Editors' Suggestion

Spin Inertia and Auto-Oscillations in Ferromagnets

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Recent experimental confirmation of spin inertia in ferromagnets positions this well-developed material class as a prime candidate for THz frequency applications. Spin-torque driven critical spin dynamics, such as auto-oscillations, play the central role in many spin-based technologies. Yet, the pressing question on spin inertia's effect on spin-torque driven dynamics in ferromagnets has remained unexplored. Here, we develop the theoretical framework of precessional auto-oscillations for ferromagnets with spin inertia. We discover and introduce the concept of nutational auto-oscillations and demonstrate that they can become pivotal for future ultrahigh frequency technologies. We conclude by revealing parallels between spin dynamics in ferrimagnets and inertial ferromagnets and derive an isomorphism that establishes a foundation for synergistic knowledge transfer between these research fields.

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Introduction.—In ferromagnets, coupling of spins to other degrees of freedom has been discussed [1–12] to apply effective inertia to the spin motion. As such, magnetic relaxation is generally accompanied by this indirect effect [13,14], termed spin inertia. It would manifest itself through nutational spin dynamics at (sub-)THz frequencies, emerging concurrently with the well-established precessional dynamics at GHz frequencies [Fig. 1(a)]. Recently, nutational spin dynamics in ferromagnets have been experimentally confirmed [15–17].

Besides its implications for the fundamental understanding of magnetism, spin inertia bears tremendous potential for ultrahigh frequency applications. The pursuit of higher-frequency spin dynamics has recently intensified interest in materials like ferrimagnets and antiferromagnets [18–22], which exhibit (sub-)THz frequency precessional dynamics even without spin inertia [18,19,23]. However, functionalization of these materials is an ongoing effort. In contrast, ferromagnets have a wide range of engineered materials and established techniques for spin-torque manipulation of magnetization [24–31]. Integrating these existing methods with the ultrahigh frequency nutational dynamics in inertial ferromagnets could be pivotal for advancing future spin-based technologies.

Spin-torque driven auto-oscillations are a prime example of critical spin dynamics with technological relevance [32,33]: auto-oscillators are used as magnetic neurons in artificial neural networks [34,35], as emitters in magnonic circuits [36,37], and as transistors in high-frequency electronics [38]. Yet, the influence of spin inertia on precessional auto-oscillations and the concept of nutational auto-oscillations have remained to be explored. This is particularly compelling, given that several widely used ferromagnetic materials are now recognized to posses substantial spin inertia [15–17].

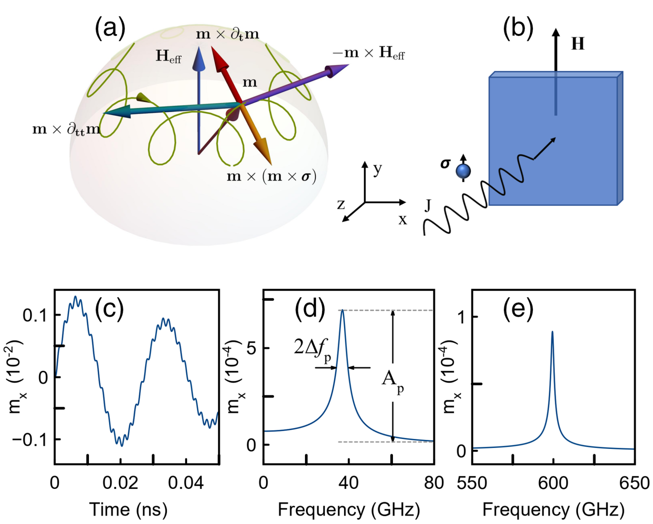

FIG. 1. (a) Magnetization trajectory in an inertial ferromagnet. In addition to the right-hand precession at a GHz frequency, the magnetization follows a left-hand nutation at a sub-THz frequency. (b) Sample model and its configuration in space. (c) Time trace of magnetization transverse component, $m_x(t)$, calculated for $H = 10$ kOe, $\eta = \bar{\eta}$, and $J = 0.7 \cdot J_n$. (d) Its Fourier transform shows Lorentzian peaks for the precessional oscillation and (e) nutational oscillation.

