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Why Does Inflation Look Single Field to Us?

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Most high-energy constructions that realize a phase of cosmic inflation contain many degrees of freedom. Yet, cosmological observations are all consistent with single-field embeddings. We show how volume selection effects explain this apparent paradox. Because of quantum diffusion, different regions of space inflate by different amounts. In regions that inflate most, and eventually dominate the volume of the Universe, a generic mechanism is unveiled that diverts the inflationary dynamics towards single-field attractors. The formalism of constrained stochastic inflation is developed to this end.

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Introduction.—Cosmic inflation [1–6] is a phase of accelerated expansion that occurred in the early Universe, during which vacuum quantum fluctuations were amplified by gravitational instability and gave rise to density fluctuations on large scales. These fluctuations constitute the seeds of all cosmological structures, and the validity of this scenario has been confirmed by a wealth of high-precision astrophysical measurements, ranging from the temperature and polarization fluctuations of the cosmic microwave background (CMB) [7], to galaxy and large-scale-structure surveys [8–10]. This makes inflation the leading paradigm to describe the early Universe.

Most physical setups that have been proposed to embed inflation contain a large number of high-energy degrees of freedom, see, e.g., [11–23]. This is because inflation is expected to occur at very high energies, ranging from GeV to 10^{15} GeV scales. The extensions to the standard model of particle physics that have been proposed to describe physics at those scales (supersymmetry, supergravity, string theory, etc.) usually come with many additional fields. The presence of multiple fields during inflation is expected to leave specific imprints, such as non-Gaussianities or entropic perturbations. However, all observations performed so far have failed to detect such features and are consistent with single-field models of inflation [24,25]. Therefore, a crucial question that remains open for inflation is: why do we observe single-field phenomenology, while inflation is expected to be realized in multiple-field setups?

In this Letter, we show how this question can be naturally answered by volume-selection effects. Because of quantum fluctuations, different regions in space inflate by different amounts. In a multiple-field landscape, it is shown that the regions that inflate the most reach an effectively single-field behavior when the scales observed in the CMB are being produced. Since such regions expand their physical volume by a larger amount, they eventually dominate the content of the universe, which explains how single-field phenomenology emerges from multifield setups.

This mechanism, which plays a central role in explaining why inflation looks single field to us, is unveiled using the formalism of stochastic inflation [26]. In this approach, as quantum fluctuations cross out the Hubble radius during inflation, they become part of the large-scale classical fields and randomly shift the background configurations. The dynamics of the fields thus become stochastic at large scales, which in practice is described by Langevin equations. The time required to terminate inflation is promoted to a random variable, and its statistics can be studied using first-passage-time techniques. It is related to the observed curvature perturbation in the stochastic-$\delta\mathcal{N}$ formalism [27–29].

In order to focus on the realisations of the Langevin equations that inflate for the longest period of time, and which thus dominate the volume of the Universe, a direct approach consists in simulating a large number of realizations numerically and keeping only those that inflate more than a certain threshold. However, this method becomes prohibitively expensive when the threshold increases since most realisations are discarded. Moreover, since it relies on numerical sampling, it is not propitious for analytical insight. This is why, in this Letter, methods from the theory of "constrained stochastic processes" [30–32] are borrowed, in order to derive modified Langevin equations that only generate realizations of a fixed duration (or a duration larger than a given bound). This allows us to study the statistics of the original stochastic process, *conditioned* to its duration.

We find that imposing a long duration for inflation leads to large realizations of the noises to be sampled at early time, which drifts the system towards the lightest field direction. This explains why, starting from initial conditions that would normally result in substantial multiple-field signatures to be produced, predictions are aligned with the single-field behavior once the selection on the duration of inflation is applied. We also show that the early phase of large-noise realizations lies in general outside the observable window, hence it does not threaten the viability of the currently preferred models of inflation.

Natural units are used with $c=\hbar=1$ and $M_\mathrm{P}=1/\sqrt{8\pi G}\simeq 2.4\times 10^{18}$ GeV denotes the reduced Planck mass.

