Can Supercooled Phase Transitions Explain the Gravitational Wave Background Observed by Pulsar Timing Arrays?

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(Received 3 August 2023; revised 15 December 2023; accepted 2 May 2024; published 28 May 2024)

Several pulsar timing array collaborations recently reported evidence of a stochastic gravitational wave background (SGWB) at nHz frequencies. While the SGWB could originate from the merger of supermassive black holes, it could be a signature of new physics near the 100 MeV scale. Supercooled first-order phase transitions (FOPTs) that end at the 100 MeV scale are intriguing explanations, because they could connect the nHz signal to new physics at the electroweak scale or beyond. Here, however, we provide a clear demonstration that it is not simple to create a nHz signal from a supercooled phase transition, due to two crucial issues that could rule out many proposed supercooled explanations and should be checked. As an example, we use a model based on nonlinearly realized electroweak symmetry that has been cited as evidence for a supercooled explanation. First, we show that a FOPT cannot complete for the required transition temperature of around 100 MeV. Such supercooling implies a period of vacuum domination that hinders bubble percolation and transition completion. Second, we show that even if completion is not required or if this constraint is evaded, the Universe typically reheats to the scale of any physics driving the FOPT. The hierarchy between the transition and reheating temperature makes it challenging to compute the spectrum of the SGWB.

DOI: 10.1103/PhysRevLett.132.221001

Introduction.—NANOGrav recently detected a stochastic gravitational wave background (SGWB) for the first time with a significance of about 4σ [1]. This was corroborated by other pulsar timing arrays (PTAs), including the CPTA [2], EPTA [3], and PPTA [4]. Although the background could originate from mergers of supermassive black holes [5,6], this explanation might be inconsistent with previous estimates of merger density and remains a topic of debate [7–10]. Thus, there is an intriguing possibility that the SGWB detected by NANOGrav could originate from more exotic sources [11]. Indeed, many exotic explanations were proposed for an earlier hint of this signal [12–14], or immediately after the announcement. These include noncanonical kinetic terms [15], inflation [16–20], first-order phase transitions (FOPTs; [21–25]), cosmic strings [26–33], domain walls [34,35], primordial black holes [36], primordial magnetic fields [37], axions and ALPs [38–44], QCD [45,46], and dark sector models [47–55].

The nanohertz (nHz) frequency of the signal indicates that any new physics explanation should naturally lie at around 100 MeV. If there are new particles around the MeV scale there are constraints from cosmology [56–59] and, in any case, from particle physics experiments. It is conceivable, however, that new physics at characteristic scales far beyond the MeV scale could be responsible for a nHz signal. This could happen, for example, if a FOPT [60–62] starts at higher temperatures but supercools down to 100 MeV. That is, the Universe remains in a false vacuum until the 100 MeV scale because a transition to the true vacuum is suppressed.

This was previously considered for an electroweak phase transition [63–70] and was discussed as a possible new physics explanation by NANOGrav [11,12]. Supercooling could help new physics explanations evade constraints on MeV-scale modifications to the standard model and connect a nHz signal to new physics and phenomenology at the electroweak scale or above.

In this Letter, however, we raise two difficulties with supercooled FOPTs. We explicitly demonstrate that these difficulties rule out one of the prominent models that explain the nHz GW signal through a supercooled FOPT used in Refs. [11,12]. First, the phase transition does not complete for the low temperatures associated with a nHz signal. This finding is consistent with brief remarks in Ref. [71] and, as mentioned there, similar to the graceful exit problem in old inflation [72]. Second, the energy released by the phase transition reheats the Universe to about the new physics scale [59] and this can rule out attempts to solve the completion problem. However, we also show that for supercooled phase transitions the temperature dependence is more complicated than naive arguments suggest, and the hierarchy between the percolation and reheating temperatures must be taken into account when computing the SGWB spectrum.

Cubic potential and benchmarks.—We consider a modification to the standard model Higgs potential to include a cubic term,

$$V_0(r) = -\frac{\mu^2}{2} r^2 + \frac{\kappa}{3} r^3 + \frac{\lambda}{4} r^4. \tag{1}$$

For further details about the model and effective potential, see Supplemental Material [73] and Refs. [63,74–77]. We define the percolation temperature, T_p, and completion temperature, T_f, of a transition as the temperatures at which the false vacuum fraction $P_f = 0.71$ and 0.01, respectively ([78–80]; see Supplemental Material [73] for further details about phase transition analysis, which includes Refs. [62,81–89]).