Here, we develop a theoretical framework for auto-oscillations in ferromagnets considering spin inertia. We examine its effects on precessional auto-oscillations and introduce the concept of spin-torque driven nutational auto-oscillations at ultrahigh frequencies. Moreover, we reveal an isomorphism between the spin dynamics in ferrimagnets and inertial ferromagnets, thus bridging these two burgeoning research fields.

Results.—Our reconciliation of the theoretical framework begins [1] with extending the equation of magnetization motion in ferromagnets by the inertial term

$$\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \partial_t \mathbf{m} + J \mathbf{m} \times (\mathbf{m} \times \boldsymbol{\sigma}) + \eta \mathbf{m} \times \partial_{tt} \mathbf{m}. \quad (1)$$

Here, γ is the gyromagnetic ratio magnitude and $\mathbf{m}$ is the normalized magnetization. With the first two right-hand terms, Eq. (1) would correspond to the Landau-Lifshitz-Gilbert equation [32] with the effective magnetic field $\mathbf{H}_{\text{eff}}$ and Gilbert damping α. The third term describes the Slonczewski spin torque, originating from inbound spin current with amplitude J and polarization unit vector $\boldsymbol{\sigma}$. The last term, containing the second time derivative of magnetization [1], describes spin inertia with the inertial parameter η.

Without spin inertia ($\eta = 0$), a spin current with the polarization parallel to the effective field, $\boldsymbol{\sigma} \upharpoonleft\upharpoonright \mathbf{H}_{\text{eff}}$, is known to drive auto-oscillations when the positive spin current amplitude reaches a critical value. The magnetization transitions into self-sustained oscillations at the frequency of the lowest precessional mode [32].

In the presence of spin inertia ($\eta > 0$), we calculate the critical current of precessional auto-oscillations by adapting the formalism by Grollier *et al.* [39]: we look for unstable solutions for the transverse components of magnetization $m_x, m_z \propto \exp(\omega t)$ and linearize Eq. (1) to their first order. This reduces Eq. (1) to a determinant equation for the oscillation frequency ω. Negative real frequencies correspond to decaying oscillations of magnetization, while positive real frequencies describe exponentially growing auto-oscillations. The real component of the oscillation frequency vanishes at the critical current

$$\text{Re}\{\omega(J_\pm)\} = 0. \quad (2)$$

We find that in inertial ferromagnets Eq. (2) yields two roots. The positive critical current J_+ corresponds to precessional auto-oscillations. The negative root J_- demonstrates that nutational auto-oscillations are possible. Figure 1(a) helps to explain the negative sign of the nutational critical current. While precessional oscillation is a right-hand motion, nutational oscillation is left-handed and requires the opposite polarity of the spin current to reach auto-oscillations [19,20].

The roots of Eq. (2) possess no reasonably concise explicit solutions. To validate our findings and derive expressions for the critical currents, we numerically time-integrate the equation of motion [Eq. (1)] without linearizations, and compute the magnetization trajectory $\mathbf{m}(t)$.

As the sample model, shown in Fig. 1(b), we use a thin ferromagnetic film with no magnetic anisotropy except for the shape anisotropy field $H_\text{d} = 10$ kOe, like in the widely used material Permalloy [40]. An external magnetic field $\mathbf{H}$ is applied within the film plane and the inbound spin current is polarized parallel to it, $\boldsymbol{\sigma} \upharpoonleft\upharpoonright \mathbf{H}$. Such configuration corresponds to a typical experiment on a spin Hall driven auto-oscillator [41]. Note that in all figures, for brevity, we may write the inertial parameter in units of $\bar{\eta} = (2\pi \cdot 560\ \text{GHz})^{-1}$, which is the experimentally determined value for Permalloy [15].

Upon a small perturbation from equilibrium, the time evolution of the transverse magnetization component $m_x(t)$ in Fig. 1(c) shows two superimposed oscillations. Its Fourier transform in Figs. 1(d),(e) has two Lorentzian peaks: precession at GHz and nutation at sub-THz frequencies.

