Power of Sequential Protocols in Hidden Quantum Channel Discrimination

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In many natural and engineered systems, unknown quantum channels act on a subsystem that cannot be directly controlled and measured, but is instead learned through a controllable subsystem that weakly interacts with it. We study quantum channel discrimination (QCD) under these restrictions, which we call hidden system QCD. We find sequential protocols achieve perfect discrimination and saturate the Heisenberg limit. In contrast, depth-1 parallel and multishot protocols cannot solve hidden system QCD. This suggests sequential protocols are superior in experimentally realistic situations.

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Introduction.—Discriminating between physical operations, often called quantum channel discrimination (QCD) in quantum information science, is a fundamental task in experiments [1–6]. In QCD, an unknown physical operation is modeled as a quantum channel C through a completely positive trace-preserving map acting on the system of interest [7]. The goal is to identify C from known alternatives using a discrimination protocol. Discrimination protocols are considered (1) efficient when a desired error probability is achieved with fewer queries than classical methods [8,9] or (2) successful when the error probability is zero [1,2,10]. For example, sequential protocols [2] involve an initial state ρ_m, and a positive operator-valued measurement (POVM) M, and N queries, each consisting of the unknown channel C and tunable unitary operations V_n ($n = 1, \ldots, N$), as shown in Fig. 1(a). Protocols including sequential and parallel protocols are able to achieve (1) and (2) when arbitrary operations of V_n and measurements M are allowed on the system [9,11,12].

While conventional QCD considers a fully controllable system, experimental systems often consist of a fully controllable subsystem, which we call the measurement system $\mathcal{M}$, and an uncontrollable subsystem, which we call the channel system $\mathcal{H}$ [13–18]. Here, $\mathcal{M}$ interacts with $\mathcal{H}$ to detect the action of C on $\mathcal{H}$. Such composite systems are used in quantum nondemolition measurements [13,14], quantum logic detection [15,16], and occur in designs of superconducting quantum devices [17].

These experiments motivate us to consider the following restrictions on system $\mathcal{H}$ in QCD: arbitrary control of $\mathcal{H}$ is not possible, measurement on $\mathcal{H}$ is not allowed, and the initialization of $\mathcal{H}$ is unreliable. The state on $\mathcal{H}$ thus evolves only under the dynamics C. The separation between $\mathcal{H}$ and $\mathcal{M}$ motivates the third restriction as one no longer has control over the state preparation on $\mathcal{H}$ and the initial state cannot be purified. We call $\mathcal{H}$ probed under these three restrictions hidden, and the associated channel discrimination problem hidden system quantum channel discrimination (HQCD). The effect of these restrictions on a conventional sequential QCD protocol is illustrated in Fig. 1(b). The restrictions become crucial when interactions between $\mathcal{H}$ and $\mathcal{M}$ have a limited ability to change the state

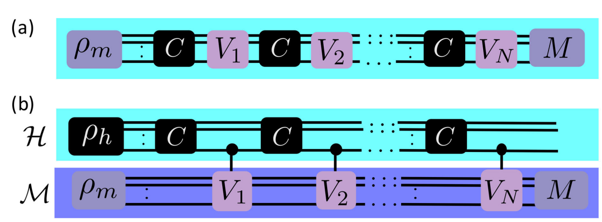

FIG. 1. Comparison between conventional quantum channel discrimination (QCD) and hidden quantum channel discrimination (HQCD). Black boxes indicate unknown channels and state. In both cases, the action of unknown channel C is inferred by selecting an input state ρ_m, applying controlled V_n operations ($n = 1, \ldots, N$), and measuring with M to minimize error probability. (a) Conventional QCD, involving direct manipulation and measurement of the system. (b) HQCD, where the physical system $\mathcal{H}$ and measurement system $\mathcal{M}$ are explicitly distinguished.

in $\mathcal{H}$ [19]. In a typical experiment, however, backaction on the channel is avoided by using a high-impedance meter. Here, we model this meter as a controlled unitary with the control on $\mathcal{H}$ and the unitary operation on $\mathcal{M}$.

