Abnormal Bifurcation of the Double Binding Energy Differences and Proton-Neutron Pairing: Nuclei Close to $N = Z$ Line from Ni to Rb

Y. P. Wang,[1] Y. K. Wang,[1] F. F. Xu,[1] P. W. Zhao,[1,*] and J. Meng[1,2,†]
[1]State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
[2]Center for Theoretical Physics, China Institute of Atomic Energy, Beijing 102413, China

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The recently observed abnormal bifurcation of the double binding energy differences δV_{pn} between the odd-odd and even-even nuclei along the $N = Z$ line from Ni to Rb has challenged the nuclear theories. To solve this problem, a shell-model-like approach based on the relativistic density functional theory is established, by treating simultaneously the neutron-neutron, proton-neutron, and proton-proton pairing correlations both microscopically and self-consistently. Without any *ad hoc* parameters, the calculated results well reproduce the observations, and the mechanism for this abnormal bifurcation is found to be due to the enhanced proton-neutron pairing correlations in the odd-odd $N = Z$ nuclei, compared with the even-even ones. The present results provide an excellent interpretation for the abnormal δV_{pn} bifurcation, and provide a clear signal for the existence of the proton-neutron pairing correlations for nuclei close to the $N = Z$ line.

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The double binding energy difference δV_{pn}, is an important mass filter for atomic nuclei, and has been frequently used to isolate the residual proton-neutron (*pn*) interaction [1,2]. It is closely related to many nuclear structure phenomena, such as the onset of collectivity and deformation [3–6], the evolution of the underlying shell structure [7], and the phase transition behavior in nuclei [4,8]. The study of δV_{pn}, particularly along the line of the nuclei with equal numbers of protons and neutrons, is of great importance to deepen our understanding of the nuclear force [9]. For example, the considerable enhancement of δV_{pn} for $N = Z$ nuclei in the light *sd* shell has been regarded as the fingerprint for the Wigner's SU(4) symmetry of the nuclear force [10]. For heavier nuclei in the *sd* shell and even lower *fp* shell, the decrease of δV_{pn} with mass number is related to the breaking of the Wigner's SU(4) symmetry due to the increasing spin-orbit and Coulomb interactions.

For the upper *fp*-shell nuclei, however, the δV_{pn} shows quite puzzling behavior. On one hand, restrengthening δV_{pn} values have been observed [11,12] for the odd-odd nuclei, and this phenomenon might be attributed to the restoration of the pseudo-SU(4) symmetry [13], the enhanced overlaps of the proton and neutron wave functions [14–16] or the nuclear deformation [6,17]. On the other hand, a recent experiment has reported an abnormal bifurcation, namely, opposite evolving trends of δV_{pn} with mass number for the even-even and odd-odd $N = Z$ nuclei from Ni to Rb [18], which cannot be understood by the aforementioned physical mechanisms and, thus, brought severe challenges to the theoretical models, including the macroscopic-microscopic models [19–23], the shell model [24], and the density functional theories (DFTs) [25–27]. Note that the macroscopic-microscopic models and DFTs are quite successful for a global description of nuclear masses over the whole nuclear chart. Their failure in describing the observed δV_{pn} bifurcation indicates that important physics may be missing in the current nuclear models.

The valence-space in-medium similarity renormalization group calculations imply that the three-nucleon force has a significant impact on the behavior of δV_{pn}, but the obtained amplitudes of the δV_{pn} bifurcation between the even-even and odd-odd $N = Z$ nuclei are dramatically overestimated [18]. The inclusion of a phenomenological Wigner term in the macroscopic-microscopic models [19–23] and DFT [27] also results in a δV_{pn} bifurcation between the even-even and odd-odd $N = Z$ nuclei, but the δV_{pn} for the odd-odd nuclei are systematically underestimated [18].

For $N = Z$ nuclei, it is very important to take into account the *pn* pairing correlations. In particular, for odd-odd $N = Z$ nuclei, the last valence neutron and proton could be paired due to the *pn* pairing correlations, which are responsible for the phenomenological Wigner terms [28–31]. To solve the puzzle of the δV_{pn} bifurcation, a microscopic model which could treat the neutron-neutron (*nn*), proton-proton (*pp*), and *pn* pairing correlations simultaneously and self-consistently is necessary. The blocking effects for odd-odd nuclei should be treated carefully.