Stochastic inflation.—On a homogeneous and isotropic background, fields are conveniently expanded into Fourier modes,

$$\boldsymbol{\phi}(\boldsymbol{x},N)=\boldsymbol{\phi}^-(\boldsymbol{x},N)+\boldsymbol{\phi}^+(\boldsymbol{x},N),$$
$$\boldsymbol{\phi}^\pm(\boldsymbol{x},N)\equiv\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\Theta[\pm(k-\sigma aH)]\boldsymbol{\phi}_{\boldsymbol{k}}(N)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}. \quad (1)$$

Here, $\boldsymbol{\phi}$ is a vector that contains all field configurations and momenta, and time is labeled with the number of e-folds N, related to cosmic time t through $dN=Hdt$, where $H=\dot{a}/a$ is the Hubble parameter and a the scale factor. The coarse-grained field $\boldsymbol{\phi}^-$ contains all wavelengths larger than the Hubble radius $1/H$ (rescaled by the constant parameter σ), while $\boldsymbol{\phi}^+$ contains smaller wavelengths. As comoving Fourier modes cross out the Hubble radius, they go from $\boldsymbol{\phi}^+$ to $\boldsymbol{\phi}^-$, and contribute a white Gaussian noise $\boldsymbol{\xi}$ to the dynamics of $\boldsymbol{\phi}^-$ (simply denoted $\boldsymbol{\phi}$ hereafter) that reads [26]

$$\frac{d\boldsymbol{\phi}}{dN}=\boldsymbol{F}(\boldsymbol{\phi})+\boldsymbol{G}(\boldsymbol{\phi})\boldsymbol{\xi}(N). \quad (2)$$

Here, $\boldsymbol{F}$ describes the homogeneous (in the limit $\sigma\ll 1$), classical dynamics of the fields, while the amplitude of the noise $\boldsymbol{G}$ is obtained from evolving quantized cosmological perturbations from the Bunch-Davies vacuum at small scales. For a scalar field ϕ in the slow-roll regime, $F=-V'/3H^2$ and $G=H/2\pi$, where $H^2=V/3M_\mathrm{P}^2$ and V is the potential energy of the fields, while the field momentum is set by the slow-roll attractor [33]. Inflation terminates when $\ddot{a}$ stops being positive. This defines a final hypersurface $\mathcal{C}$ in the field space on which absorbing boundary conditions are imposed.

The Langevin equation (2) gives rise to a Fokker-Planck equation for the probability density $P(\boldsymbol{\phi},N)$ of $\boldsymbol{\phi}$ at time N,

$$\frac{\partial P}{\partial N}=\left(-\boldsymbol{\nabla}\cdot\boldsymbol{F}+\frac{1}{2}\boldsymbol{\nabla}\otimes\boldsymbol{\nabla}:\boldsymbol{G}^2\right)P. \quad (3)$$

Hereafter, Itô's convention is adopted for explicitness (although extensions to other discretization conventions are straightforward [34]) and Frobenius inner product's notation $\boldsymbol{\nabla}\otimes\boldsymbol{\nabla}:\boldsymbol{G}^2=(\partial^2/\partial\phi^i\partial\phi^j)G^j_kG^{ki}$ is used where $\boldsymbol{\nabla}$ is the field space gradient.

Starting from the initial condition $\boldsymbol{\phi}$, the number of e-folds elapsed until the first crossing of $\mathcal{C}$ is a random variable denoted by $\mathcal{N}$ and referred to as the first-passage time. Its distribution function, $P_\mathrm{FPT}(\boldsymbol{\phi},\mathcal{N})$, obeys the adjoint Fokker-Planck equation [29,35]

$$\frac{\partial P_\mathrm{FPT}}{\partial\mathcal{N}}=\left(\boldsymbol{F}\cdot\boldsymbol{\nabla}+\frac{1}{2}\boldsymbol{G}^2:\boldsymbol{\nabla}\otimes\boldsymbol{\nabla}\right)P_\mathrm{FPT}\equiv\mathcal{L}P_\mathrm{FPT}. \quad (4)$$

This partial differential equation needs to be solved with the boundary condition $P_\mathrm{FPT}(\boldsymbol{\phi},\mathcal{N})=\delta_\mathrm{D}(\mathcal{N})$ for $\boldsymbol{\phi}\in\mathcal{C}$, where δ_D is the Dirac distribution. At large $\mathcal{N}$, the upper tail of P_FPT decays exponentially with $\mathcal{N}$ [35,36], which makes the sampling of long-lasted realizations numerically challenging.

