Beating the Standard Quantum Limit Electronic Field Sensing by Simultaneously Using Quantum Entanglement and Squeezing

X. N. Feng,[1] M. Zhang,[2] and L. F. Wei[1]

[1]*Information Quantum Technology Laboratory, International Cooperation Research Center of China Communication and Sensor Networks for Modern Transportation, School of Information Science and Technology, Southwest Jiaotong University, Chengdu 610031, China*

[2]*School of Physical Science and Technology, Southwest Jiaotong University, Chengdu 610031, China*

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Quantum entanglement and quantum squeezing are two typical approaches to beat the standard quantum limit (SQL) for the sensitive phase estimations in quantum metrology. Each of them has already been utilized individually and sequentially to improve the sensitivity of electric field sensing with the trapped ion platform. However, the upper bound of the demonstrated sensitivity gain is still limited, i.e., the theoretical 6 dB and experimental 3 dB over the corresponding SQL, for electric field sensing. By simultaneously using the internal (spin)-external (oscillator) state entanglement and the oscillator squeezing to effectively amplify the accumulation phase, we show here that such a theoretical sensitivity gain upper bound can be significantly surpassed. The proposal provides a novel approach to implement the stronger beat of the SQL and even approach the Heisenberg limit, for the sensitive sensings of the desired electric field and also the other metrologies.

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Introduction.—In recent years, using hybrid quantum interferometers, composed of different degrees of freedom, to improve the sensitivity of parameter estimation has been paid much attention [1–9]. Specially, the hybrid quantum interferometer with the experimental trapped ion platform had demonstrated the sensitive sensings of the weak electronic fields, which is applied to drive the external vibration of the ion [7–17]. The sensitivity of the implemented weak electric force can reach at $1yN/\sqrt{\mathrm{Hz}}$ level [12], which, however, is still limited by the standard quantum limit (SQL) scaling as $1/\sqrt{\bar{n}}$ (with $\bar{n}$ being the mean phonon number of the vibration [18–20]). Therefore, how to beat the SQL for further improving the measurement sensitivity with limited quantum resource is an important issue in quantum metrology.

Basically, the quantum resources typically such as entanglement and squeezing can be used to implement the sensitive weak electric field sensing beyond the SQL. Indeed, it has been shown that, by using either the oscillator squeezing or the spin-oscillator entanglement [8–10] in a hybrid interferometer, the sub-SQL sensitivity of the weak electric field sensing can be achieved. However, the demonstrated sensitivity gain is only 3 dB experimentally, although the one predicted theoretically can reach 6 dB [21], over the SQL. Also, one can prove that, such a theoretical sensitive gain limit cannot be surpassed, even the spin-oscillator entanglement and oscillator squeezing are *sequentially* utilized [9].

Alternatively, by *simultaneously* using the spin-oscillator entanglement and the oscillator squeezing, we show in this Letter that the theoretical 6 dB sensitivity gain over the SQL can be surpassed for the weak electric field sensing, with the well-known trapped ion platform. This is because, different from the *sequential* evolutions used in the previous schemes [9,10], here the state preparation (by using the entangled and squeezed operations) and also the phase accumulation (i.e., the applying of the electric field) are implemented *simultaneously*. As a result, the strength of the spin-oscillator coupling and also the applied electric field can be amplified simultaneously, and thus the phase accumulation can be speeded up greatly. Therefore, the higher sensitivity gain over the SQL can be realized for the same evolution duration and mean phonon number, compared with the previous schemes.

Model and method.—Let us consider a hybrid quantum system shown in Fig. 1, wherein a one-dimensional

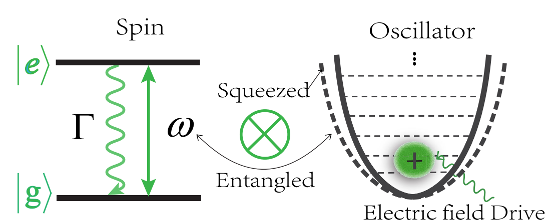

FIG. 1. A trapped ion system utilized to implement the sensitive electric field sensing, wherein the external vibrational squeezing and the spin-oscillator entangling operations are applied simultaneously. The spin is represented by the inner states of trapped ion, i.e., $|g\rangle$ and $|e\rangle$, with decoherence rate Γ and resonant frequency ω. And the vibration of the ion is driven by the electric field, whose strength is expected to be sensitive measured.

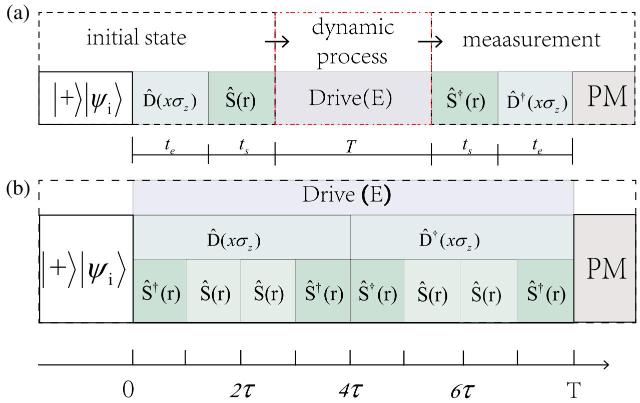

FIG. 2. (a) A typical operational protocol to implement the sensitive electric field sensing by using the ion-trap interferometer. Here, all the desired quantum operations are applied sequentially. (b) An alternative protocol to implement the electric field sensing by *simultaneously* using spin-oscillator entanglement and oscillator squeezing resources, wherein the squeezing and antisqueezing operations are applied alternately. Also, the electric field is applied continuously before the spin-state projective measurements.

harmonic oscillator (i.e., the external vibration of the ion) is coupled to an internal two-levels (called as the $1/2$ spin afterwards). The external vibration of the trapped ion is served as the probe to sense the driving electric field force (to be estimated), which can be effectively amplified by squeezing the external vibration of the ion.