We consider two benchmark points to highlight the challenges of fitting a nHz signal with this cubic potential. These benchmarks are selected to probe two criteria: (i) realistic percolation, that is, having a percolation temperature and that the physical volume of the false vacuum is decreasing at the onset of percolation; and (ii) having a completion temperature, that is, a temperature at which the false vacuum fraction falls to below 1%. These benchmarks are

$$\text{BP1}: \kappa = -117.96\,\text{GeV}, \quad \text{BP2}: \kappa = -118.67\,\text{GeV}. \tag{2}$$

BP1 resulted in the most supercooling for which the transition satisfies both criteria, though it fails to supercool to sub-GeV temperatures. For BP1, the physical volume of the false vacuum starts decreasing at exactly the percolation temperature. Increasing supercooling any further thus violates our first criteria. BP2 resulted in stronger supercooling with a nominal percolation temperature of 100 MeV but no completion temperature. However, although BP2 was chosen so that percolation was estimated to begin at 100 MeV, it violates our first criteria and the space between bubbles continues to expand below 100 MeV. Thus, despite a nominal percolation temperature, percolation could be unrealistic. Without significant percolation of bubbles, the phase transition would not generate a SGWB. The benchmarks are sensitive to uncertainties; for example, changing the Higgs mass by 1σ, 0.17 GeV [90], changes the value of κ below which percolation is unrealistic (BP1) and the associated percolation temperature by about 0.5 GeV. Our conclusions and results, however, would be qualitatively unchanged.

Challenges.—Challenge 1: Percolation and completion: As discussed, supercooling was proposed to achieve a peak frequency at the nHz scale. However, in many models, a first-order electroweak phase transition has bubbles nucleating at around the electroweak scale. There is then an extended period of bubble growth and expansion of space. If bubbles grow too quickly compared to the expansion rate of the Universe, the bubbles will percolate before sufficient supercooling. Yet if bubbles grow too slowly the transition may never percolate or complete due to the space between bubbles inflating [71,85,89]; this effect can cause both the realistic percolation condition and the condition for a completion temperature to fail. Thus, while it is possible to tune model parameters to achieve a nominal percolation temperature at sub-GeV temperatures, true percolation and completion of the transition become less likely as supercooling is increased.

We find that a completion temperature is impossible for the cubic potential if $T_p \lesssim 1$ GeV. The same arguments apply to the models considered in Ref. [85]. In the cubic potential, strong supercooling implies a Gaussian bubble nucleation rate peaking at $T_\Gamma \sim 50$ GeV. (It might be possible to evade this argument in models that predict a non-Gaussian nucleation rate, e.g., conformal models [91–94].)

In Ref. [11], the cubic potential is suggested as a candidate model for a strongly supercooled phase transition that could explain the detected SGWB. The Universe was assumed to be radiation dominated in the original investigation [63] of detecting GWs from the cubic potential with PTAs. However, a more careful treatment of the energy density during strong supercooling shows that the Universe becomes vacuum dominated [71]. This leads to a prolonged period of rapid expansion that hinders bubble percolation and completion of the transition. In fact, one must check not only that $P_f < 0.01$ eventually, but also that the physical volume of the false vacuum is decreasing at T_p [71,85,89].

The SGWB from a FOPT should not be computed at the nucleation temperature T_n, as this will generally give a very different result compared to computing it at lower temperatures where bubbles are actually colliding [95]. The percolation temperature is a much better choice [85]. By definition, we anticipate the formation of a cluster of connected bubbles at the percolation temperature and thus bubble collisions and the generation of GWs are expected to begin at around this time. Figure 1 demonstrates the large difference between T_n and T_p in supercooled phase transitions. In BP1 the difference is $\mathcal{O}(10\text{ GeV})$. In BP2 there is no nucleation temperature—one might be tempted to assume GWs cannot be produced because of this. However, percolation and completion are possible even without a nucleation temperature [85].