The nuclear DFT starts from a universal density functional and has achieved great successes in describing many nuclear phenomena [32–37]. It is a promising framework to consider the *pn* pairing correlations in a microscopic way.

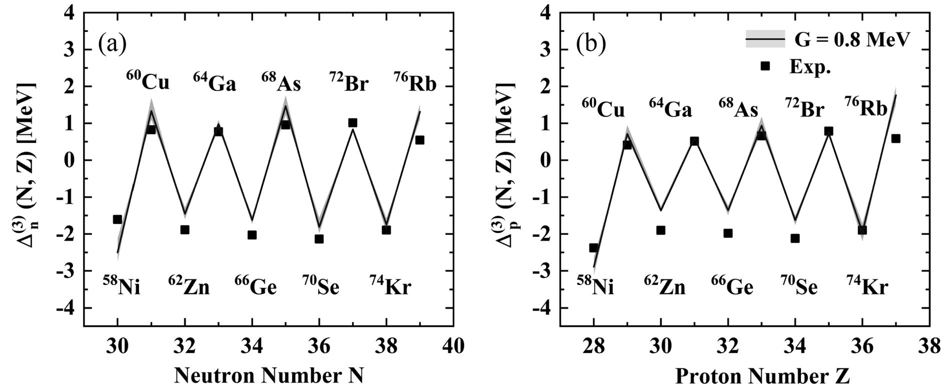

FIG. 1. Odd-even mass differences $\Delta_n^{(3)}$ (a) and $\Delta_p^{(3)}$ (b) for the $N = Z + 2$ nuclei from Ni to Rb calculated by RDFT-SLAP (lines) in comparison with the data (symbols). The gray bands correspond to the odd-even mass differences for the pairing strength G varying by 10%.

In the conventional Bardeen-Cooper-Schrieffer and Bogoliubov methods, the particle number conservation is violated and the Pauli blocking effects in odd-nucleon systems cannot be treated exactly [38]. The shell-model-like approach (SLAP), also known as particle number conserving method, treats the pairing correlations and the blocking effects exactly by diagonalizing the many-body Hamiltonian in a properly truncated many-particle configuration (MPC) space with good particle number [39,40]. It has been implemented in the relativistic [41–46] and nonrelativistic [47,48] DFTs to treat the nn and pp pairing correlations and widely used to investigate both the nuclear ground-state and excited-state properties. Nevertheless, a self-consistent treatment of the pn pairing correlations is missing.

In this Letter, based on the relativistic DFT (RDFT), a SLAP is developed, which allows a microscopic and self-consistent treatment of the nn, pp, and pn pairing correlations simultaneously. The developed approach will be applied to investigate the abnormal δV_{pn} bifurcation for the $N = Z$ nuclei from Ni to Rb.

In the SLAP based on RDFT (RDFT-SLAP), the many-body Hamiltonian reads

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pair}}, \tag{1}$$

where $\hat{H}_0$ is the one-body part, and $\hat{H}_{\text{pair}}$ is the pairing part. The one-body part reads

$$\hat{H}_0 = \sum_{k>0} [\varepsilon_k^{\pi}(a_k^{\dagger} a_k + a_{\bar{k}}^{\dagger} a_{\bar{k}}) + \varepsilon_k^{\nu}(b_k^{\dagger} b_k + b_{\bar{k}}^{\dagger} b_{\bar{k}})], \tag{2}$$

where $\bar{k}$ represents the time conjugate of the state k, and $\varepsilon_k^{\pi(\nu)}$ are the single-proton (neutron) energies obtained from the Dirac equation,

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta(m + S) + V]\psi_k = \epsilon_k \psi_k. \tag{3}$$