It is then natural to ask if discrimination with zero error probability or fewer queries than classical methods is possible under these restrictions. This is a difficult task using conventional QCD techniques. For example, discrimination of a unitary channel is impossible when the input state is maximally mixed in conventional QCD [20]. Nevertheless, we give an affirmative answer to this question by studying hidden binary channel discrimination (HBCD), which is a minimal two-qubit binary HQCD problem shown in Fig. 2, and by constructing concrete measurement protocols with desired performance. The new protocols inherit ideas from conventional QCD, including sequential, parallel, and multishot strategies [11,21]. Inspired by previous work showing entanglement-free protocols can achieve similar performance to parallel protocols [22], the input states in our new sequential and multishot protocols are entanglement free [2,9,39–43]. Our parallel protocol can utilize an entangled initial state. Nevertheless, surprising performance differences arise.

In this Letter, we demonstrate that for the HBCD problem, sequential protocols outperform nonsequential protocols, including parallel and multishot protocols, in terms of the number of queries required to achieve a desired error probability. Furthermore, we prove sequential protocols achieve perfect discrimination with zero error. In contrast, we show a case where nonsequential protocols fail to solve HBCD when C is applied once before measurement. We extend the quantum metrology concepts of standard quantum limit (SQL) and Heisenberg limited scaling (HLS) to HBCD. The number of queries needed to solve HBCD by sequential protocols is proven to be asymptotically optimal using an information-theoretic bound and saturates HLS, whereas nonsequential protocols achieve only SQL. These advantages of sequential protocols over parallel protocols in QCD are reported for the first time to the best of our knowledge. Finally, we illustrate how HBCD restrictions arise in an experimental example.

Problem statement.—In our HBCD problem, we consider a two-qubit system composed of a one-qubit hidden system $\mathcal{H}$ on which the unknown channel C acts and a one-qubit measurement system $\mathcal{M}$ used to learn C.

Definition 1.—Unknown channel C.Let $\alpha \in (0, 2\pi)$, and θ_C be a Bernoulli random variable taking values in $\{0, \alpha\}$ with probability $P_{\theta_C}(0) = P_{\theta_C}(\alpha) = 1/2$. The unknown quantum channel acting on $\mathcal{H}$ is then $C = e^{i\theta_C \sigma_x}$.

Definition 2.—Query. A query $Q(\psi, \phi)$ is a unitary operation acting on the two-qubit system composed of $\mathcal{H}$ and $\mathcal{M}$, and is parametrized by a pair of phases $\{\psi, \phi\}$. The circuit of $Q(\psi, \phi)$ is depicted in Fig. 2(a). It involves three components: (i) the unknown channel C, (ii) a controlled rotation on $\mathcal{M}$ by ψ along the z axis with the control qubit in $\mathcal{H}$, and (iii) a single-qubit rotation on $\mathcal{M}$ by ϕ along the x axis.

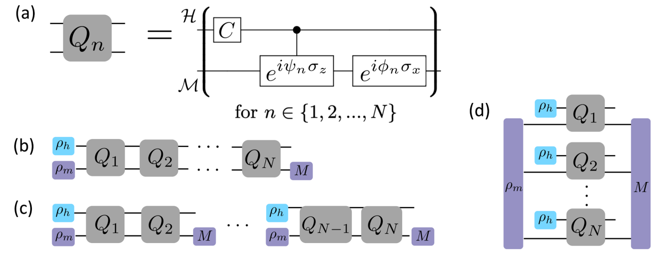

FIG. 2. The query and sequential-multishot-parallel protocols. (a) Query Q_n (Definition 2) with phases ψ_n and ϕ_n specified independently. The upper (hidden) qubit undergoes unitary evolution every round and at the end we measure the lower (measurement) qubit. (b)–(d) Discrimination protocol S: (b) sequential protocol, (c) multishot protocol with depth $d = 2$, (d) parallel protocol. ρ_m can be a highly entangled state for the parallel protocol.

The query as defined above is inspired from quantum signal processing (QSP) [44] and lends to the success of the constructed protocols. Connections to QSP are elaborated in [22]. We now define our HBCD problem.