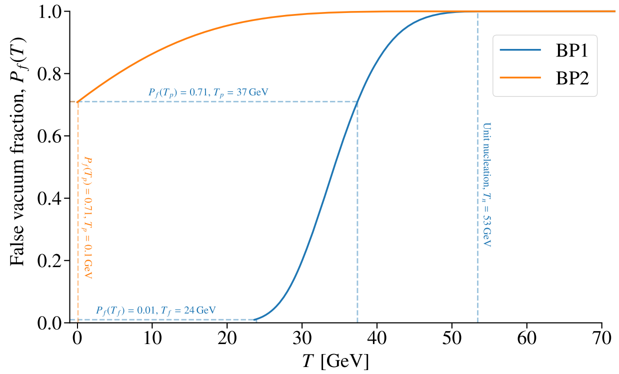

FIG. 1. The false vacuum fraction as a function of temperature for BP1 (blue, right-most solid curve) and BP2 (orange, left-most solid curve). The nucleation, percolation, and completion temperatures are shown for BP1. However, BP2 only has a percolation temperature at $T_p \approx 100$ MeV.

Another large source of error is the use of β/H for estimating the timescale of the transition. The mean bubble separation can be used instead as described in Supplemental Material [73] for thermal parameters, which includes Refs. [62,71,85,96–101].

Challenge 2: Reheating: Even if the completion constraints can be avoided, a second issue was recently observed [59]. While strong supercooling can lower the percolation temperature down to $T_p \approx 100$ MeV as in BP2 or even lower, the energy released from the phase transition reheats the plasma, creating a hierarchy $T_{\rm reh} \gg T_p$. Indeed, the reheating and percolation temperatures are approximately related by [71]

$$T_{\rm reh} \simeq [1 + \alpha(T_p)]^{1/4} T_p, \tag{3}$$

where α is the transition strength and is related to the latent heat released during the phase transition. The substantial latent heat in a strongly supercooled transition, $\alpha \gg 1$, thus implies that $T_{\rm reh} \gg T_p$. Reference [59] approximates $\alpha \approx \Delta V/\rho_R$ from the free energy difference (ΔV) and the radiation energy density (ρ_R) and shows that in the Coleman-Weinberg model the latent heat is so large that the Universe reheats well above the percolation temperature and back to the scale of new physics.

A simple scaling argument suggests that this observation—that supercooled FOPTs reheat to the scale of new physics, M—is generic. The new physics creates a barrier between minima so we expect $\Delta V \sim M^4$, and because the radiation energy density goes like T_p^4, we expect the latent heat may go like $\alpha \sim M^4/T_p^4$. This leads to

$$T_{\rm reh} \sim \left(\frac{M^4}{T_p^4}\right)^{\frac{1}{4}} T_p = M. \tag{4}$$

It is possible that reheating to $T_{\rm reh} \ll M$ could be achieved, however, by avoiding $\Delta V \sim M^4$. For example, by fine-tuning couplings in the potential such that, despite new physics at a scale M creating a second minima separated by a barrier, the relative depth of the minima at $T_p \ll M$ is much less than M^4 such that $\Delta V \ll M^4$.

The arguments leading to Eq. (4), however, rely on the simple approximation of the reheating temperature in Eq. (3) and crude dimensional analysis. We now confirm that this problem exists and is unavoidable in a careful analysis of the example model we consider. This careful treatment is general and can be used in other models. We assume that the reheating occurs instantaneously around the time of bubble percolation, and use conservation of energy so that the reheating temperature can be obtained from [83,85]

$$\rho[\phi_f(T_p), T_p] = \rho[\phi_t(T_{\rm reh}), T_{\rm reh}], \tag{5}$$

where ϕ_f and ϕ_t are the false and true vacua, respectively, and ρ is the energy density. For BP1, the percolation temperature is $T_p \approx 37.4$ GeV, and the transition completes and reheats the Universe to $T_{\rm reh} \approx 44.1$ GeV. The reheating temperature exceeds the percolation temperature due to the energy released from the supercooled phase transition, though they remain of the same order of magnitude. For BP2, however, the percolation temperature drops to $T_p \approx 100$ MeV, whereas the reheating temperature is $T_{\rm reh} \approx 35.6$ GeV; more than 2 orders of magnitude larger.

We show the behavior of the reheating temperature as a function of percolation temperature in Fig. 2. We clearly see that the reheating temperature tends towards a constant value $T_{\rm reh} \approx 36$ GeV for $T_p \to 0$. As we now discuss, the fact that $T_\Gamma \sim T_{\rm reh} \gg T_p$ breaks assumptions typically made when computing the SGWB.