Here, the scalar field S and vector field V are connected in a self-consistent way to the scalar and vector densities, for details see Ref. [41]. The pairing part reads

$$\hat{H}_{\text{pair}} = \sum_{T_z=0,\pm1} \hat{H}_{\text{pair}}^{T_z}, \qquad \hat{H}_{\text{pair}}^{T_z} = -G \sum_{k,k'>0}^{k\neq k'} P_{k,T_z}^{\dagger} P_{k',T_z}, \tag{4}$$

where G is the effective pairing strength, and $k \neq k'$ means that the self-scattering for the nucleon pairs is forbidden [41]. The nn and pp pair creation operators are $P_{k,1}^{\dagger} = b_k^{\dagger} b_{\bar{k}}^{\dagger}$ and $P_{k,-1}^{\dagger} = a_k^{\dagger} a_{\bar{k}}^{\dagger}$ for $T_z = \pm 1$, and the pn pair creation operator is $P_{k,0}^{\dagger} = (1/\sqrt{2})(b_k^{\dagger} a_{\bar{k}}^{\dagger} + a_k^{\dagger} b_{\bar{k}}^{\dagger})$ for $T_z = 0$.

The nuclear wave functions are expressed as

$$|\Psi\rangle = \sum_{i,\{s_k\}} C_i^{\{s_k\}} |\text{MPC}_i^{\{s_k\}}\rangle. \tag{5}$$

The many-particle configurations $|\text{MPC}_i^{\{s_k\}}\rangle$ with exact proton number Z and neutron number N are expressed as $|l_1 l_2 \cdots l_N m_1 m_2 \cdots m_Z\rangle = b_{l_1}^{\dagger} b_{l_2}^{\dagger} \cdots b_{l_N}^{\dagger} a_{m_1}^{\dagger} a_{m_2}^{\dagger} \cdots a_{m_Z}^{\dagger} |0\rangle$, and the corresponding configuration energy is denoted as E_i. Here, s_k represents the eigenvalue of the seniority operator $\hat{s}_k$ for the state k [49], and it is a good quantum number. The expansion coefficients $C_i^{\{s_k\}}$ and, thus, the wave functions are determined by diagonalizing the Hamiltonian $\hat{H}$ in the MPC space.

Note that the obtained wave function $|\Psi\rangle$ is used to determine the occupation probabilities for the single-particle states, and thus the nucleon densities should be updated, which in turn determines the scalar and vector fields S and V in the Dirac equation (3). Therefore, the full framework should be solved iteratively to achieve the self-consistency [41,44,45]. Once a self-consistent solution is obtained, one can calculate the pairing energy,

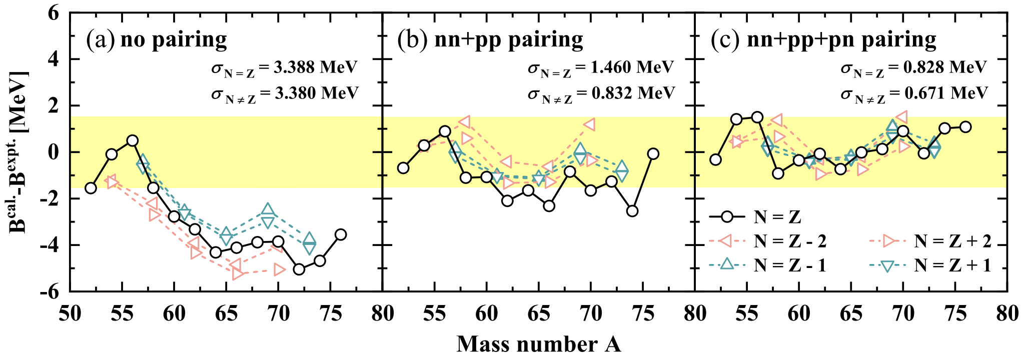

FIG. 2. Differences between the calculated binding energies and the data for the $N = Z, Z \pm 1, Z \pm 2$ nuclei around Ni and Rb region without pairing (a), with the *n-n* and *pp* pairing (b), and with the *nn*, *pp*, and *pn* pairing (c). The root-mean-square deviation for the $N = Z$ ($N \neq Z$) nuclei $\sigma_{N=Z}$ ($\sigma_{N\neq Z}$) is also given.

$$E_{\text{pair}} = \langle\Psi|\hat{H}_{\text{pair}}|\Psi\rangle = \sum_{ij} C_i^* C_j \langle \text{MPC}_i|\hat{H}_{\text{pair}}|\text{MPC}_j\rangle, \tag{6}$$

which is added to the total energy of RDFT.

