Tunable Non-Fermi Liquid Phase from Coupling to Two-Level Systems

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We study a controlled large-N theory of electrons coupled to dynamical two-level systems (TLSs) via spatially random interactions. Such a physical situation arises when electrons scatter off low-energy excitations in a metallic glass, such as a charge or stripe glass. Our theory is governed by a non-Gaussian saddle point, which maps to the celebrated spin-boson model. By tuning the coupling strength we find that the model crosses over from a Fermi liquid at weak coupling to an extended region of non-Fermi liquid behavior at strong coupling, and realizes a marginal Fermi liquid at the crossover. Our results are valid for generic space dimensions $d > 1$.

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Introduction.—Understanding scenarios in which strong interactions between itinerant electrons and collective quantum fluctuations invalidate the conventional Landau Fermi-liquid (FL) paradigm is a central problem in the field of correlated metals [1–5]. A prominent example of such a scenario is the enigmatic "strange metal" (SM) behavior found in high-T_c superconductors and other quantum materials [6–9]. SMs are commonly defined by an anomalous T-linear scaling of the dc resistivity at arbitrarily low temperatures, which is in sharp contrast to the T^2 dependence predicted by FL theory. While SM behavior is often associated with an underlying quantum critical point (QCP) at $T = 0$ [1,10–14], there are multiple examples, such as cuprates [15–18], twisted bilayer graphene [9,19], and twisted transition metal dichalcogenides [20], where it appears to persist over an extended region, suggesting the interesting possibility of a critical, non-Fermi liquid phase at zero temperature. Extended critical behavior has also been observed in MnSi with $T^{3/2}$ scaling of the resistivity [21–24] and in CePdAl with a T^n scaling with n varying from ∼1.4 to 2 [25].

The absence of a quasiparticle description in such systems, known as non-Fermi liquids (NFLs) [11,26,27] or marginal Fermi liquids (MFLs) [10,11], makes well-controlled theoretical investigations challenging. Nevertheless, recent years have witnessed a proliferation of illuminating solvable models, largely facilitated by the use of large-N and Sachdev-Ye-Kitaev (SYK) techniques [2,5,28,29]. In particular, the extensive analysis of a class of Yukawa-SYK theories [30–39], where a Fermi surface is coupled to critical bosons, has demonstrated that strange metallicity can emanate from a QCP.

Non-Fermi liquid behavior is ultimately tied to an anomalous spectrum of gapless excitations that invalidate the phase-space arguments that protect FLs. Such a spectrum occurs at a QCP. In contrast, a controlled microscopic theory hosting a stable NFL phase, free of fine-tuning, remains mostly absent within the existing literature. This is surprising as such extended metallic critical phases seem to be allowed within holographic setups [40,41], and are also supported by numerical evidence [42].

A gapless spectrum can also be the result of quantum fluctuations of localized, gapped modes, if the distribution of the local gaps has a sufficiently large weight near zero. Indeed, in the presence of disorder, a system may host such an anomalous spectrum of localized modes due to quantum Griffiths effects, where rare, large droplets have a gap that is exponentially small in the droplet volume [43–45].

Here we take a different route and show that a NFL phase can arise due to coupling of electrons to the low-energy two-level excitations of an underlying glassy state [46,47]. While individual two-level systems (TLSs) are governed by a distribution function that may vanish rapidly near zero energy, strong local quantum fluctuations renormalize the excitation gap down. This changes the shape of the excitation spectrum drastically, resulting in a breakdown of FL behavior.

To this end we formulate and solve in a controlled large-N limit a model of N fermion species coupled to $M \sim N$ dynamical TLSs per site via spatially random interactions. The theory is governed by a non-Gaussian saddle point, which maps to the spin-boson (SB) model. While the electrons constitute an Ohmic bath for the TLSs for all coupling strengths, the backaction of the TLSs gives rise to a tunable exponent $\gamma(\alpha)$ in the electronic self-energy $\mathrm{Im}\Sigma(\omega) \sim |\omega|^\gamma$. Here α is the dimensionless coupling constant of the problem. Notably, the low-temperature dc resistivity obeys $\rho(T) = \rho_0 + CT^\gamma$, with a tunable exponent $0 < \gamma < 2$ that depends on the coupling. This behavior offers an alternative viewpoint to the conventional form: $\rho(T) = \rho_0 + AT + BT^2$, that is often used in the interpretation of

strange metal behavior [8,15,16,48–52]. A summary of our results for a representative model is shown in Fig. 1.