Constrained stochastic processes.—Consider the subset of realisations of the stochastic process (2) starting from $\boldsymbol{\phi}_0$ that realize a fixed number of e-folds N_F. At time N, they follow a distribution function that is denoted by $\mathcal{P}(\boldsymbol{\phi},N|N_\mathrm{F})$. Using Bayes theorem, this can be written in terms of quantities defined above in the unconstrained process,

$$\mathcal{P}(\boldsymbol{\phi},N|N_\mathrm{F})=\frac{P_\mathrm{FPT}(\boldsymbol{\phi},N_\mathrm{F}-N)P(\boldsymbol{\phi},N)}{P_\mathrm{FPT}(\boldsymbol{\phi}_0,N_\mathrm{F})}. \quad (5)$$

Here, we used that, for Markovian processes, the probability to realize a total N_F e-folds if the system is at $\boldsymbol{\phi}$ at time N, is equal to the probability to realize $N_\mathrm{F}-N$ e-folds starting from $\boldsymbol{\phi}$. Since P and P_FPT satisfy (3) and (4), respectively, the above leads to [37]

$$\frac{\partial\mathcal{P}}{\partial N}=\left(-\boldsymbol{\nabla}\cdot\tilde{\boldsymbol{F}}+\frac{1}{2}\boldsymbol{\nabla}\otimes\boldsymbol{\nabla}:\boldsymbol{G}^2\right)\mathcal{P}, \quad (6)$$

where

$$\tilde{\boldsymbol{F}}(\boldsymbol{\phi},N)\equiv\boldsymbol{F}(\boldsymbol{\phi})+\boldsymbol{G}^2(\boldsymbol{\phi})\boldsymbol{\nabla}\ln P_\mathrm{FPT}(\boldsymbol{\phi},N_\mathrm{F}-N). \quad (7)$$

One notices that (6) is of the same form as (3), i.e., it is a Fokker-Planck equation, except that the drift function $\tilde{\boldsymbol{F}}$ now explicitly depends on time. As such, (6) can equivalently be written in the Langevin form

$$\frac{d\boldsymbol{\phi}}{dN}=\tilde{\boldsymbol{F}}(\boldsymbol{\phi},N)+\boldsymbol{G}(\boldsymbol{\phi})\boldsymbol{\xi}(N), \quad (8)$$

which is known as the Doob's transformation of (2) [41].

The additional term in (7) is an effective force induced by the selection effect. When $N<N_\mathrm{F}$, the boundary condition, $P_\mathrm{FPT}=\delta_\mathrm{D}$ on $\mathcal{C}$, implies that the induced force is infinitely repelling on the final surface, preventing realizations from finishing before N_F. As N approaches N_F, since $P_\mathrm{FPT}(\boldsymbol{\phi},\mathcal{N})\propto\exp[-f(\boldsymbol{\phi})/\mathcal{N}]$ for small time arguments,

where f grows with the distance between $\boldsymbol{\phi}$ and $\mathcal{C}$ [37], the induced force becomes infinitely attracting towards the final surface and inflation necessarily terminates at N_F. This guarantees that the duration of inflation is indeed fixed in the constrained process. Let us stress that sampling (2) and keeping only realizations that produce N_F e-folds is mathematically equivalent to sampling (8).