In this work, the relativistic density functional PC-PK1 [50] is adopted. The Dirac equation (3) is solved in the three-dimensional harmonic oscillator basis in Cartesian coordinates [51] with ten major shells, and the quadru pole deformation including triaxiality is considered self-consistently.

The dimension of the MPC space and the corresponding pairing strength G are determined by the odd-even mass differences of the $N = Z + 2$ nuclei from Ni to Rb. In Fig. 1, the calculated odd-even mass differences $\Delta_{\text{n}}^{(3)}(N,Z) = [B(N-1,Z)+B(N+1,Z)]/2-B(N,Z)$ and $\Delta_{\text{p}}^{(3)}(N,Z) = [B(N,Z-1) + B(N,Z+1)]/2 - B(N,Z)$ [52] are shown, in comparison with the experimental ones extracted from AME'20 [53,54]. The experimental odd-even mass differences are well reproduced by the calculation with the pairing strength $G = 0.8$ MeV. The corresponding MPC space is truncated by $E_{\text{cut}} = 16$ MeV, which means only the MPCs with the energies $E_i \leq 16$ MeV are included in the model space. In addition, a variation of the pairing strength by 10% does not change the odd-even mass differences $\Delta_{\text{n}}^{(3)}$ and $\Delta_{\text{p}}^{(3)}$ significantly.

With the pairing strength G thus determined, the binding energies for the $N = Z, Z \pm 1, Z \pm 2$ nuclei around Ni and Rb region are calculated. The differences between the calculated binding energies and the data [53,54] are shown in Fig. 2. As shown in Fig. 2(a), without the pairing correlations, the deviations between the calculated binding energies and the data are large. The root-mean-square (rms) deviations are 3.388 MeV for $N = Z$ nuclei and 3.380 MeV for $N \neq Z$ ones. After the inclusion of the *nn* and *pp* pairing, as shown in Fig. 2(b), the descriptions of the binding energies are improved. The rms deviation for $N = Z$ nuclei changes to 1.460 MeV, and for $N \neq Z$ nuclei changes to 0.832 MeV. After including the *pn* pairing, as shown in Fig. 2(c), the agreements become better. The rms deviation is 0.832 MeV for $N = Z$ nuclei and 0.671 MeV for $N \neq Z$ nuclei [55]. These results illustrate the importance of the pairing correlations for nuclei near the $N = Z$ line, in particular the *pn* pairing correlations.

From the binding energies, the δV_{pn} can be extracted as [10]

$$\delta V_{pn}^{\text{ee}}(N,Z) = \frac{1}{4}[B(N,Z) - B(N-2,Z) - B(N,Z-2) + B(N-2,Z-2)], \tag{7}$$

for even-even nuclei with $N = Z$, and

$$\delta V_{pn}^{\text{oo}}(N,Z) = [B(N,Z) - B(N-1,Z) - B(N,Z-1) + B(N-1,Z-1)], \tag{8}$$

for odd-odd nuclei with $N = Z$. The extracted theoretical results and data [18] are shown in Fig. 3. The experimental $\delta V_{pn}^{\text{ee}}$ (red circles) decrease smoothly with the mass number, while the $\delta V_{pn}^{\text{oo}}$ (blue squares) exhibit a distinct tendency; this is the challenging puzzle of the abnormal δV_{pn} bifurcation observed in Ref. [18]. When the *nn*, *pp*, and *pn* pairing correlations are taken into account simultaneously (solid lines), the calculated results well reproduce the evolution of δV_{pn} for both the odd-odd and even-even nuclei. The agreement remains even by changing the pairing strength by 10%. In contrast, if the *pn* pairing correlations are switched off (dashed lines), the calculated results cannot reproduce the bifurcation. As shown clearly in Fig. 3, the successful reproducing for the abnormal δV_{pn} bifurcation is due to the enhancement of the δV_{pn} for the odd-odd $N = Z$ nuclei by the *pn* pairing correlations.