Our theory draws inspiration from the intricate phase diagrams of high-T_c superconductors which often exhibit a competition between multiple frustrated orders that could lead to metallic glassiness [53,54]. In the cuprates, there is plenty of evidence for short-range charge-density-wave and nematic order [55–69], that can account for such glassy behavior. It is worth noting that such a scenario can arise independently of extrinsic quenched disorder, namely, due to frustration-induced self-generated randomness [70]. For example, the dynamics of helical magnetism in MnSi is governed by soft bosonic excitations on a surface in momentum space, a setting prone to lead to self-generated glassiness [71].

Model.—We consider a model of N species of electrons hopping on a d-dimensional lattice with dispersion $\varepsilon_{\boldsymbol{k}}$. Each site hosts M TLSs (i.e., spin-1/2 Pauli operators) subject to random fields $h_{l,\boldsymbol{r}}$, which we locally describe in their eigenbases:

$$H = \sum_{\boldsymbol{k},i<N} \varepsilon_{\boldsymbol{k}} c^{\dagger}_{i\boldsymbol{k}} c_{i\boldsymbol{k}} - \sum_{\boldsymbol{r},l<M} h_{l,\boldsymbol{r}} \sigma^{z}_{l,\boldsymbol{r}} + H_{\mathrm{int}}. \tag{1}$$

Having multiple TLSs per site ($M \gg 1$) could arise from a coarse grained description of mesoscale collective degrees of freedom of an underlying glassy state [72,73], each of which interacts with many electrons. The electronic orbitals are extended as well, such that each electron interacts with many TLSs. The coupling between the electrons and TLSs is given by

$$H_{\mathrm{int}} = \frac{1}{N} \sum_{\boldsymbol{r},ijl} \boldsymbol{g}_{ijl,\boldsymbol{r}} \cdot \boldsymbol{\sigma}_{l,\boldsymbol{r}} c^{\dagger}_{i\boldsymbol{r}} c_{j\boldsymbol{r}}, \tag{2}$$

with random couplings $\boldsymbol{g}_{ijl,\boldsymbol{r}}$. Adding a potential scattering term $H_{\mathrm{pot}} = (1/\sqrt{N}) \sum_{\boldsymbol{r},ij<N} V_{ij,\boldsymbol{r}} c^{\dagger}_{i\boldsymbol{r}} c_{j\boldsymbol{r}}$ induces elastic scattering of the itinerant electrons, while leaving the TLS properties unaffected.

For $\boldsymbol{g}_{ijl,\boldsymbol{r}} \sim (0,0,1)$ the TLSs are static and the problem reduces to a single-particle one and can be solved exactly. Electrons cause quantum transitions whenever there are coupling constants that differ from the above choice. We have analyzed the case of general coupling constants and find that the case where $\boldsymbol{g}_{ijl,\boldsymbol{r}} \sim (g_{ijl,\boldsymbol{r}},0,0)$ describes all the important aspects that occur in the generic case [74]. Finally, we need to specify the distribution function of the random coupling constants and fields. For the former we chose a Gaussian distribution with zero mean and variance $\overline{g_{ijl,\boldsymbol{r}} g_{ijl,\boldsymbol{r}'}} = g^2 \delta(\boldsymbol{r}-\boldsymbol{r}')$. For the latter, consider that each TLS represents two nearly degenerate configurations of some local collective mode. The level splitting and tunneling rate, characterizing each TLS, can be considered as independently distributed random variables due to their disordered nature. For the eigenvalues h in Eq. (1) this yields a distribution function $\mathcal{P}(h) \propto |h|$ for $|h| < h_c$, where h_c is the maximum splitting of the TLSs, assumed to be much smaller than the Fermi energy, E_F. We further consider $\mathcal{P}_\beta(h) \propto |h|^\beta$ that also includes the case of a constant distribution for $\beta = 0$ (which could describe a physical situation where tunneling is significantly smaller than the energy splitting), and keep $\beta \geq 0$ arbitrary. The qualitative behavior is similar for all values of $\beta \geq 0$.