Definition 3.—HBCD problem. Suppose $\epsilon \in [0, 1/2]$, and ρ_h is the initial one-qubit mixed state on $\mathcal{H}$. Let C be the unknown channel from Definition 1 with θ_C determined at the start of the experiment and which remains constant for all subsequent queries. Then $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$ defines the problem of learning an estimate $\hat{\theta}_C$ of the unknown θ_C with error probability $P(\hat{\theta}_C \neq \theta_C) \leq \epsilon$.

We would ideally like to solve an HBCD problem using minimal queries. We assume ρ_h is known for simplicity. Generally for unknown ρ_h, HBCD is no more difficult than when ρ_h is maximally mixed and no easier than when ρ_h is a pure state. To learn θ_C, we design discrimination protocols denoted by Σ which involve specifying the initial state ρ_m to $\mathcal{M}$, POVM measurement M acting on $\mathcal{M}$, and N queries $\{Q_1, ..., Q_N\}$. We denote the corresponding vector of phases as $\Phi \equiv (\psi_1, ..., \psi_N, \phi_1, ..., \phi_N) \in [0, 2\pi)^{2N}$.

Definition 4.—Discrimination protocols.Given a problem $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$, we define a discrimination protocol $\Sigma(N, d, Z, S)$ where N is the total number of the queries used, depth d is the number of concatenated queries before measurement, $Z = (\rho_m, \Phi, M)$ is the collection of specified settings with Φ being the vector of phases specifying the N queries, and S defines the type of protocol which can be sequential, multishot or parallel. The circuit corresponding to Σ for different S is shown in Figs. 2(b)–2(d).

Our discrimination protocols are designed using knowledge of ρ_h and α. The depth d takes the value of N when S is sequential, N/m when S is a multishot protocol using m shots and 1 when S is a parallel protocol over an N-qubit measurement system $\mathcal{M}$ interacting with N copies of $\mathcal{H}$ [see Fig. 2(d)]. Our sequential protocol uses one probe qubit [45]. In conventional QCD, these have weaker discrimination performance than parallel protocols [48,49]. We compare their performance in HBCD. The multishot protocol allows for adaptive choice of Z, but we do not explore this here [50].

We now define the discrimination error associated with each protocol. Suppose $\mathbf{y} = (y^1, \ldots, y^m) \in \{0,1\}^m$ is the vector of m POVM outcomes. Given $\mathbf{y}$, an estimator $\hat{\theta}_C(\mathbf{y})$ will output either 0 or α. The error probability of a protocol Σ, denoted by $\mathcal{E}(\Sigma)$ is then

$$\begin{aligned} \mathcal{E}(\Sigma) &= P(\hat{\theta}_C(\mathbf{y}) \neq \theta_C; \Sigma) \\ &= \frac{1}{2}\left[P_{\hat{\theta}_C(\mathbf{y})|\theta_C}(\alpha|0;\Sigma) + P_{\hat{\theta}_C(\mathbf{y})|\theta_C}(0|\alpha;\Sigma)\right], \end{aligned} \tag{1}$$

where we noted prior probabilities satisfy $P(\theta_C = 0) = P(\theta_C = \alpha) = 1/2$. We design the estimators such that $\hat{\theta}_C(0) = 0$ and $\hat{\theta}_C(1) = \alpha$. The error is then $\mathcal{E}(\Sigma) = \frac{1}{2}[P_{y|\theta_C}(0|\alpha;\Sigma) + P_{y|\theta_C}(1|0;\Sigma)]$. Hence, $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$ is solved when

$$\mathcal{E}(\Sigma) \leq \epsilon, \tag{2}$$

is satisfied. For the multishot and parallel protocols, we use the likelihood ratio test (LRT) as our estimator. An overview of estimators is given in [22].

Another performance metric is the scaling of the minimal number of queries N required to solve $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$. As α decreases, solving HBCD becomes more difficult and N increases. We define two scaling limits of N with α.

Definition 5.—Standard quantum limit and Heisenberg limited scaling in HBCD.Suppose $\epsilon \in [0, 1/2)$, ρ_h, S, and d are given. For $0 < \alpha \ll 1$, let N be the number of queries required to solve $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$ by $\Sigma(N, d, Z, S)$. A depth-d S protocol achieves the standard quantum limit (SQL) if $N = \Theta(\alpha^{-2})$ and Heisenberg limited scaling (HLS) if $N = \Theta(\alpha^{-1})$.