A regime of physical interest below is when the noise amplitude $\boldsymbol{G}$ is small. In that limit, $P_\mathrm{FPT}(\mathcal{N})$ is approximately Gaussian,

$$P_\mathrm{FPT}(\boldsymbol{\phi},\mathcal{N}) \simeq \frac{1}{\sqrt{2\pi\sigma^2(\boldsymbol{\phi})}}\exp\left\{-\frac{[\mathcal{N}-\mu(\boldsymbol{\phi})]^2}{2\sigma^2(\boldsymbol{\phi})}\right\}, \quad (9)$$

where μ is the mean number of e-folds and σ is its standard deviation. From (4), they satisfy $\mathcal{L}\mu = -1$ and $\mathcal{L}\sigma^2 = -\boldsymbol{G}^2 : (\boldsymbol{\nabla}\mu) \otimes (\boldsymbol{\nabla}\mu)$, with $\mathcal{L} \simeq \boldsymbol{F}\cdot\boldsymbol{\nabla}$ at leading order in the noise. This gives rise to the induced drift

$$\tilde{\boldsymbol{F}} = \boldsymbol{F} + \frac{N_\mathrm{F}-N-\mu}{\sigma^2}\boldsymbol{G}^2\cdot\left(\boldsymbol{\nabla}\mu + \frac{N_\mathrm{F}-N-\mu}{2\sigma^2}\boldsymbol{\nabla}\sigma^2\right). \quad (10)$$

Here, σ scales like $\boldsymbol{G}$, hence the induced force is independent of the noise amplitude. In that regime, the noise term in (8) can thus be neglected [37], and the constrained dynamics becomes quasi-deterministic. Note that, from the point of view of the unconstrained dynamics, the noise plays a crucial role in the realizations that produce N_F e-folds, especially if N_F differs substantially from the mean first-passage time. However, in the regime described here, statistical fluctuations *among the subset of constrained realizations* are negligible. This is why the noise can be neglected in (8), but obviously not in (2).

Stochastic processes of a fixed duration are dubbed "excursions" [31], but other constrained processes can be sampled similarly, e.g., "meanders" in which only a lower bound on the duration of inflation, $\mathcal{N} \geq N_\mathrm{F}$, is imposed. In that case, one still obtains a modified Langevin equation of the form (8), except that $P_\mathrm{FPT}(\boldsymbol{\phi}, N_\mathrm{F}-N)$ in (7) needs to be replaced with $\int_{N_\mathrm{F}-N}^{\infty} d\mathcal{N} P_\mathrm{FPT}(\boldsymbol{\phi},\mathcal{N})$ [37]. In the low-diffusion limit, whether the volume selection effect during inflation is implemented via excursions or meanders leads to similar conclusions [37], hence in the following we focus on excursions.

Single-field phenomenology from multifield inflation.— Let us now apply the formalism presented above to multiple-field models of inflation. For concreteness, a double quadratic potential is considered [42–45],

$$V(\boldsymbol{\phi}) = \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2, \quad (11)$$

where $\sqrt{r} \equiv m_2/m_1 > 1$ is assumed without loss of generality. Upon introducing the rescaled variables, $x \equiv \phi_1/M_\mathrm{P}$, $y \equiv \phi_2/M_\mathrm{P}$, and $v_0 \equiv m_1^2/24\pi^2 M_\mathrm{P}^2$, in the slow-roll regime the unconstrained Langevin equation (2) reads

$$\frac{d}{dN}\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{2}{x^2+ry^2}\begin{pmatrix} x \\ ry \end{pmatrix} + \sqrt{v_0(x^2+ry^2)}\boldsymbol{\xi}(N). \quad (12)$$

Inflation terminates on the contour $\mathcal{C}$ defined by $2(x^2+r^2y^2) = (x^2+ry^2)^2$, on which an absorbing boundary is placed. When the stochastic noise is neglected in (12), inflation proceeds along the classical lines $y = cx^r$, where c is conserved and depends on the initial condition. This leads to the classical number of e-folds, $\mu(x,y) = (x^2+y^2-x_\mathrm{F}^2-y_\mathrm{F}^2)/4$, where $(x_\mathrm{F}, y_\mathrm{F})$ is the intersection of the classical trajectory with $\mathcal{C}$, and thus implicitly depends on the initial conditions x and y.