To further understand why the *pn* pairing correlations have more significant influence on the δV_{pn} for the odd-odd nuclei as compared to the even-even ones, in

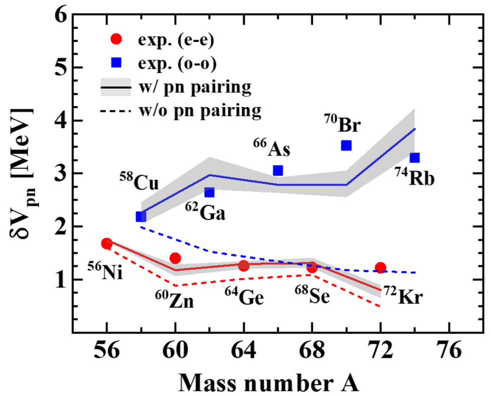

FIG. 3. Calculated δV_{pn} for the even-even (red lines) and odd-odd (blue lines) $N = Z$ nuclei from Ni to Rb with (solid lines) and without (dashed lines) the *pn* pairing, in comparison with the data (symbols) [18]. The gray bands correspond to the results with the pairing strength G varying by 10%.

Fig. 4(a), the calculated *pn* pairing energies, $E_{\text{pair}}^{pn} = \langle\Psi|\hat{H}_{\text{pair}}^{T_z=0}|\Psi\rangle = \sum_{ij} C_i^* C_j \langle \text{MPC}_i|\hat{H}_{\text{pair}}^{T_z=0}|\text{MPC}_j\rangle$, are shown as functions of the sums of the configuration energies for $N = Z$ odd-odd nucleus ^{66}As and even-even nucleus ^{64}Ge. For ^{66}As, the nonvanishing value of E_{pair}^{pn} starts at about 3.5 MeV, while for ^{64}Ge, about 6.9 MeV. This can be understood from the lowest MPC and the lowest excitation contributing to the *pn* pairing energy for ^{66}As and ^{64}Ge as shown in Figs. 4(b) and 4(c). With the increase of $E_i + E_j$, the *pn* pairing energy for ^{66}As is significantly larger than that for ^{64}Ge. Changing the pairing strength G by 10% will influence the *pn* pairing energy by around 1 MeV for ^{66}As, and by around 0.4 MeV for ^{64}Ge. These results in Fig. 4 suggest that the *pn* pairing correlations have more influence on the odd-odd nuclei than on the even-even nuclei, and this explains why the *pn* pairing correlations would significantly enhance the δV_{pn} for the odd-odd $N = Z$ nuclei in Fig. 3, and thus result in the abnormal δV_{pn} bifurcation.

In summary, a shell-model-like approach is developed based on the relativistic density functional theory, which allows a microscopic and self-consistent treatment of the neutron-neutron, proton-proton, and proton-neutron pairing correlations simultaneously. The challenging puzzle of the abnormal δV_{pn} bifurcation between the odd-odd and even-even nuclei along the $N = Z$ line from Ni to Rb is found to be originated from the proton-neutron pairing correlations. The proton-neutron pairing correlations would significantly enhance the δV_{pn} for odd-odd $N = Z$ nuclei, and thus result in the δV_{pn} bifurcation. The proton-neutron pairing correlations improve the description of the masses not only for $N = Z$ nuclei, but also for these nuclei near the $N = Z$ line. These conclusions remain true even if the pairing strength is changed by 10%. The present results provide an excellent interpretation to the challenging puzzle of the abnormal δV_{pn} bifurcation, and provide a clear signal for the existence of the proton-neutron pairing correlations for $N = Z$ nuclei.

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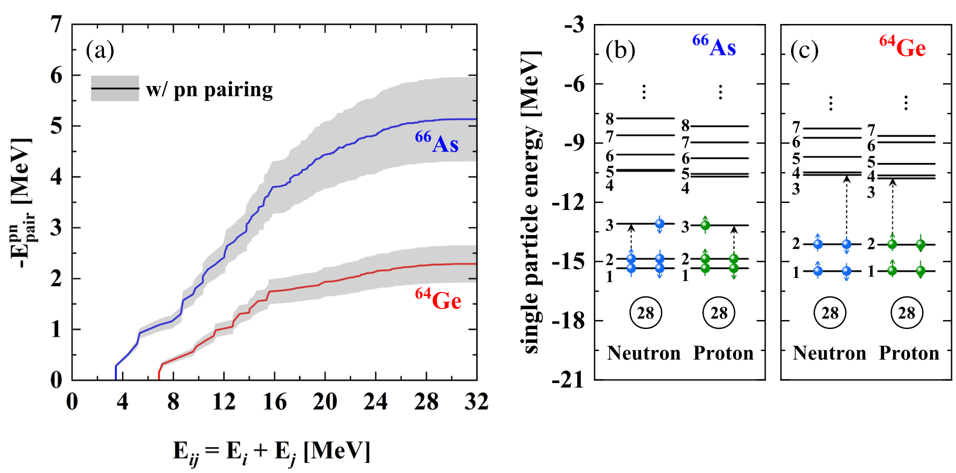

