significantly affect our qualitative analysis.

[56] This includes the possibility that the symmetry is enlarged at a critical point, as when $\lambda_1^{\mathrm{crit}} \neq 0$ and $\lambda_2^{\mathrm{crit}} = 0$. But in computing in the renormalization group flow, both couplings need to be included.

[57] Lattice QCD finds that in measuring quantities related to topologically nontrivial fluctuations, that the autocorrelation times are much larger than in measuring mass spectra [54]. Our analysis suggests that relative to standard hadronic mass scales, that quantum instantons are small and/or overlap strongly. Presumably much finer lattices are required to measure topological fluctuations.

[58] B.-J. Schaefer and M. Wagner, The three-flavor chiral phase structure in hot and dense QCD matter, Phys. Rev. D **79**, 014018 (2009).

[59] S. Resch, F. Rennecke, and B.-J. Schaefer, Mass sensitivity of the three-flavor chiral phase transition, Phys. Rev. D **99**, 076005 (2019).

[60] V. V. Skokov, Mean field analysis of the chiral phase transition in a chiral matrix model (unpublished).

[61] We note that the approximation used in Ref. [59] is known to overestimate mesonic fluctuations that tend to soften the phase transition [62]. But this effect alone cannot explain why m_π^{crit} is so small in Ref. [59].