Following [46], the prevalence of multifield effects can be assessed by comparing the rate at which the trajectory turns in field space, $\eta_\perp$, with the rate at which it accelerates, $\eta_\parallel$. In other words, if $\boldsymbol{v} = (dx/dN, dy/dN)^\mathrm{T}$ denotes the field-space velocity, $\eta_\parallel$ is the component of $d\boldsymbol{v}/dN$ that is parallel to $\boldsymbol{v}$, and $\eta_\perp$ is its orthogonal component. In the model (11), one finds

$$\lambda \equiv \frac{\eta_\perp}{\eta_\parallel} = r(r-1)\frac{w(1+rw^2)}{r^4w^4-(r^3-4r^2+r)w^2+1}, \quad (13)$$

where $w \equiv y/x$. In what follows, quasi single-field phenomenology will be associated with regions where $\lambda < \lambda_\mathrm{c}$, with λ_c a fixed threshold on which our conclusions do not depend. When the two fields have the same mass, $\lambda = 0$ and the setup is effectively single field. For a large mass ratio, $r \gg 1$, single-field phenomenology requires $w < \lambda_\mathrm{c}/r^2$, which delineates a smaller field-space region as r increases.

From a given initial condition, one can integrate the classical dynamics and compute λ when the scales probed in the CMB emerge, i.e., 60 e-folds before the end of inflation. If $\lambda < \lambda_\mathrm{c}$, this initial condition is said to yield quasi single-field phenomenology, and is displayed in blue in the left panels of Fig. 1. One can see that, as r increases, the space of initial conditions compatible with single-field phenomenology is enlarged, although the single-field condition becomes more stringent as mentioned above. This is because, when the heavy field is heavier, it gets more quickly suppressed during inflation, and the system is more efficiently attracted towards the light field direction. Nonetheless, a fair fraction of the initial conditions yield multifield effects.

Let us now turn on the selection effect, and impose that N_F e-folds are realized. In practice, CMB measurements [24] constrain v_0 to be of order 10^{-13}, hence the noise amplitude in (12) is highly suppressed and the Gaussian approximation (9) can be used. There, μ was given above,