FIG. 4. (a) Calculated *pn* pairing energies E_{pair}^{pn} as functions of the sums of the configuration energies for the ith and jth MPC, $E_i + E_j$, for ^{66}As and ^{64}Ge. The gray bands correspond to the results for the pairing strength G varying by 10%. (b) Single-particle energies for the odd-odd nucleus ^{66}As. The single-proton levels are renormalized to the first single-neutron level above the $N = Z = 28$ shell. The lowest-energy MPC and the lowest excitation with nonvanishing contribution to the *pn* pairing energy E_{pair}^{pn} are schematically shown. (c) Same as (b), but for the even-even nucleus ^{64}Ge.

*pwzhao@pku.edu.cn
†mengj@pku.edu.cn

[1] J. Y. Zhang, R. F. Casten, and D. S. Brenner, Phys. Lett. B **227**, 1 (1989).
[2] D. S. Brenner, C. Wesselborg, R. F. Casten, D. D. Warner, and J. Y. Zhang, Phys. Lett. B **243**, 1 (1990).
[3] I. Talmi, Rev. Mod. Phys. **34**, 704 (1962).
[4] P. Federman and S. Pittel, Phys. Lett. **69B**, 385 (1977).
[5] R. F. Casten, Phys. Rev. Lett. **54**, 1991 (1985).
[6] R. B. Cakirli and R. F. Casten, Phys. Rev. Lett. **96**, 132501 (2006).
[7] K. Heyde, P. Van Isacker, R. F. Casten, and J. L. Wood, Phys. Lett. **155B**, 303 (1985).
[8] P. Federman and S. Pittel, Phys. Lett. **77B**, 29 (1978).
[9] D. D. Warner, M. A. Bentley, and P. Van Isacker, Nat. Phys. **2**, 311 (2006).
[10] P. Van Isacker, D. D. Warner, and D. S. Brenner, Phys. Rev. Lett. **74**, 4607 (1995).
[11] P. Schury, C. Bachelet, M. Block, G. Bollen, D. A. Davies *et al.*, Phys. Rev. C **75**, 055801 (2007).
[12] I. Mardor *et al.*, Phys. Rev. C **103**, 034319 (2021).
[13] P. Van Isacker, O. Juillet, and F. Nowacki, Phys. Rev. Lett. **82**, 2060 (1999).
[14] D. S. Brenner, R. B. Cakirli, and R. F. Casten, Phys. Rev. C **73**, 034315 (2006).
[15] L. Chen, Y. A. Litvinov, W. R. Plass, K. Beckert, P. Beller *et al.*, Phys. Rev. Lett. **102**, 122503 (2009).
[16] R. B. Cakirli, K. Blaum, and R. F. Casten, Phys. Rev. C **82**, 061304(R) (2010).
[17] D. Bonatsos, S. Karampagia, R. B. Cakirli, R. F. Casten, K. Blaum, and L. A. Susam, Phys. Rev. C **88**, 054309 (2013).
[18] M. Wang, Y. H. Zhang, X. Zhou, X. H. Zhou, H. S. Xu *et al.*, Phys. Rev. Lett. **130**, 192501 (2023).
[19] J. Duflo and A. P. Zuker, Phys. Rev. C **52**, R23(R) (1995).
[20] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[21] T. Kawano, S. Chiba, and H. Kouba, J. Nucl. Sci. Technol. **43**, 1 (2006).
[22] N. Wang, M. Liu, X. Wu, and J. Meng, Phys. Lett. B **734**, 215 (2014).
[23] P. Möller, A. J. Sierk, T. Ichikawa, and H. Sagawa, At. Data Nucl. Data Tables **109–110**, 1 (2016).
[24] M. Honma, T. Otsuka, B. Brown, and T. Mizusaki, Eur. Phys. J. A **25**, 499 (2005).
[25] L. S. Geng, H. Toki, and J. Meng, Prog. Theor. Phys. **113**, 785 (2005).