In Fig. 2(a), the precessional frequency f_p is shown as a function of magnetic field. For $\eta = 0$, it follows [42] the Kittel equation $2\pi f_\text{p}^{\eta=0} = \gamma\sqrt{H(H + H_\text{d})}$. With increasing inertial parameter, the precessional frequency falls [43]. As shown in Fig. 2(b), this redshift is in excellent agreement with the recently developed inertial Smit-Beljers formalism [44]. It allows [45] for calculating the frequencies of inertial ferromagnets

$$2\pi f_\text{p} = \sqrt{a - \sqrt{a^2 - b^2}} \quad (3)$$

$$a = \frac{1}{2\eta^2} + \frac{\pi}{\alpha\eta} \Delta f_\text{p}^{\eta=0}, \qquad b = \frac{2\pi}{\eta} f_\text{p}^{\eta=0}, \quad (4)$$

using noninertial parameters of frequency $f_\text{p}^{\eta=0}$ and (half-width at half-maximum) linewidth $\Delta f_\text{p}^{\eta=0}$.

The precessional linewidth Δf_p, defined in Fig. 1(d), can also be used to assess the effective damping of the precessional mode [42]. First, the linewidth is usually translated into the field domain via $\Delta H_\text{p} = \Delta f_\text{p}(\partial f_\text{p}/\partial H)^{-1}$. This procedure involves the first derivative of the frequency-field relation, which itself—as we have seen in Fig. 2(a)—depends on the inertial parameter η. Despite this explicit dependence, we find the linewidth to follow a linear trend as a function of frequency, independently of the inertial parameter. For all η, the line fit $\Delta H_\text{p}\gamma/2\pi = \alpha_{\text{eff}} f_\text{p}$ returns the same slope of the effective damping. At zero spin current, it is equal to $\alpha_{\text{eff}} = \alpha = 0.005$, the Gilbert damping used in our simulations.

With increasing positive spin current, the linewidth decreases linearly; the effective damping, shown in Fig. 2(d), extrapolates to zero at the critical current J_p.