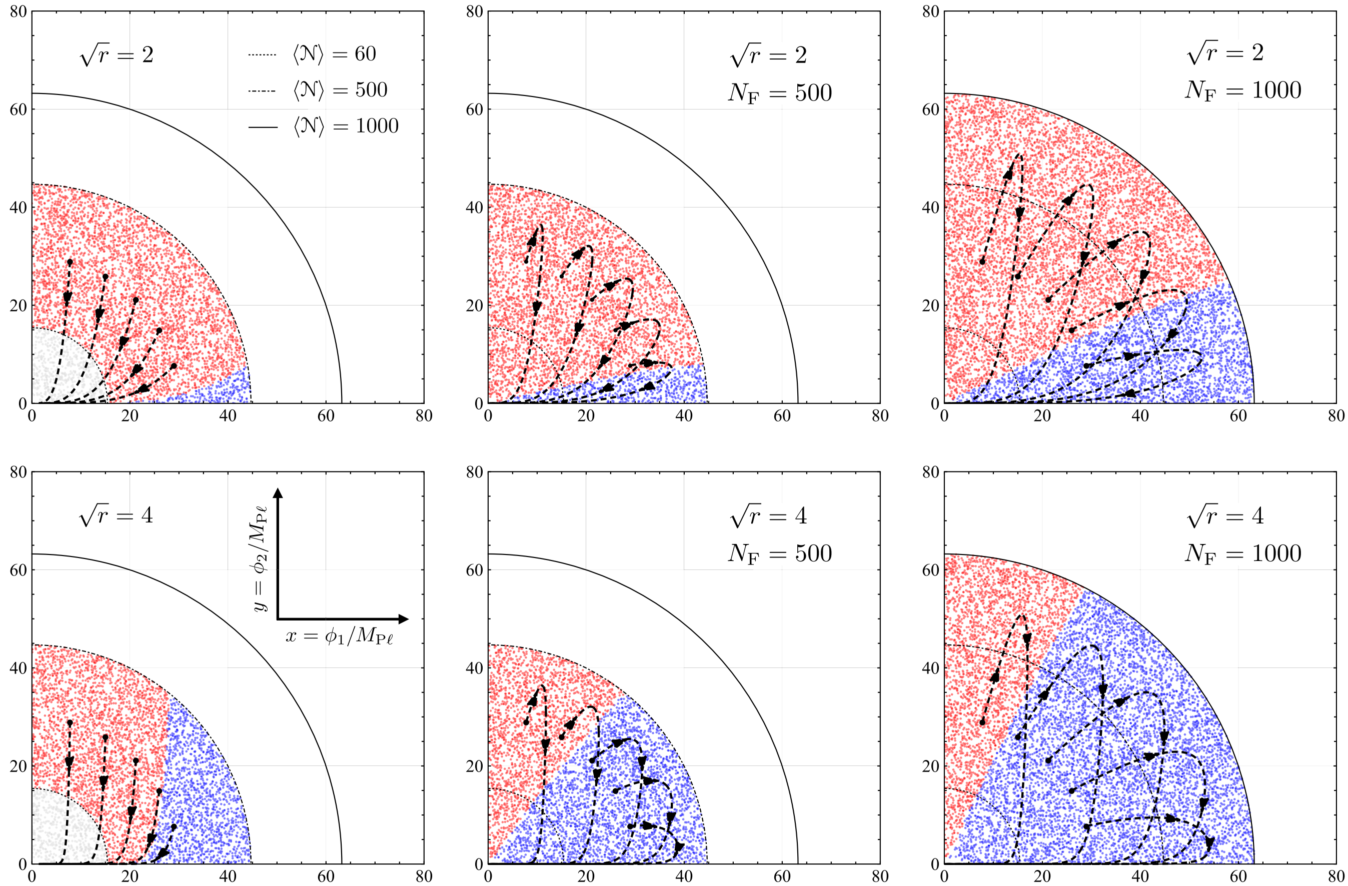

FIG. 1. Random initial conditions are drawn inside a contour of equal e-folds, from which the inflationary dynamics is solved. λ is computed 60 e-folds before the end of inflation, and the initial condition is marked in blue if $\lambda < \lambda_c = 0.1$ (single-field phenomenology), and in red otherwise. A few examples of inflationary trajectories are displayed with the black thick dashed lines, the initial conditions of which are set on the contour $x^2 + y^2 = 30^2$ (hence $\mu = \langle \mathcal{N} \rangle = 225.5$ e-folds are classically realized). The left panels correspond to the unconstrained setup, where the gray dots realize less than 60 e-folds. The middle and right panels show the constrained setup. When N_F increases, more initial conditions give rise to single-field phenomenology (see also Fig. S5 in [37]).

and one finds $\sigma^2 \simeq v_0[f(x,y) - f(x_F, y_F)]/48$, where $f(x,y) = x^6 + ry^6 + 3[r(r+2)/(2r+1)]x^2y^4 + 3[(2r+1)/(r+2)]x^4y^2$. At leading order in v_0, the constrained dynamics is deterministic, so one can apply the same procedure as in the classical unconstrained setup and the result is displayed in the middle and right panels of Fig. 1. One can see that, if one imposes $\mathcal{N} = N_F$, the realizations of the noise that are selected divert the system towards a large detour at larger-field values. This detour circles clockwise, such that the system is much closer to the light-field direction when it approaches the end of inflation than what it would be without selection effects. As a consequence, one notices that more initial conditions give rise to single-field phenomenology once selection effects are turned on.

When N_F increases, the detour is wider and the preference for single-field phenomenology is more pronounced. Although this effect is shown explicitly in the two-field model (11), we expect it to be generic. Indeed, in the unconstrained setup, the stochastic noise may either take the system closer to the heavy-field direction or to the light-field direction. If the system gets closer to the heavy field, the subsequent number of e-folds it realizes is smaller than if it gets closer to the light field. As a consequence, imposing a large duration of inflation necessarily results in a biased sampling in favor of those noise realizations that bring the system closer to the light field, i.e., to the field-space regions with stronger single-field phenomenology. This generic mechanism is present in any multiple-field model.