Effective action.—We start from a coherent-state path integral for the fermions and TLSs, average over the random couplings by using the replica trick, and enforce the identities

$$G_{\boldsymbol{r},\boldsymbol{r}'}(\tau,\tau') = \frac{1}{N} \sum_i \bar{c}_{i\boldsymbol{r}}(\tau) c_{i\boldsymbol{r}'}(\tau'), \tag{3}$$

$$\chi_{\boldsymbol{r}}(\tau,\tau') = \frac{1}{M} \sum_l \sigma^{x}_{l,\boldsymbol{r}}(\tau) \sigma^{x}_{l,\boldsymbol{r}}(\tau'), \tag{4}$$

via corresponding Lagrange multiplier fields $\Sigma_{\boldsymbol{r},\boldsymbol{r}'}(\tau,\tau')$ and $\Pi_{\boldsymbol{r}}(\tau,\tau')$. We integrate out the fermions and obtain the effective action:

$$\begin{aligned} S_{\mathrm{eff}} = {} & S_{\mathrm{TLS}} - N \mathrm{tr} \log\left(G_0^{-1} - \Sigma\right) - N \mathrm{tr} G\Sigma + \frac{M}{2} \mathrm{tr} \chi \Pi \\ & + \frac{M}{2} g^2 \sum_{\boldsymbol{r}} \int_{\tau,\tau'} G_{\boldsymbol{r}}(\tau,\tau') G_{\boldsymbol{r}}(\tau',\tau) \chi_{\boldsymbol{r}}(\tau,\tau'), \end{aligned} \tag{5}$$

where the trace is taken over space and time. In contrast to the usual Yukawa-SYK approach [30–39], where the role of the TLSs is played by Gaussian bosonic modes, there is no Wick theorem for spin variables and the TLSs cannot be integrated out. Instead, they are governed by the action $S_{\mathrm{TLS}} = \sum_{\boldsymbol{r},l} S_{\mathrm{SB}}[\boldsymbol{\sigma}_{l,\boldsymbol{r}}]$, which is a sum of individual spins with fermion-induced dynamics:

$$\begin{aligned} S_{\mathrm{SB}}[\boldsymbol{\sigma}_{l,\boldsymbol{r}}] = {} & - \int_{\tau,\tau'} \Pi_{\boldsymbol{r}}(\tau'-\tau) \sigma^{x}_{l,\boldsymbol{r}}(\tau) \sigma^{x}_{l,\boldsymbol{r}}(\tau') \\ & - \int_{\tau} h_{l,\boldsymbol{r}} \sigma^{z}_{l,\boldsymbol{r}}(\tau) + S_{\mathrm{Berry}}[\boldsymbol{\sigma}_{l,\boldsymbol{r}}]. \end{aligned} \tag{6}$$

See Supplemental Material [75] for details of the derivation of Eq. (6). The last term is merely the Berry phase that occurs from the spin coherent state representation. $S_{\mathrm{SB}}[\boldsymbol{\sigma}_{l,\boldsymbol{r}}]$ is a highly nontrivial local many-body problem. One recognizes that it is equivalent to the spin-boson problem of a localized spin coupled to an environment of bosons with spectral function leading to the bath function $\Pi_{\boldsymbol{r}}(\tau'-\tau)$ [78–81]. In our case the bath is due to electronic particle-hole excitations. In the large-N, M limit the theory is governed by the saddle point with respect to G, Σ, and χ, given, respectively, by

$$\begin{aligned} \Sigma_{\boldsymbol{r},\boldsymbol{r}'}(\tau) &= \delta_{\boldsymbol{r},\boldsymbol{r}'} \frac{M}{N} g^2 G_{\boldsymbol{r},\boldsymbol{r}}(\tau) \chi_{\boldsymbol{r}}(\tau), \\ G_{\boldsymbol{r},\boldsymbol{r}'}(i\omega) &= \left(G_0^{-1}(i\omega) - \Sigma(i\omega)\right)^{-1}|_{\boldsymbol{r},\boldsymbol{r}'}, \\ \Pi_{\boldsymbol{r}}(\tau) &= -g^2 G_{\boldsymbol{r},\boldsymbol{r}}(\tau) G_{\boldsymbol{r},\boldsymbol{r}}(-\tau). \end{aligned} \tag{7}$$

The saddle with respect to Π replaces the right-hand side of Eq. (4) by its expectation value. Equations (7) are similar to the Yukawa-SYK problem, where one obtains essentially a set of coupled Eliashberg equations. The crucial difference is that in our case, Π and χ are not the self-energy and propagator of a boson, related by a Dyson equation. Instead Π is the bath function of a spin-boson problem whose solution determines χ.