[62] J. M. Pawlowski and F. Rennecke, Higher order quark-mesonic scattering processes and the phase structure of QCD, Phys. Rev. D **90**, 076002 (2014).
[63] A. Bazavov, H. T. Ding, P. Hegde, F. Karsch, E. Laermann, S. Mukherjee, P. Petreczky, and C. Schmidt, Chiral phase structure of three flavor QCD at vanishing baryon number density, Phys. Rev. D **95**, 074505 (2017).
[64] Y. Kuramashi, Y. Nakamura, H. Ohno, and S. Takeda, Nature of the phase transition for finite temperature $N_{\mathrm{f}} = 3$ QCD with nonperturbatively O(a) improved Wilson fermions at $N_{\mathrm{t}} = 12$, Phys. Rev. D **101**, 054509 (2020).
[65] F. Cuteri, O. Philipsen, and A. Sciarra, On the order of the QCD chiral phase transition for different numbers of quark flavours, J. High Energy Phys. 11 (2021) 141.
[66] L. Dini, P. Hegde, F. Karsch, A. Lahiri, C. Schmidt, and S. Sharma, Chiral phase transition in three-flavor QCD from lattice QCD, Phys. Rev. D **105**, 034510 (2022).
[67] F. Giacosa, G. Kovacs, P. Kovacs, R. D. Pisarski, and F. Rennecke, The order of the chiral phase transition and anomalous correlations in a low-energy model (to be published).
[68] G. Fejös and A. Patkos, Backreaction of mesonic fluctuations on the axial anomaly at finite temperature, Phys. Rev. D **105**, 096007 (2022).
[69] Y. Nakayama and T. Ohtsuki, Conformal bootstrap dashing hopes of emergent symmetry, Phys. Rev. Lett. **117**, 131601 (2016).
[70] G. 't Hooft, Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking, NATO Sci. Ser. B **59**, 135 (1980).
[71] K. Fujikawa and K. Umetsu, Berry's phase and chiral anomalies, Prog. Part. Nucl. Phys. **128**, 103992 (2023).
[72] A. Butti, A. Pelissetto, and E. Vicari, On the nature of the finite temperature transition in QCD, J. High Energy Phys. 08 (2003) 029.
[73] R. D. Pisarski and D. L. Stein, Critical behavior of linear ϕ^4 models with $G \times G'$ symmetry, Phys. Rev. B **23**, 3549 (1981).
[74] R. D. Pisarski and D. L. Stein, The renormalisation group and global G × G' theories about four dimensions, J. Phys. A **14**, 3341 (1981).
[75] P. Calabrese and P. Parruccini, Five loop epsilon expansion for $\mathrm{U(n)} \times \mathrm{U(m)}$ models: Finite temperature phase transition in light QCD, J. High Energy Phys. 05 (2004) 018.
[76] A. O. Sorokin, First-order and pseudo-first-order transition in the high dimensional $O(N) \otimes O(M)$ model, arXiv:2105.00072.
[77] A. O. Sorokin, Phase transition in the three-dimensional $O(N) \otimes O(M)$ model: A Monte Carlo study, arXiv:2205.07199.
[78] Y. Nakayama and T. Ohtsuki, Bootstrapping phase transitions in QCD and frustrated spin systems, Phys. Rev. D **91**, 021901(R) (2015).
[79] J. Henriksson, S. R. Kousvos, and A. Stergiou, Analytic and numerical bootstrap of CFTs with $O(m) \times O(n)$ global symmetry in 3D, SciPost Phys. **9**, 035 (2020).
[80] S. R. Kousvos and A. Stergiou, CFTs with $\mathbf{U(m)} \times \mathbf{U(n)}$ global symmetry in 3D and the chiral phase transition of QCD, SciPost Phys. **15**, 075 (2023).
[81] J. Bernhardt and C. S. Fischer, QCD phase transitions in the light quark chiral limit, Phys. Rev. D **108**, 114018 (2023).
[82] B. B. Brandt, A. Francis, H. B. Meyer, O. Philipsen, D. Robaina, and H. Wittig, On the strength of the $U_A(1)$ anomaly at the chiral phase transition in $N_f = 2$ QCD, J. High Energy Phys. 12 (2016) 158.
[83] P. de Forcrand and M. D'Elia, Continuum limit and universality of the Columbia plot, Proc. Sci., LATTICE2016 (**2017**) 081 [arXiv:1702.00330].
[84] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz, and K. K. Szabo, The order of the quantum chromodynamics transition predicted by the standard model of particle physics, Nature (London) **443**, 675 (2006).
[85] A. Bazavov *et al.* (HotQCD Collaboration), Chiral crossover in QCD at zero and non-zero chemical potentials, Phys. Lett. B **795**, 15 (2019).
[86] S. Borsanyi, Z. Fodor, J. N. Guenther, R. Kara, S. D. Katz, P. Parotto, A. Pasztor, C. Ratti, and K. K. Szabo, QCD crossover at finite chemical potential from lattice simulations, Phys. Rev. Lett. **125**, 052001 (2020).
[87] V. Dick, F. Karsch, E. Laermann, S. Mukherjee, and S. Sharma, Microscopic origin of $U_A(1)$ symmetry violation in the high temperature phase of QCD, Phys. Rev. D **91**, 094504 (2015).
[88] A. Tomiya, G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, and J. Noaki, Evidence of effective axial U(1) symmetry restoration at high temperature QCD, Phys. Rev. D **96**, 034509 (2017); **96**, 079902(A) (2017).
[89] H. T. Ding, S. T. Li, S. Mukherjee, A. Tomiya, X. D. Wang, and Y. Zhang, Correlated Dirac eigenvalues and axial anomaly in chiral symmetric QCD, Phys. Rev. Lett. **126**, 082001 (2021).
[90] S. Aoki, Y. Aoki, G. Cossu, H. Fukaya, S. Hashimoto, T. Kaneko, C. Rohrhofer, and K. Suzuki (JLQCD Collaboration), Study of the axial $U(1)$ anomaly at high temperature with lattice chiral fermions, Phys. Rev. D **103**, 074506 (2021).
[91] S. Aoki, Y. Aoki, H. Fukaya, S. Hashimoto, C. Rohrhofer, and K. Suzuki (JLQCD Collaboration), Role of the axial U(1) anomaly in the chiral susceptibility of QCD at high temperature, Prog. Theor. Exp. Phys. **2022**, 023B05 (2022).
[92] O. Kaczmarek, L. Mazur, and S. Sharma, Eigenvalue spectra of QCD and the fate of UA(1) breaking towards the chiral limit, Phys. Rev. D **104**, 094518 (2021).
[93] E. V. Shuryak, Which chiral symmetry is restored in hot QCD?, Comments Nucl. Part. Phys. **21**, 235 (1994).
[94] T. D. Cohen, The high temperature phase of QCD and U(1)-A symmetry, Phys. Rev. D **54**, R1867 (1996).
[95] S. H. Lee and T. Hatsuda, U-a(1) symmetry restoration in QCD with N(f) flavors, Phys. Rev. D **54**, R1871 (1996).
[96] N. J. Evans, S. D. H. Hsu, and M. Schwetz, Topological charge and U(1)-A symmetry in the high temperature phase of QCD, Phys. Lett. B **375**, 262 (1996).