When do selection effects take place?—Volume selection diverts the inflationary dynamics towards single-field looking regions, and a natural question is whether or not this detour leaves observable effects. Indeed, the predictions of inflationary models are usually derived along the unconstrained trajectory, and one may wonder how they change when computed along the constrained ones. This question already arises in single-field models, so for simplicity ϕ_2 is set to 0 in (11).

In that case, the above formulas can be used with setting $y = 0$. The first-passage-time problem can be solved

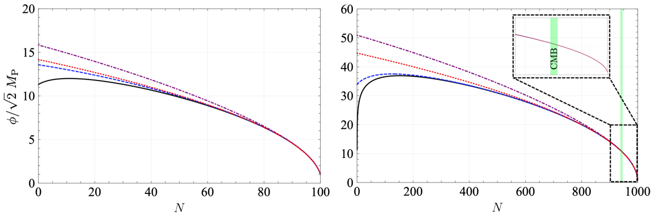

FIG. 2. Constrained inflationary trajectories, which become deterministic in the regime $v_0 \ll 1$, from several initial conditions when imposing $N_{\rm F} = 100$ (*left*) and 1000 (*right*). The red dotted line corresponds to the unconstrained trajectory that leads to $N_{\rm F}$ e-folds of inflation.

semi-analytically [37], but when $v_0 \ll 1$, the Gaussian approximation (9) applies and the constrained dynamics becomes deterministic. It is displayed in Fig. 2, where one can see that by 60 e-folds before the end of inflation, the constrained realizations have all collapsed to the unconstrained one. This means that the part of the inflationary era that is probed in cosmological surveys is not affected by selection effects, and confirms that standard results apply.

The reason why selection effects mostly take place early on is because quantum diffusion is more prominent at early stages of inflation (in the present model, $\boldsymbol{F}$ decays as $1/x$ while $\boldsymbol{G}$ grows as x). Since it is more likely for the system to fluctuate away from the mean path when the amplitude of the noise is larger, this explains why the selection detour is imprinted at early stages, i.e., at scales larger than those observed [47]. In the vast majority of single-field models [17] quantum diffusion is larger at earlier times, but there exist potentials that feature a transient phase of large diffusion at late time, and which are relevant to primordial black hole production for instance. The presence of observable imprints from selection effects in these setups would be interesting to further explore.

Conclusion.—We have developed the formalism of constrained random processes in the context of stochastic inflation. This allowed us to derive effective Langevin equations that select the realizations where quantum diffusion leads to the longest duration of inflation. The corresponding space-time regions dominate the volume of the universe at late time and thus represent the most likely past history of a given observer.

We have found that, in multiple-field setups, these regions are efficiently diverted to single-field attractors. As a consequence, the set of initial conditions that leads to single-field phenomenology is much larger than in the absence of volume-selection effects. This mechanism is generic and helps to explain why, although inflation is most commonly realized in high-energy constructions that involve multiple additional degrees of freedom, cosmological observations are compatible with single-field models. If multifield signatures are detected in the future, it would point towards the limited class of models where, 60 e-folds before the end of inflation, multifield effects are produced everywhere in field space.

We have also shown that selection effects take place at early time and barely affect the last 60 e-folds of inflation. This implies that the standard predictions of inflation are unaffected once the single-field attractor has been reached (although possible observable imprints of selection effects in models with substantial quantum diffusion towards the end of inflation [50,51], or in the case of "just-enough inflation" [52,53], require further investigations).

Note that, contrary to previous attempts to implement volume-weighting procedures [54–58], our results do not depend on a choice of a volume measure. Instead, it relies on selecting the realizations that inflate the most, which are shown to reach single-field attractors at late time. Single-field phenomenology would therefore emerge for any volume measure, only the strength of the attractor would depend on the details of that measure.

Finally, let us mention that the formalism of constrained random processes can be used more generally to sample rare realizations, in complement with other importance-sampling methods [59]. This may be of practical interest, e.g., in situations leading to the formation of primordial black holes.

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