The saddle-point equations (7) together with the solution of the spin-boson problem S_{SB} of (6) are valid for a given disorder configuration $\{h_{l,\boldsymbol{r}}\}$ of the random field. Hence, the system is not translation invariant and correlation functions like $\langle \sigma^x_{l,\boldsymbol{r}}(\tau)\sigma^x_{l,\boldsymbol{r}}(0)\rangle$ fluctuate in space. However, to determine the self-energy in Eq. (7) we only need to know the average of this correlation function over the M flavors. To proceed we employ the large-M limit to replace sums over the TLS flavors with an average over the TLS splitting distribution: $(1/M)\sum_{l=1}^{M}\cdots \to \int \mathcal{P}(h)\cdots dh$. Since the distribution is independent of position we obtain a statistical translation invariance for the average of interest. Hence, $\chi(\tau)$ and therefore the bath function $\Pi(\tau)$ are independent of $\boldsymbol{r}$, implying that the local fermionic Green's function and self-energy are position independent as well.

The theory is therefore governed by a momentum-independent self-energy. Standard manipulations for such a self-energy, valid in the regime where the typical bosonic energies are smaller than the electronic bandwidth, yield $\Pi(i\omega) = (\rho_F^2 g^2/2\pi)|\omega|$. That is, the electrons constitute an Ohmic bath for the TLSs, independent of the form of the self-energy. To proceed, we obtain the TLS susceptibility $\chi(h_l,\omega)$ (i.e. for a fixed h_l) by solving its corresponding SB problem $S_{\mathrm{SB}}[\boldsymbol{\sigma}_{l,r}]$, and average over the random field configurations $\chi(\omega) = \int dh \mathcal{P}(h)\chi(h,\omega)$. Then the fermionic self-energy of Eq. (7) can at $T=0$ be written as $\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega) = -\rho_F g^2 (M/N)\int_0^{|\omega|} d\omega' \mathrm{Im}\chi_{\mathrm{ret}}(\omega')$ (hereafter "ret" denotes the retarded function).

Results.—The solution of the SB model is a classic problem in many-body physics of impurity models [79,81]. For our purposes we use two established facts: (i) the low-energy physics of the problem is governed by renormalization group equations

$$\frac{d\alpha}{d\ell} = -\alpha\tilde{h}^2 \quad \text{and} \quad \frac{d\tilde{h}}{d\ell} = (1-\alpha)\tilde{h} \tag{8}$$

for the dimensionless coupling constant $\alpha = g^2\rho_F^2/\pi^2$ and field $\tilde{h} = h/E_F$, where ℓ measures the logarithm of the characteristic energy. Its solution yields renormalized fields and coupling constants $h_R(h,\alpha)$ and $\alpha_R(h,\alpha)$, respectively [up to $\mathcal{O}(1)$ numerical coefficients which can be determined by alternative methods [82–85]]. (ii) The correlation function obeys a scaling form in terms of the renormalized field

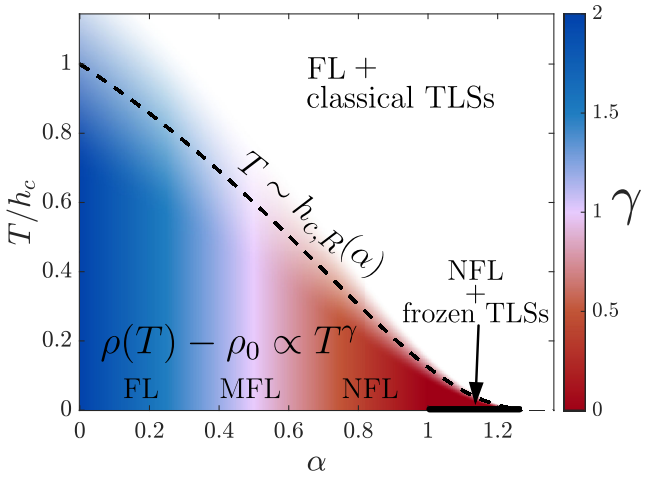

FIG. 1. Phase diagram for a system with a linear field distribution ($\beta = 1$). α is the dimensionless coupling strength and $h_c \ll E_F$ is the bare cutoff energy of the TLS distribution. The colored area represents the low-T dynamical regime, characterized for $\alpha < 1$ by the exponent $\gamma = 2(1-\alpha)$ that governs the inelastic scattering rate. We find FL behavior for $\alpha < 1/2$, a MFL at $\alpha = 1/2$ and a NFL for $\alpha > 1/2$. As α crosses 1, the TLSs partially freeze at $T = 0$, indicated by a thick black line. At temperatures above the renormalized cutoff $T \gg h_{c,R}$, the TLSs act as classical elastic scatterers with T-independent scattering rate. The crossover to this region is denoted by the fading out of the color. Here $h_c/E_F = 0.4$.