Gravitational wave spectra: The frequencies of a SGWB created at a percolation temperature T_p would be redshifted from the reheating temperature $T_{\rm reh}$ to the current

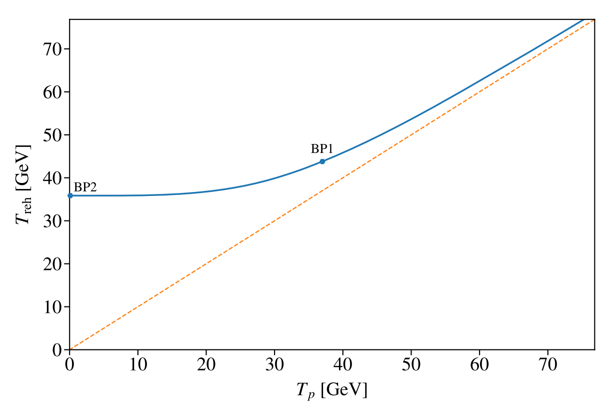

FIG. 2. The reheating temperature $T_{\rm reh}$ against percolation temperature T_p as κ varied. The dashed orange line corresponds to $T_{\rm reh} = T_p$. We see that $T_{\rm reh} \gtrsim 36$ GeV even when $T_p \to 0$. Our two benchmark points are labeled.

temperature $T \simeq 2.725$ K [62]. The redshifted peak frequency of the SGWB today would be

$$f_p \approx 10 \text{ nHz}\left(\frac{g_*(T_{\rm reh})}{100}\right)^{\frac{1}{6}}\left(\frac{T_{\rm reh}}{100 \text{ MeV}}\right)\left(\frac{1}{R_* H(T_{\rm reh})}\right), \quad (6)$$

where R_* is the mean bubble separation, H is the Hubble parameter and g_* is the number of effective degrees of freedom. (We apply suppression factors from Ref. [102] to the degrees of freedom of each particle when estimating g_*. This incorporates the effects of particles decoupling from the thermal bath as the temperature drops below their respective mass. The peak frequency and amplitude depend only weakly on g_* such that mismodeling g_* cannot dramatically change the SGWB.) In the absence of supercooling we anticipate that $T_{\rm reh} \sim T_p$, such that $R_* H(T_{\rm reh}) \sim R_* H(T_p)$ and since the bubbles would not have a long time to grow $R_* H(T_p) \lesssim 1$. Thus, in the absence of supercooling, we expect $T_p \sim 100$ MeV to lead to a ~10 nHz signal.

In this cubic model, however, $T_p \sim 100$ MeV requires strong supercooling, so we now consider an analysis more appropriate for this scenario. At the time of the phase transition the peak frequency $f_{p,*}$ is set by the mean bubble separation R_* via $f_{p,*} \sim 1/R_*$ [60]. After redshifting, the peak frequency of the SGWB today scales as

$$f_p \sim \frac{1 \text{ GeV}}{R_*(T_p) s_t(T_{\rm reh})^{1/3}}, \quad (7)$$

where s_t is the true vacuum entropy density (see Supplemental Material [73]). Because radiation domination is a valid assumption in the true vacuum, the entropy density scales as $s_t(T) \sim T^3$.

One can show that $R_* \sim 1 \text{ GeV}/(T_\Gamma T_p N^{1/3})$ if bubbles nucleate simultaneously at T_Γ, where N is the total number of bubbles nucleated per Hubble volume throughout the transition. (Simultaneous nucleation is an extreme case of Gaussian nucleation, found to be a good approximation in this model [85].) Combining this with Eq. (7), we obtain

$$f_p \sim T_p\left(\frac{T_\Gamma N^{\frac{1}{3}}}{T_{\rm reh}}\right). \quad (8)$$

Numerically, we find that $N^{\frac{1}{3}}$, T_Γ and $T_{\rm reh}$—and thus the right-most factor—depend only weakly on the amount of supercooling (see Fig. 2). Thus, for supercooling we find the relationship $f_p \sim T_p$. This suggests that one can obtain an arbitrarily low peak frequency by fine-tuning the percolation temperature.