The scalings SQL and HLS are defined in quantum metrology for parameter estimation in terms of the number of accesses to an unknown physical system of interest. This corresponds to the number of interactions N between $\mathcal{H}$ and $\mathcal{M}$. In parameter estimation, a protocol is said to achieve SQL when the number of queries N required to achieve an estimation error α_{PE} scales as $N \sim \alpha_{\mathrm{PE}}^{-2}$ and HLS when $N \sim \alpha_{\mathrm{PE}}^{-1}$ [9,12,53]. Similarly, we can model the problem of discriminating the value of θ_C from $\{0, \alpha\}$ in HBCD as estimating the value of θ_C. We succeed if the estimation error is smaller than half of the angle difference ($\alpha/2$). Definition 5 is then evident. We note that HLS in QCD is strictly weaker than that in parameter estimation. A future direction would be to show if sequential protocols can achieve HLS in parameter estimation problems on hidden systems.

Perfect discrimination.—We now discuss advantages of using sequential protocols in HBCD. The proofs of the theorems presented below are in [22].

Theorem 1.—Perfect discrimination in HBCD.For any $\alpha \in (0, 2\pi)$, there exists a sequential protocol $\Sigma(N, d = N, Z, S = \text{sequential})$ that solves $\mathsf{HBCD}(\alpha, \epsilon = 0, \rho_h)$ with at most $N = 24\lceil \pi/2\beta \rceil$ queries.

Here, β is an effective rotation angle on the measurement qubit, which is given by $\beta = -i \log[\tilde{a}^2 - \tilde{a}(1+\tilde{a})(3+\tilde{a})\cos^2\alpha + (1+\tilde{a})^3 \cos^4\alpha]$ with $\tilde{a} = \tan^2\alpha + i\sqrt{1 - \tan^4\alpha}$ [22]. To prove the theorem, we construct a diagonal unitary matrix from four queries. The controlled rotation then becomes a single-qubit R_Z gate on $\mathcal{M}$ with its rotation angle being either $-\beta$ for $\theta_C = 0$ or β for $\theta_C = \alpha$. Using this rotation on $\mathcal{M}$, we accumulate the phase $\pm\beta$ on the measurement qubit so that the measurement qubit is $|0\rangle$ for $\theta_C = 0$ and $|1\rangle$ for $\theta_C = \alpha$ [54,55]. By expanding β around small α, we observe this particular sequential protocol only achieves SQL. Later, however, our numerical results and information-theoretic bound show properly designed sequential protocols achieve HLS.

Weakness of the nonsequential protocol.—Since any entanglement improves conventional QCD [48,49], one expects parallel protocols to outperform sequential protocols. Indeed, this occurs when the channel is noisy and error correction is unavailable [56]. However, when the channel is noiseless but the state is noisy, the opposite is true; sequential protocols outperform parallel protocols. We show discrimination is impossible for nonsequential protocols with query depth $d = 1$ and the initial state in $\mathcal{H}$ is maximally mixed.

Theorem 2.—Impossible case for depth-1 nonsequential protocols.Suppose $\rho_h = (\mathbb{I}/2)$, and S is multishot or parallel. For any $\epsilon \in [0, 1/2)$ and $\alpha \in (0, 2\pi)$, the protocols $\Sigma(N, d = 1, Z, S)$ cannot solve $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$. That is, Σ does not obtain any information on θ_C through M.

Critically, the maximally mixed state remains invariant under single-qubit rotations. Therefore, the state ρ_M on $\mathcal{M}$ before measurement is independent of θ_C. However, if $d \geq 2$, ρ_M correlates with θ_C through the controlled interactions. Thus, protocols with $d \geq 2$ queries are strictly better than nonsequential protocols with $d = 1$.

Next, we comment on the asymptotic query complexity required to solve HBCD for $d = 2$. The multishot protocol with a fixed depth cannot achieve HLS (Theorem 3), which is illustrated numerically later.

Theorem 3.—SQL in HBCD by multishot protocol. For all $\epsilon \in [0, 1/2)$ and ρ_h, depth-2 multishot protocol achieves SQL.