$$\mathrm{Im}\chi_{\mathrm{ret}}(h,\omega,T) = \frac{1}{\omega} f_\alpha\left(\frac{\omega}{h_R}, \frac{T}{h_R}\right). \tag{9}$$

The scaling function f_α is known numerically and, in several limiting regimes, analytically. To perform the average over field configurations, it is convenient to work with the distribution function of renormalized fields:

$$\mathcal{P}_{R,\beta}(h_R) \equiv \frac{dh}{dh_R}\mathcal{P}_\beta(h). \tag{10}$$

The renormalized distribution function follows from the solution of the flow equations. In Fig. 2 we show the evolution of the corresponding renormalized distribution function as a function of the coupling constant α for the case of $\beta = 1$ (i.e., a linear bare distribution).

We first consider $\alpha < 1$. In this case the solution of Eqs. (8) yields $h_R = E_F \tilde{h}^{[1/(1-\alpha)]}$ [78–81], which yields that $\mathcal{P}_{R,\beta}(h_R) \propto h_R^{\beta-\alpha(1+\beta)}$. The downwards renormalization of the excitation energies of the TLSs leads to a strong weight transfer in the distribution function, making it significantly more singular at low energies. As soon as α reaches $\alpha_{\mathrm{MFL}} \equiv \beta/(1+\beta)$ the renormalized distribution function remains finite at arbitrarily small h_R, for $\alpha > \alpha_{\mathrm{MFL}}$ it is even divergent in the low-energy limit. We can now straightforwardly perform the average with the renormalized distribution. We find that at $T = 0$, the low-energy ($\omega \ll h_{c,R}$)

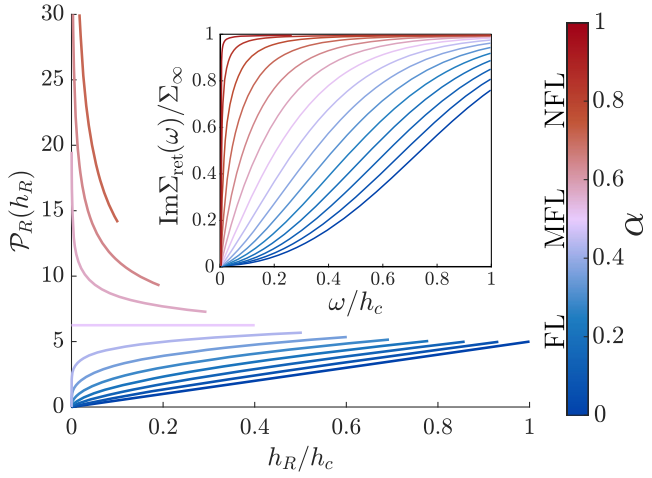

FIG. 2. Main: Evolution of the renormalized distribution $\mathcal{P}_R(h_R)$ [see Eq. (10)] is shown for increasing dimensionless coupling α with $\beta = 1$ and $h_c/E_F = 0.4$. Inset: Frequency dependence of $\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega)$ at $T = 0$ for different α. $\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega \ll h_{c,R}) \sim |\omega|^{2(1-\alpha)}$, while it saturates to the value $\Sigma_\infty \sim \alpha E_F$ for $\omega \gg h_{c,R}$ due to the finite Hilbert space of TLSs.