In the cubic model, these arguments are surprisingly accurate. Indeed, we find numerically that

$$\frac{1}{R_* H(T_{\rm reh})} \simeq 1.1\left(\frac{T_p}{T_{\rm reh}}\right)\left(\frac{T_\Gamma N^{\frac{1}{3}}}{T_{\rm reh}}\right). \quad (9)$$

Assuming radiation domination in the true vacuum for $H(T_{\rm reh})$ and that $g_* \approx 100$, Eqs. (9) and (6) lead to

$$f_p \approx 10 \text{ nHz}\left(\frac{T_p}{100 \text{ MeV}}\right)\left(\frac{T_\Gamma N^{\frac{1}{3}}}{T_{\rm reh}}\right), \quad (10)$$

in agreement with the scaling anticipated in Eq. (8). The rightmost factor in Eqs. (9) and (10) is $\mathcal{O}(1)$ and approximately independent of the amount of supercooling. Thus, to achieve a redshifted peak frequency of 10 nHz, we require $T_p \approx 100$ MeV.

Comparing Eq. (10) with the result in the absence of supercooling Eq. (6), supercooling and subsequent substantial reheating redshift the frequency more than usual. However, assuming radiation domination Eq. (9) leads to

$$R_* H(T_p) \approx \frac{T_\Gamma}{T_p}. \quad (11)$$

This increase in bubble radius caused by the delay between nucleation and percolation partially offsets the impact of additional redshifting.

Our findings are contrary to the claim in Refs. [59,71] that reheating makes it difficult to reach GW frequencies relevant for PTAs. However, we do agree with the finding in Ref. [71] that completion poses an issue for nHz GW signals in this model. As found earlier, a percolation temperature of $T_p = 100$ MeV would not result in a successful transition. Not only would the majority of the Universe remain in the false vacuum even today, the true vacuum bubbles would not actually percolate due to the inflating space between the bubbles.

We now consider the SGWB predictions. We use the pseudotrace [98] to avoid assumptions about the speed of sound and the equation of state that can break down in realistic models. We also use the mean bubble separation rather than proxy timescales derived from the bounce action that are invalid for strongly supercooled phase transitions. For a full description, see Supplemental Material [73] which includes Refs. [60–62,103–114].

In this model we find that the bubbles mostly nucleate at temperatures around $T_\Gamma \sim 50$ GeV. We thus expect that friction from the plasma is sufficient to prevent runaway bubble walls, despite the large pressure difference. This implies that the SGWB from bubble collisions is negligible and that all the available energy goes into the fluid, resulting in a SGWB from sound waves and turbulence.

In Fig. 3 we show the predicted SGWB spectrum for both BP1 (upper panel; the model with maximal supercooling while guaranteeing percolation and completion) and BP2 (lower panel; the model with a percolation temperature at 100 MeV but questionable percolation