[26] M. Kortelainen, J. McDonnell, W. Nazarewicz, P.-G. Reinhard, J. Sarich, N. Schunck, M. V. Stoitsov, and S. M. Wild, Phys. Rev. C **85**, 024304 (2012).
[27] S. Goriely, N. Chamel, and J. M. Pearson, Phys. Rev. C **88**, 061302(R) (2013).
[28] W. Satuła and R. Wyss, Phys. Lett. B **393**, 1 (1997).
[29] W. Satuła, D. Dean, J. Gary, S. Mizutori, and W. Nazarewicz, Phys. Lett. B **407**, 103 (1997).
[30] I. Bentley, K. Neergård, and S. Frauendorf, Phys. Rev. C **89**, 034302 (2014).
[31] D. Negrea and N. Sandulescu, Phys. Rev. C **90**, 024322 (2014).
[32] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
[33] J. Dechargé and D. Gogny, Phys. Rev. C **21**, 1568 (1980).
[34] P. Ring, Prog. Part. Nucl. Phys. **37**, 193 (1996).
[35] D. Vretenar, A. V. Afanasjev, G. A. Lalazissis, and P. Ring, Phys. Rep. **409**, 101 (2005).
[36] T. Nikšić, D. Vretenar, and P. Ring, Prog. Part. Nucl. Phys. **66**, 519 (2011).
[37] *Relativistic Density Functional for Nuclear Structure*, edited by J. Meng (World Scientific, Singapore, 2016).
[38] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980).
[39] J. Y. Zeng and T. S. Cheng, Nucl. Phys. **A405**, 1 (1983).
[40] A. Volya, B. A. Brown, and V. Zelevinsky, Phys. Lett. B **509**, 37 (2001).
[41] J. Meng, J. Y. Guo, L. Liu, and S. Q. Zhang, Front. Phys. China **1**, 38 (2006).
[42] W. C. Chen, J. Piekarewicz, and A. Volya, Phys. Rev. C **89**, 014321 (2014).
[43] L. Liu, Z. H. Zhang, and P. W. Zhao, Phys. Rev. C **92**, 044304 (2015).
[44] Z. Shi, Z. H. Zhang, Q. B. Chen, S. Q. Zhang, and J. Meng, Phys. Rev. C **97**, 034317 (2018).
[45] Y. P. Wang and J. Meng, Phys. Lett. B **841**, 137923 (2023).
[46] F. F. Xu and Y. P. Wang, Int. J. Mod. Phys. E **32**, 2340007 (2023).
[47] N. Pillet, P. Quentin, and J. Libert, Nucl. Phys. **A697**, 141 (2002).
[48] W. Y. Liang, C. F. Jiao, Q. Wu, X. M. Fu, and F. R. Xu, Phys. Rev. C **92**, 064325 (2015).
[49] R. W. Richardson, Phys. Rev. **144**, 874 (1966).
[50] P. W. Zhao, Z. P. Li, J. M. Yao, and J. Meng, Phys. Rev. C **82**, 054319 (2010).
[51] T. Nikšić, N. Paar, D. Vretenar, and P. Ring, Comput. Phys. Commun. **185**, 1808 (2014).
[52] M. Bender, K. Rutz, P. G. Reinhard, and J. A. Maruhn, Eur. Phys. J. A **8**, 59 (2000).
[53] M. Wang, W. J. Huang, F. G. Kondev, G. Audi, and S. Naimi, Chin. Phys. C **45**, 030003 (2021).
[54] F. G. Kondev, M. Wang, W. J. Huang, S. Naimi, and G. Audi, Chin. Phys. C **45**, 030001 (2021).
[55] The expectation values for the isospin operator $\hat{T}$ [38] are calculated for the ground states of all these nuclei, and they agree with the experimental data, except for ^{58}Cu. For ^{58}Cu, consistent with the data, the state with $\langle\Psi|\hat{T}|\Psi\rangle = 0$ is chosen as the ground state.