TLS susceptibility and the electronic self-energy are characterized by a coupling-constant-dependent exponent $\gamma = (1+\beta)(1-\alpha)$:

$$\mathrm{Im}\chi_{\mathrm{ret}}(\omega) = \mathrm{sign}(\omega)\frac{\gamma A_\alpha}{h_{c,R}}\left|\frac{\omega}{h_{c,R}}\right|^{\gamma-1},$$
$$\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega) = -\rho_F g^2 A_\alpha \frac{M}{N}\left|\frac{\omega}{h_{c,R}}\right|^{\gamma}. \tag{11}$$

$h_{c,R}$ is the renormalized value of the upper cutoff h_c. Importantly, the exact form of the scaling function $f_\alpha(x,0)$ of Eq. (9) only affects the $\mathcal{O}(1)$ coefficient A_α, given in the Supplemental Material [75]. At finite temperatures Eq. (9) implies that the averaged TLS-correlation function and hence the self energy obey ω/T scaling. Since the exponent γ is a continuous function of the coupling strength α, the self-energy can be tuned to realize a MFL form: $\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega) \sim |\omega|$, provided that $\alpha = \alpha_{\mathrm{MFL}}$. The MFL behavior arises at generic couplings and is not associated with a QCP [14], rather, the system smoothly crosses over between a Fermi liquid for $\alpha < \alpha_{\mathrm{MFL}}$, where the lifetime is large compared to the excitation energy, and a non-Fermi liquid for $\alpha > \alpha_{\mathrm{MFL}}$. The coefficient in the self-energy defines an energy scale $\omega_0 \sim h_{c,R}(\alpha M E_F/N h_{c,R})^{[1/(1-\gamma)]}$. This scale is significant relative to the natural scale $h_{c,R}$ provided that $M/N \gtrsim h_{c,R}/E_F$. Because of the large density of states of the TLSs, the system exhibits pronounced NFL behavior even in a limit of dilute TLSs, i.e., when $M/N \ll 1$.

We proceed to consider transport properties of the theory using the Kubo formula. With vanishing vertex corrections due to spatially uncorrelated interactions, the T dependence of the conductivity follows from the frequency dependence of the self-energy. We find

$$\rho(T \ll h_{R,c}) - \rho_0 \propto T^\gamma, \tag{12}$$

where the residual resistivity, ρ_0, is due to the potential scattering term H_{pot}. The same reasoning for the thermal conductivity implies that the Wiedemann-Franz law is obeyed in the $T \to 0$ limit [86]. For $T \gtrsim h_{R,c}$, $\rho(T)$ saturates to a T-independent constant, up to a small T^2/E_F correction. Our results agree with the T-linear resistivity previously obtained in the weak coupling limit with $\beta = 0$ [87]. The optical conductivity is given by $\sigma(\Omega) = \sigma_{\mathrm{el}}(\Omega) + \sigma_{\mathrm{TLS}}(\Omega)$, where the first term denotes the electronic contribution and the second term is due to dipole excitations of TLSs, with magnitude proportional to the typical TLS-dipole moment (for more details see the Supplemental Material [75]). At $T = 0$, in the absence of potential scatterers we find $\sigma_{\mathrm{el}} \propto \Omega^{-\min(2-\gamma,\gamma)}$ [$\gamma = 1$ admits a logarithmic correction $\sim\Omega^{-1}\log^{-2}(\Omega)$], while the TLS term reads $\sigma_{\mathrm{TLS}}(\Omega) \propto \overline{d^2}\Omega\mathrm{Im}\chi_{\mathrm{ret}}(\Omega)$, $\overline{d^2}$ being the typical squared magnitude of the TLS-dipole moment, and hence may give rise to a nonmonotonic Ω dependence of the optical response. The frequency dependence of $\sigma(\Omega)$ with a small potential scattering term included is shown in Fig. 3.

A similar analysis can be performed for a coupling constant $\alpha = 1$ and $\alpha > 1$. For $\alpha = 1$ the renormalized field is exponentially small, $h_R \sim E_F \exp[-(\pi/2\tilde{h})]$, which yields a renormalized distribution function $\mathcal{P}_R(h_R) \sim h_R^{-1}\log^{-(2+\beta)}(E_F/h_R)$ and we find for the TLS propagator and fermionic self-energy the highly singular behavior

$$\mathrm{Im}\chi_{\mathrm{ret}}(\omega) \sim \frac{1}{\omega\log^{2+\beta}\left(\frac{E_F}{|\omega|}\right)},$$
$$\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega) \sim \frac{1}{\log^{1+\beta}\left(\frac{E_F}{|\omega|}\right)}, \tag{13}$$