The theorem implies HBCD becomes challenging with decreasing α, and the minimum distinguishable value of α scales as $\alpha \sim N^{-1/2}$ with increasing N.

The advantages of sequential protocols over nonsequential protocols in HBCD are evident from Theorems 1–3. The sequential protocol alone enables perfect discrimination.

Additionally, nonsequential protocols with $d = 1$ cannot learn θ_C regardless of the number of queries, while sequential protocols can.

Heisenberg limit in HBCD.—The possibility of achieving HLS (Definition 5) is still unanswered. We first derive a lower bound on N required to solve HBCD.

Theorem 4.—Fundamental limit of HBCD. Any protocol $\Sigma(N, d, Z, S)$ with $N < [1/\sqrt{2(1-\cos\alpha)}]$ cannot solve $\mathsf{HBCD}(\alpha, \epsilon = 0, \rho_h)$.

Expanding the bound on N in Theorem 4 around $\alpha \ll 1$ indicates HLS is indeed the optimal scaling, i.e., $N = \Omega(\alpha^{-1})$.

We now present numerical evidence that HLS is achieved by sequential protocols while multishot protocols using constant depth queries achieves SQL. We solve the HBCD problem through measurements on $\mathcal{M}$ shown in Figs. 2(b) and 2(c) using a maximally mixed state ($\rho_h = (\mathbb{I}/2)$) on $\mathcal{H}$. The state on $\mathcal{M}$ depends on the specified phase sequence Φ. If some Φ of length N sets the state of $\mathcal{M}$ to be $|1\rangle$ for $\theta_C = \alpha$ and $|0\rangle$ for $\theta_C = 0$, then $\mathsf{HBCD}(\alpha, \epsilon = 0, \rho_h)$ is solved with one shot.

For the sequential protocol, we attempt to solve HBCD by measuring once and with error probability $\epsilon \in [0, 1/2)$. The goal is to set the outcome y of measuring $\mathcal{M}$ in the computational basis such that Eq. (2) is satisfied. Thus, to determine Φ, we solve the following optimization problem

$$\arg\min_{\Phi} \big(1 - P_{y|\theta_C}(1|\alpha;\Phi) + P_{y|\theta_C}(1|0;\Phi)\big)^2, \qquad (3)$$

with additional constraints $\psi_n = \psi$, $\forall\, n \in [N]$. Optimization details are in [22]. We claim Φ succeeds in $\mathsf{HBCD}(\alpha, \epsilon, \rho_h)$ if the solution to Eq. (3) satisfies Eq. (2). Given α, we determine the minimal number of queries required by starting with $N = 1$ and incrementing the value of N by one until the solution to Eq. (3) satisfies Eq. (2).

For the multishot protocol with constant depth-d queries, we use a phase sequence Φ of length d but may measure $\mathcal{M}$ $m \geq 1$ times to solve HBCD with error probability $\epsilon \in [0, 1/2)$. Given α, we determine Φ of length d by solving the optimization problem of Eq. (3). We determine $m^\star$ or the smallest number of shots required to achieve an error ϵ by evaluating Eq. (1), considering the estimator based on the likelihood-ratio test over the measurement outcomes [22]. The total number of queries required is then $N = dm^\star$.

Figure 3 shows numerically determined trends of N required by the sequential and multishot protocols to solve $\mathsf{HBCD}(\alpha, \epsilon, \rho_h = \mathbb{I}/2)$. As expected, N increases as α decreases and approaches zero for both protocols. Notably, we observe a scaling of SQL for multishot protocols but crucially HLS $N \sim O(\alpha^{-1})$ for sequential protocols.

HBCD example.—To demonstrate the advantage of the sequential protocol in HBCD in a realistic setting, consider the discrimination of an unknown channel describing the presence or absence of a birefringent slab which rotates the polarization of incident single photons by an angle α. Naively, the discrimination of this slab is formulated as a conventional QCD in Fig. 1(a) by propagating a polarization qubit. Sequential protocols are known to solve conventional QCD efficiently [9,54,57].