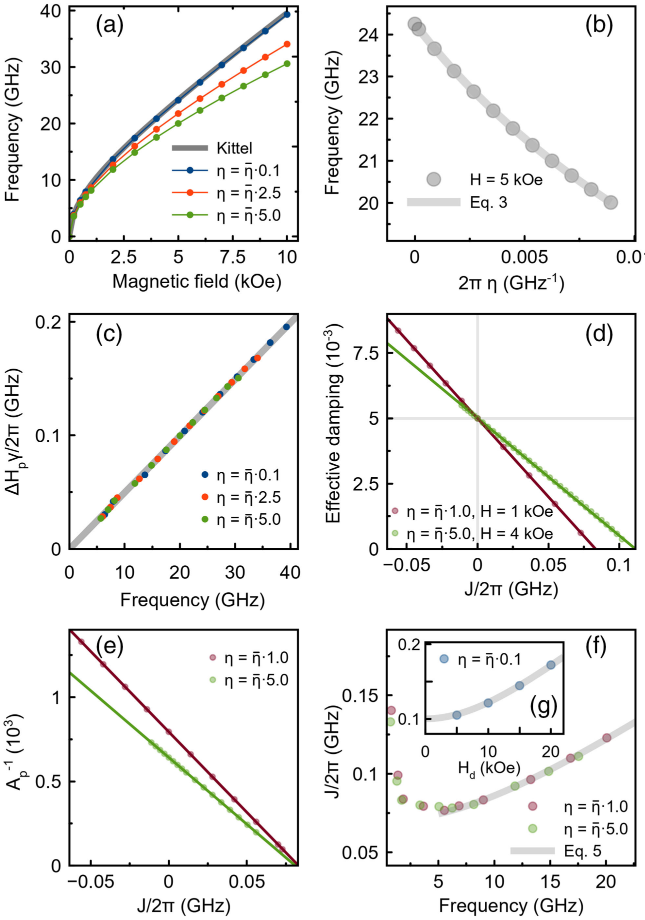

FIG. 2. (a) Precessional frequency as a function of magnetic field follows the Kittel equation for $\eta = 0$. With increasing inertial parameter, the frequencies experience a redshift. (b) The redshift and the inertial Smit-Beljers formalism of Eq. (3) are in excellent agreement. (c) Field-domain precessional linewidth grows linearly with frequency, with a slope that does not depend on the inertial parameter. The slope corresponds to the effective damping and can be tuned by the spin current J. (d) Effective damping extrapolates to zero at a critical current J_p. (e) Inverse precessional amplitude extrapolates linearly to zero at the same critical current as for the precessional linewidth. (f) The critical current for precessional auto-oscillation does not depend on the inertial parameter, despite the frequency redshift. The high-frequency region and Eq. (5) are in a very good agreement. (g) The critical current as a function of the shape anisotropy field follows Eq. (5) at 20 GHz (high-frequency region).

The precession amplitude A_p grows with increasing spin current and shows a sudden jump with the onset of auto-oscillations at the critical current. In the subcritical regime, $J < J_p$, the inverse amplitude A_p^{-1} falls linearly with increasing spin current (an experimentally observed behavior [46]). The inverse amplitude extrapolates to zero [Fig. 2(e)] at the same critical current J_p as in the case of the linewidth.

Without considering spin inertia, it has been a common practice to treat the critical current as an explicit function of external and anisotropy fields [39,47]. We find that such approach no longer holds in the presence of spin inertia. All critical currents, obtained for various inertial parameters and shown in Fig. 2(f), can be described by the same function of the precession frequency—in spite of the frequency redshifts caused by inertia. This counterintuitive result suggests that the critical current should be treated as an explicit function of the frequency.

As shown in Fig. 2(f), this function is nonmonotonic with a low-frequency and a high-frequency region. We find that the latter can be described by

$$J_p \approx \alpha\gamma\sqrt{(2\pi f_p/\gamma)^2 + (H_d/2)^2}. \quad (5)$$

The dependence of the critical current on the shape anisotropy field H_d is confirmed in Fig. 2(g). We further find that by substituting Eq. (3) into Eq. (2), we can eliminate the explicit dependence on η. The linearized approximation of J_+ is then in excellent agreement with our heuristically defined critical current J_p.

We now turn to the nutational oscillations [Fig. 1(e)]. The nutational frequency, shown in Fig. 3(a), falls with increasing inertial parameter and grows with magnetic field. This behavior is described well by the aforementioned inertial Smit-Beljers formalism

$$2\pi f_n = \sqrt{a + \sqrt{a^2 - b^2}}. \quad (6)$$

As shown in Fig. 3(b), the amplitude of the nutational oscillation is susceptible to spin current. It grows with increasing magnitude of the negative current. In the subcritical regime, the inverse amplitude A_n^{-1} approaches a linear trend and extrapolates to zero at the nutational critical current J_n.

The critical current magnitude increases with increasing Gilbert damping and falls with increasing inertial parameter [compare different scenarios in Fig. 3(b)]. It furthermore shows a weak dependence on magnetic field. With increasing magnetic field, the nutational frequency experiences a blueshift and the critical current magnitude grows with the frequency, as shown in Fig. 3(c).

Figure 3(d) summarizes the behavior of the nutational critical current. In general, we find it to depend linearly on the inverse inertial parameter with the negative slope equal to the Gilbert damping. Moreover, application of magnetic field leads to a minor increase of the critical current magnitude. These findings can be approximated by

$$J_n = -\alpha\eta^{-1} - \mathcal{O}(H) \approx -\alpha 2\pi f_n. \quad (7)$$

We compare this heuristically defined critical current with the linearization of Eq. (1), J_-, and find them to agree within 1% for the parameter space discussed here.