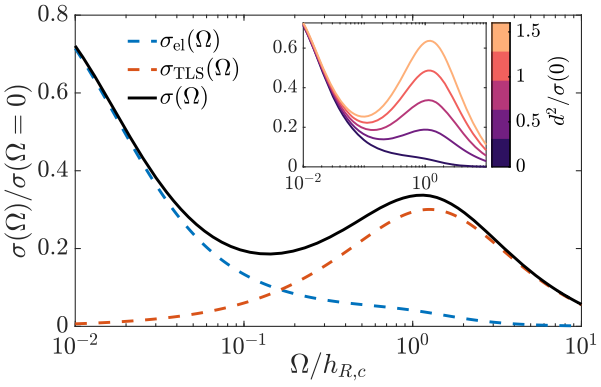

FIG. 3. Representative plot of the optical conductivity at the MFL point ($\gamma = 1$). The electronic and TLS contributions are shown in dashed lines. Inset shows the total optical conductivity for different values of the TLS-dipole moment strength d^2.

where the single-particle scattering rate vanishes extremely slowly as $\omega \to 0$, a behavior owed to the fact that there are many exponentially small excitations of the ensemble of TLSs (this is reminiscent of the behavior found in [88]).

For $\alpha > 1 + h_c/E_F$ the flow equations imply that the field $\tilde{h}$ flows to zero and the TLS freezes due to the Caldeira-Leggett mechanism [79,81,89–91]. This yields a contribution $\delta(h_R)$ to the renormalized distribution function and a corresponding term $\sim\delta(\omega)$ in $\mathrm{Im}\chi(\omega)$, giving rise to a constant elastic scattering term in $\mathrm{Im}\Sigma(\omega)$. The frozen TLSs behave like additional potential scatterers and give an extra contribution to the residual resistivity, i.e., FL behavior is reinstated. In the narrow regime $1 < \alpha < 1 + h_c/E_F$ we must divide the TLSs into those that are still dynamical ($h/E_F > \alpha - 1$), and those that are frozen ($h/E_F \leq \alpha - 1$). While the frozen ones give rise to a constant $\mathrm{Im}\Sigma_{\mathrm{ret}}(\omega)$, the contribution of the dynamical ones will be similar to that for $\alpha = 1$ given in Eq. (13).

Discussion.—In conclusion, we formulated and solved a model for TLSs coupled to electrons that are expected to emerge in a glassy metallic state. We find a critical phase with exponents that change as one varies microscopic parameters. Electrons form an Ohmic bath for the TLS, while the backaction of TLSs yields the variable exponent for electrons. We then find a sequence of crossovers from a FL via a MFL to a NFL state as one increases the coupling strength; once the interaction becomes too strong, all TLSs freeze. We verified that these results are robust if we include more generic coupling constants $\boldsymbol{g}_{ijl,\boldsymbol{r}}$, provided that the dominant bath is not aligned with the direction of the field [74] (i.e., that the z component of $\boldsymbol{g}^2$ is not the largest of the three).

Our model is solvable in the large-N, M limit. It is important to discuss physical effects that may arise at finite N, M. One such effect is RKKY-like interactions between TLSs (mediated by itinerant electrons), which are suppressed by $1/N$ due to the frustrated nature of the interactions. Physically, such frustration could occur if the TLSs are spatially extended, and interact with several channels of incident itinerant electrons, each with a random interaction (corresponding to the many electron "flavors" in our model). Moreover, if the TLSs are dilute (corresponding to $M \ll N$ in our model), the RKKY interactions between them are small. Nevertheless including this effect, we find that each TLS modifies the bath felt by other TLSs [92,93], and its Ohmic behavior breaks down below a small energy scale $\omega_\star \sim h_{c,R}(M/N^2)^{[1/(2-\gamma)]}$. In this low-energy regime, we expect the theory of Ref. [94] to be of relevance, which still yields deviations from $\gamma = 2$ FL behavior.

Another property of the large-M limit is the self-averaging of the electronic self-energy, physically related to the fact that each electron may undergo multiple scattering events with different TLSs. Considering finite M corrections we find that the self-averaging assertion holds down to a small energy scale $\omega_{\star\star} \sim h_{c,R}/M^{(1/\gamma)}$, below which fluctuations are expected to dominate. We defer further investigation of such effects to future study [74].

Our theory is applicable in arbitrary space dimensions $d > 1$, i.e., the quantum critical phase is not due to strong long-wave fluctuations that are particularly pronounced in low dimensions, but due to the very slow dynamics present in an ensemble of local degrees of freedom. It is conceivable that such slow local fluctuations of a glassy state are responsible for some of the strange metal behavior observed in strongly correlated materials.

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