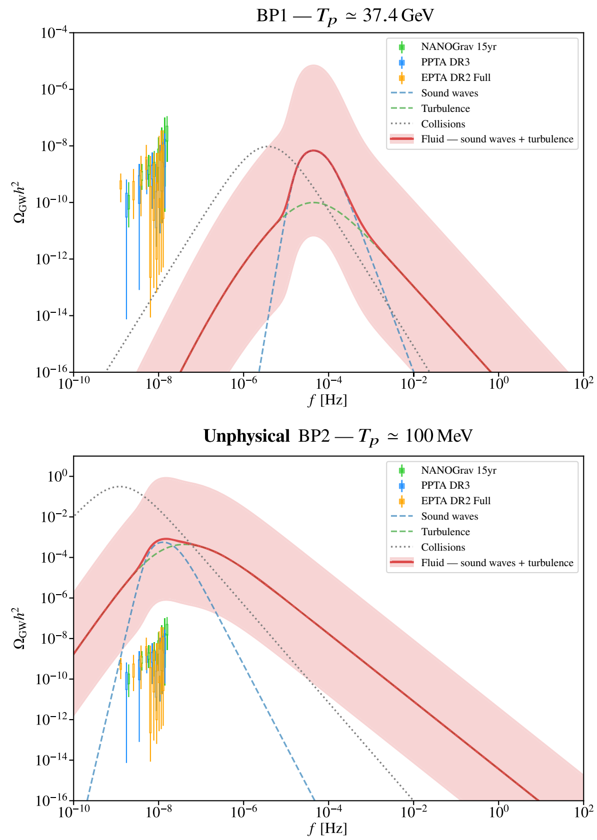

FIG. 3. The SGWB from BP1 (top panel; strongest supercooling for which the FOPT completed) and the unphysical SGWB from BP2 (bottom panel; strongest supercooling for which the FOPT has a percolation temperature though it does not complete and percolation is questionable). We show the 50% and 95% bands for the PTA observations (box plots). The BP1 and BP2 predictions fail to match the PTA observations, even when allowing for a factor $\mathcal{O}(10^3)$ uncertainty (shaded red band). For our BPs, the total SGWB (solid red) comes only from sound waves (dashed blue) and turbulence (dashed green). For comparison, we show the SGWB from a vacuum transition where bubble collisions would be the only source of GWs (dotted gray).

and no completion). The peak frequencies are about 4×10^4 and 15 nHz for BP1 and BP2, respectively. BP1 represents the lowest peak frequency that can be obtained for realistic scenarios in this model because for more supercooling the transition does not complete and percolation becomes questionable. To compare the BP1 predictions with the PTA signals, we must consider the theoretical uncertainties. In our analysis we used daisy resummation and full one-loop corrections to the effective potential. While this approach suffers from substantial theoretical uncertainties, leading to a factor $\mathcal{O}(10^3)$ uncertainty in the predicted GW amplitude [115], the BP1 predictions lie more than 7 orders of magnitude below the NANOGrav signal at nHz frequencies. Thus this model *cannot* explain the nHz signal observed by PTA experiments despite various optimistic statements from the literature. For comparison we show the SGWB prediction if one were to assume vacuum transitions (dotted gray curves). This assumption is not realistic for this model and in any case does not result in agreement with the observed spectrum.

If one ignores the percolation and completion requirements, BP2 shows that the peak frequency can be reduced to match the nHz signal observed by PTA experiments, though the amplitude is several orders of magnitude higher than the PTA observations. Caution should be taken interpreting the SGWB predictions for such strong supercooling because it is well beyond what has been probed in simulations. These predictions are somewhat unphysical because, despite a nominal percolation temperature, bubbles are not expected to percolate as the false vacuum between them is inflating. Without percolation, GWs would not be generated. Lastly, we note that points between BP1 and BP2 may exist in which the low-frequency tail of the SGWB passes through the PTA observations. The transitions for such points, however, would not complete.

Conclusions.—Supercooled FOPTs are an intriguing explanation of the nHz SGWB recently observed by several PTAs, as they could connect a nHz signal to the electroweak scale. Indeed, they were mentioned as a possibility [1,11]. However we demonstrate two major difficulties that can affect supercooled explanations. First, percolation and completion of the transition are hindered by vacuum domination. We demonstrate with an explicit numerical calculation that this rules out the possibility of explaining the PTA signal in the supercooling model of Ref. [63] mentioned as a prototypical example in Refs. [1,11].

Second, the Universe typically reheats to the scale of any physics driving the transition, splitting the percolation and reheating temperatures significantly. This makes it challenging to compute the signal from a supercooled transition because factors often implicitly neglected must be carefully included in fit formulae and the thermal parameters are well beyond those in hydrodynamical simulations on which fit formulas are based. The correct scaling, Eqs. (8) and (10), shows that for supercooled phase transitions that do not complete, the peak frequency could be reduced to nHz. In contrast, completing the phase transition by increasing the nucleation rate at late stages would *not* lead to a nHz signal due to a higher bubble number density, ruling out solutions similar to those proposed for the graceful exit problem [116,117]. We anticipate that these issues are quite generic and they should be carefully checked in supercooled explanations.

P. A., C. L. and L. W. are supported by the National Natural Science Foundation of China (NNSFC) under Grant No. 12335005. P. A. is also supported by NNSFC under Grants No. 12150610460 and by the supporting fund for foreign experts Grant No. wgxz2022021L. A. F. was supported by RDF-22-02-079. L. M. was supported by an

Australian Government Research Training Program (RTP) Scholarship and a Monash Graduate Excellence Scholarship (MGES). Y. W. is supported by NNSFC under Grant No. 12305112 and also by a starting grant from the Nanjing Normal University.

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