It should be noted that the term "nutation" is rather ambiguous in that it suggests that nutation could exist only

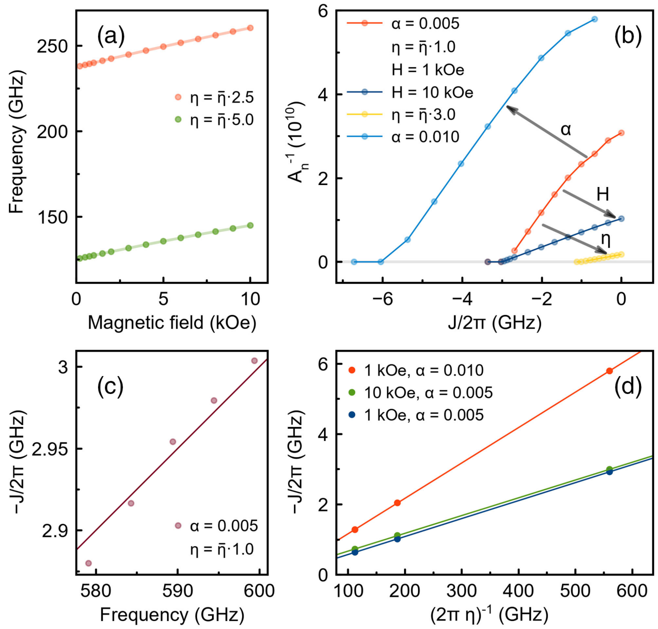

FIG. 3. (a) Nutation frequency falls with increasing inertial parameter, exhibits a blueshift with increasing magnetic field, and agrees well with Eq. (6) (solid lines). (b) Inverse amplitude of nutation extrapolates, nearly linearly, to the negative critical current J_n. Starting from the red curve, the arrows demonstrate various scenarios where only one of the parameters is increased: Gilbert damping leads to an increase of the critical current magnitude, the inertial parameter—to a decrease. (c) Magnetic field leads to frequency blueshift, which in turn leads to a minor increase of the critical current magnitude in agreement with Eq. (7) (solid line). (d) The nutational critical current is proportional to the inverse inertial parameter, with a slope approximately equal to the Gilbert damping.

on top of a precessional motion. Our study shows that both precession and nutation undergo auto-oscillations; these are largely independent oscillation modes. We shall stick to the historically established nomenclature of nutation, but we will reconsider the interpretation of the magnetization trajectory visualized in Fig. 1(a). In fact, two largely independent oscillation modes with opposite chiralities are strongly reminiscent of spin dynamics in ferrimagnets [20,48].

Ferrimagnets consist of typically two magnetic sublattices with the magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$, which are coupled antiferromagnetically through exchange field $H_{\mathrm{ex}} = E_{\mathrm{ex}}/(M_1 + M_2)$. We define the order parameters of the system—magnetization and Néel vectors—via

$$\mathbf{m} = \frac{\mathbf{M}_1 + \mathbf{M}_2}{M_1 + M_2}, \qquad \mathbf{l} = \frac{\mathbf{M}_1 - \mathbf{M}_2}{M_1 + M_2}, \tag{8}$$

which are subject to the constraints

$$\mathbf{m}\cdot\mathbf{l} = \nu, \qquad \mathbf{m}^2 + \mathbf{l}^2 = 1 + \nu^2, \tag{9}$$

where parameter $\nu = (M_1 - M_2)/(M_1 + M_2)$ describes the imbalance between the sublattices.

Omitting the dissipative terms of damping and spin current at first, we rewrite two equations of motion—one for each sublattice—into coupled equations for the order parameters [48]

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma\left[\mathbf{m}\times\mathbf{H}_{\mathrm{eff}}^{(m)}\right] - \gamma\left[\mathbf{l}\times\mathbf{H}_{\mathrm{eff}}^{(l)}\right], \tag{10}$$

$$\frac{\partial \mathbf{l}}{\partial t} = -\gamma\left[\mathbf{l}\times\mathbf{H}_{\mathrm{eff}}^{(m)}\right] - \gamma\left[\mathbf{m}\times\mathbf{H}_{\mathrm{eff}}^{(l)}\right], \tag{11}$$

where $\mathbf{H}_{\mathrm{eff}}^{(m,l)} = -[1/(M_1 + M_2)]\{\delta W/\delta(\mathbf{m},\mathbf{l})\}$ are effective fields, calculated from the magnetic energy $W[\mathbf{l},\mathbf{m}]$ expressed through $\mathbf{m}$ and $\mathbf{l}$.