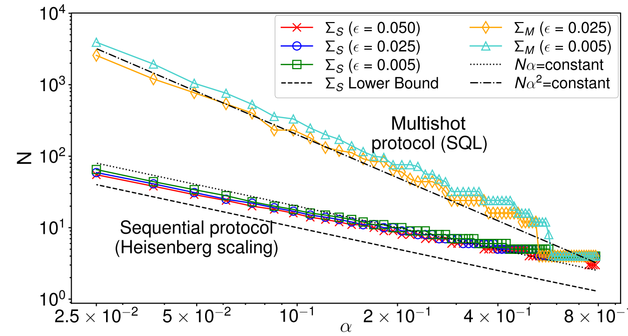

FIG. 3. Number of queries N sufficient for solving $\mathsf{HBCD}(\alpha, \epsilon, \rho_h = \mathbb{I}/2)$. For the sequential protocol (Σ_S), we measure only once and use a phase sequence Φ of length N. For the multishot protocol (Σ_M), we use queries of depth $d = 4$ and measure multiple times. Trends for different values of $\epsilon \in \{0.005, 0.025, 0.05\}$ are shown for Σ_S and $\epsilon \in \{0.025, 0.005\}$ for Σ_M.

However, in the free-space setting, optical elements implementing $\{V_n\}$ cannot be reconfigured at the timescale of a round-trip [58,59]. Consequently, a sequential protocol without an adjoint measurement system requires the number of optical elements to increase with the depth of the protocol. Thus, the propagating photonic polarization qubit should be considered a hidden system $\mathcal{H}$ for small α.

To address this problem, we introduce a measurement system $\mathcal{M}$ consisting of a cavity QED system (see Fig. 4) which enables the implementation of reconfigurable single-qubit gates, and improves measurement efficiency and fidelity [22,60]. Queries Q involving interactions between $\mathcal{H}$ and $\mathcal{M}$ are realized by the proposal in [61]. With this implementation, we could show sequential

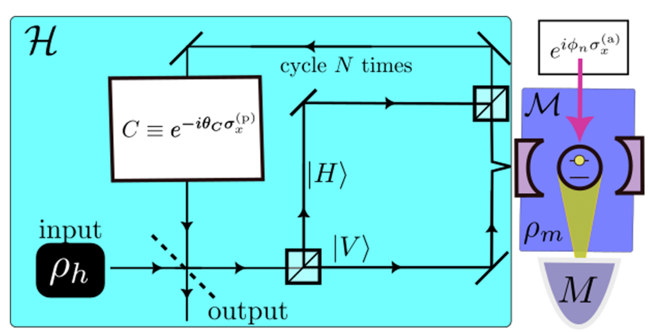

FIG. 4. Schematic for optical experiment for measurements through hidden channel discrimination. The color code is identical to Figs. 1(a) and 2(a), with the photonic (p) and atomic (a) degrees of freedom realizing systems $\mathcal{H}$ and $\mathcal{M}$, respectively. See [22] for implementation details.

protocols achieve perfect discrimination (Theorem 1) and HLS (Fig. 3).

Although decoherence may prevent achieving HLS in practice, we show the proposed HBCD sequential protocol achieves higher probability of detection and lower errors even at shallower depths with weaker requirements on query fidelity [22].

Conclusion.—In this Letter, we proposed the HQCD problem and analyzed the performance of different protocols on HBCD. We showed sequential protocols outperform multishot and parallel protocols in HBCD. Notably, our work shows HLS can be achieved with a sequential protocol in HQCD for a single-qubit channel. We expect these results to have interesting implications for other learning tasks on hidden quantum systems. Theoretically, one question is whether our results can be extended to problems of discriminating multiqubit channels on hidden systems or learning multiqubit channels with continuous parameters. If the hidden quantum channels correspond to unitary Hamiltonian evolution, could we learn the Hamiltonian [62,63] at HLS? Our results suggest the guiding principle for measuring the properties of the hidden system requires transmitting information through interactions between the hidden and measurement systems when they are separated. The sequential protocol conveys information through repeated interactions and could achieve HLS. Conversely, the parallel protocol fails to estimate the channel because the initial state does not have entanglement across the two systems. We hope this interpretation could be proved for other learning problems on hidden systems.

Code for the different discrimination protocols in solving HBCD numerically and data are available on GitHub [64].

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