[97] A. Lahiri, Aspects of finite temperature QCD towards the chiral limit, Proc. Sci., LATTICE2021 (**2022**) 003 [arXiv:2112.08164].

[98] A. Bazavov *et al.* (HotQCD Collaboration), The chiral transition and $U(1)_A$ symmetry restoration from lattice QCD using domain wall fermions, Phys. Rev. D **86**, 094503 (2012).

[99] M. I. Buchoff *et al.*, QCD chiral transition, U(1)A symmetry and the dirac spectrum using domain wall fermions, Phys. Rev. D **89**, 054514 (2014).

[100] T. Bhattacharya *et al.*, QCD phase transition with chiral quarks and physical quark masses, Phys. Rev. Lett. **113**, 082001 (2014).

[101] O. Kaczmarek, F. Karsch, A. Lahiri, L. Mazur, and C. Schmidt, QCD phase transition in the chiral limit, arXiv:2003.07920.

[102] O. Kaczmarek, R. Shanker, and S. Sharma, Eigenvalues of QCD Dirac matrix with improved staggered quarks in the continuum limit, Phys. Rev. D **108**, 094501 (2023).

[103] K. I. Ishikawa, Y. Iwasaki, Y. Nakayama, and T. Yoshie, Nature of chiral phase transition in two-flavor QCD, arXiv:1706.08872.

[104] M. Asakawa and K. Yazaki, Chiral restoration at finite density and temperature, Nucl. Phys. **A504**, 668 (1989).

[105] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Signatures of the tricritical point in QCD, Phys. Rev. Lett. **81**, 4816 (1998).

[106] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Event-by-event fluctuations in heavy ion collisions and the QCD critical point, Phys. Rev. D **60**, 114028 (1999).

[107] A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, and N. Xu, Mapping the phases of quantum chromodynamics with beam energy scan, Phys. Rep. **853**, 1 (2020).

[108] H. Adhikary *et al.* (NA61/SHINE Collaboration), Excess of charged over neutral K meson production in high-energy collisions of atomic nuclei, arXiv:2312.06572.

[109] W. Brylinski, M. Gazdzicki, F. Giacosa, M. Gorenstein, R. Poberezhnyuk, S. Samanta, and H. Stroebele, Large isospin symmetry breaking in kaon production at high energies, arXiv:2312.07176.

[110] S. E. Vance, T. Csorgo, and D. Kharzeev, Partial UA1 restoration from Bose-Einstein correlations, Phys. Rev. Lett. **81**, 2205 (1998).

[111] T. Csorgo, R. Vertesi, and J. Sziklai, Indirect observation of an in-medium η' mass reduction in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions, Phys. Rev. Lett. **105**, 182301 (2010).

[112] R. Vertesi, T. Csorgo, and J. Sziklai, Significant in-medium η' mass reduction in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at the BNL relativistic heavy ion collider, Phys. Rev. C **83**, 054903 (2011).

[113] A. Adare *et al.* (PHENIX Collaboration), Lévy-stable two-pion Bose-Einstein correlations in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions, Phys. Rev. C **97**, 064911 (2018).

[114] M. Vargyas, T. Csörgő, and R. Vertesi, Effects of chain decays, radial flow and $U_A(1)$ restoration on the low-mass dilepton enhancement in $\sqrt{s_{NN}} = 200$ GeV Au + Au reactions, Central Eur. J. Phys. **11**, 553 (2013).

[115] G. Agakishiev *et al.* (HADES Collaboration), Statistical hadronization model analysis of hadron yields in p + Nb and Ar + KCl at SIS18 energies, Eur. Phys. J. A **52**, 178 (2016).

[116] J. Adamczewski-Musch *et al.* (HADES Collaboration), Probing dense baryon-rich matter with virtual photons, Nat. Phys. **15**, 1040 (2019).

[117] We note that other effects can also contribute to the η meson abundance, cf., e.g., Ref. [118].

[118] A. B. Larionov and L. von Smekal, Effects of chiral symmetry restoration on meson and dilepton production in relativistic heavy-ion collisions, Phys. Rev. C **105**, 034914 (2022).

[119] D. Bodeker and W. Buchmuller, Baryogenesis from the weak scale to the grand unification scale, Rev. Mod. Phys. **93**, 035004 (2021).

[120] G. Fejos and T. Hatsuda, Order of the $SU(N_f) \times SU(N_f)$ chiral transition via the functional renormalization group, arXiv:2404.00554.