We should note that while ferrimagnets and antiferromagnets may, in principle, possess spin inertia [9,49–51], we have intentionally disregarded it here and thus omitted the explicit second time-derivative term in the equations of motion. Even without spin inertia, spin dynamics in these materials show inertial features, as observed experimentally [23,52]. The underlying mechanism, however, resides distinctly in the exchange coupling of the magnetic sublattices, as opposed to the coupling of spins to extrinsic degrees of freedom [13,14] in inertial ferromagnets.

For a small imbalance $\nu \ll 1$, we use the approximation of small $\mathbf{m}$. Consequently, we keep this vector only in the uniform exchange energy term with $\mathbf{m}^2$ and write all other terms through the vector $\mathbf{l}$, which can further be treated as a unit vector, $\mathbf{l}^2 = 1$. The energy density becomes [48]

$$w(\mathbf{m},\mathbf{l}) = \frac{E_{\mathrm{ex}}}{2}\mathbf{m}^2 + w(\mathbf{l}). \tag{12}$$

In Eq. (11), all terms are bilinear over $\mathbf{m}$ and $\mathbf{l}$, and we thus keep only the dominant exchange term,

$$\mathbf{m} = \nu\mathbf{l} + \frac{1}{\gamma H_{\mathrm{ex}}}\left(\frac{\partial \mathbf{l}}{\partial t}\times\mathbf{l}\right). \tag{13}$$

Substituting this expression for $\mathbf{m}$ into Eq. (10) and restoring the dissipative terms, we arrive at the equation of motion for the normalized Néel vector $\mathbf{l}$,

$$\nu\partial_t\mathbf{l} = -\gamma\mathbf{l}\times\mathbf{H}_{\mathrm{eff}}^{(l)} + \alpha\mathbf{l}\times\partial_t\mathbf{l} + (\gamma H_{ex})^{-1}\mathbf{l}\times\partial_{tt}\mathbf{l} + J\mathbf{l}\times(\mathbf{l}\times\boldsymbol{\sigma}). \tag{14}$$

Equation (14) describes ferrimagnets near spin compensation point $\nu \ll 1$ but, more generally, it is also valid qualitatively for $\nu \leq 1$. In particular, it reveals the presence of two precession modes: a right-hand mode at low frequencies and a left-hand mode with the characteristic frequency [18,19,48] of the order of $\omega_{\mathrm{ex}} = \gamma H_{\mathrm{ex}}$. These modes are similar to the two modes of opposite chirality, precessional and nutational, in inertial ferromagnets.

This phenomenological similarity is further substantiated by comparing Eq. (14) with Eq. (1). Defining a map between the constituents of these equations,

$$(\mathbf{m}, \mathbf{H}_{\mathrm{eff}}, \gamma, \alpha, J, \eta) \leftrightarrow \left(\mathbf{l}, \mathbf{H}_{\mathrm{eff}}^{(l)}, \gamma/\nu, \alpha/\nu, J/\nu, \frac{1}{\omega_{\mathrm{ex}}\nu}\right), \qquad (15)$$

allows us to establish an isomorphism between spin dynamics in ferrimagnets and inertial ferromagnets. Advances in knowledge made in one area of research [8,18,19] can now be synergistically transferred to the other one.

Conclusions.—We have investigated the effect of spin inertia on critical spin dynamics in ferromagnets using macrospin models and arrived at three major conclusions: (i) spin inertia causes a redshift of precessional frequencies and, as a consequence, a tangible reduction of effective damping and critical current for a given magnetic field. However, the relation of these two experimentally pertinent parameters to the frequency remains universally valid, regardless of the inertial parameter. (ii) Nutational auto-oscillations are of left-hand chirality and can be achieved with spin current polarity opposite to that of precessional auto-oscillations. The critical current magnitude $\alpha 2\pi f_{\mathrm{n}}$ is a few ten times higher than that of precessional auto-oscillations. Reaching nutational auto-oscillations thus appears to be a challenging but reasonably achievable task, in particular given the existing advanced methods of materials and spin-torque engineering in ferromagnets and the prospect of using them in future ultrahigh frequency applications. (iii) Spin dynamics in inertial ferromagnets is isomorphic to that in ferrimagnets near compensation point. Knowledge advances for these materials can be directly transferred, thus mutually benefiting these burgeoning research fields.

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