Conventionally, a general procedure for the parameter estimation may be divided roughly into three sequential steps: the preparation of the initial state, the dynamical evolution (i.e., phase accumulation), and the measurements [23,24]. Specifically, Fig. 2(a) shows how to implement the electric field parameter estimation by using the above ion-trap quantum interferometer with the spin-dependent squeezed cat-state initial state, $|\psi_1\rangle = \hat{S}(r)\hat{D}(x\sigma_z)|+\rangle|\psi_i\rangle$ [4,9]. For this state preparation, the spin-oscillator entanglement (SOE) operation $\hat{D}(x\sigma_z)$ and the oscillator squeezing one $\hat{S}(r)$ were utilized sequentially. This can be achieved by applying the Hamiltonian $\hat{H}_E = (\alpha\hat{a}^\dagger + \alpha^*\hat{a})\sigma_z$ and $\hat{H}_S = g(\hat{a}^{\dagger 2}e^{i\theta} + \hat{a}^2 e^{-i\theta})/2$ for duration t_e and t_s [8,9], respectively. Thus, $x = \alpha t_e$ and $r = gt_s$ with $|\alpha|$, g and θ denoting the spin-oscillator coupling strength, the squeezing strength and squeezing phase and, respectively. Then, applying the weak electric field to deliver the interferometer undergoes the evolution $\hat{U}_\eta(T) = \exp[-i\eta T(\hat{a}^\dagger + \hat{a})]$, for encoding the information of the applied electric field into the spin-oscillator entangled state. Finally, after applying the reverse squeezing operations $\hat{S}^\dagger(r)$ and also the reverse disentangling operation $\hat{D}^\dagger(x\sigma_z)$, the information of the electric field can be further transferred into the spin of the final state $|\psi_f\rangle$ for the detection, which is achieved by performing the relevant spin-state projective measurements, i.e., $\hat{P}_\downarrow = |\langle\psi_f|+\rangle|^2$, after a $\pi/2$ pulse.

Theoretically, the optimal sensitivity for the electric field sensing with the initial state $|\psi_1\rangle$ can be estimated as $\Delta\eta = 1/\sqrt{F_Q(|\psi_1\rangle)}$, wherein $F_Q(|\psi_1\rangle) \approx 1/(16T^2\bar{n})$ is the quantum Fisher information (QFI) with $\bar{n} \approx \sinh^2(r) + |\alpha t_e|^2 e^{2gt_s}$ being mean phonon number. Generally, it can be further proved that, for arbitrary input spin-oscillator states $\hat{\rho}_{as}$, such a sensitivity is limited by

$$\Delta\eta \geq \frac{1}{\sqrt{F_Q(\hat{\rho}_{as})}} \geq \frac{1}{4T\sqrt{\bar{n} + 1/2}}, \tag{1}$$

which is bounded by 6 dB for $\bar{n} \gg 1$ (see Supplemental Material [22] for the proof, which includes the Refs. [25–27]). Indeed, only a 3 dB gain of the sensitivity, i.e., $\Delta\eta \approx 1/(2T\sqrt{\bar{n}})$, has been realized experimentally [8–10]. This implies that the protocol for the sensitive electric field sensing can be optimized further.

We now investigate how to further beat the SQL for the electric field sensing by using the spin-oscillator entanglement and oscillator squeezing resources *simultaneously*. The protocol designed here is shown in Fig. 2(b), wherein the duration T is separated by a series of the shorter intervals. Different from the previous sequential evolution schemes [see, e.g., Fig. 2(a)], here either the sequeezing or antisequeezing operation of the oscillator is always on, during the corresponding SOE operation $\hat{D}(x\sigma_z)$ and the spin-oscillator disentangled operation $\hat{D}^\dagger(x\sigma_z)$. Also, the weak electric field is applied continuously. Again, the projective measurements of the spin states are performed to estimate the electric field parameter η. As we can see below that, the squeezing and antisqueezing operations applied here, within the durations $(jT/2, (j+1)T/2)$ (with $j = 0$, 1), can amplify the spin-oscillator interaction strength (i.e., the α-parameter) and the electric field strength (i.e., η parameter) simultaneously, and thus speed up the phase accumulation much faster. As a consequence, the SQL can be further beaten, compared with the previous protocols by only using the entanglement or squeezing resource and even by using both of them sequentially.

Basically, the advantage of the present protocol benefits from the following Hamiltonian:

$$\hat{H}_{k,\pm} = \eta(\hat{a} + \hat{a}^\dagger) \pm i\alpha(\hat{a}^\dagger - \hat{a})\sigma_z + (-1)^{n(k)}\frac{ig}{2}(\hat{a}^{\dagger 2} - \hat{a}^2), \tag{2}$$

which can be really engineered in the present trap-ion interferometer for $n(k) = -\mathrm{mod}\,(k-3, 3)$ with $1 \leq k \leq 8$. During the duration $[k\tau, (k+1)\tau]$, the evolution of the system can be described by the operator (see the Supplemental Material [22] for the detail)

$$\hat{V}_{k\pm}(\tau) = e^{\pm is\phi_0\sigma_z}\hat{D}(\alpha, \eta, g, \tau, s)\hat{S}(sg\tau), \tag{3}$$

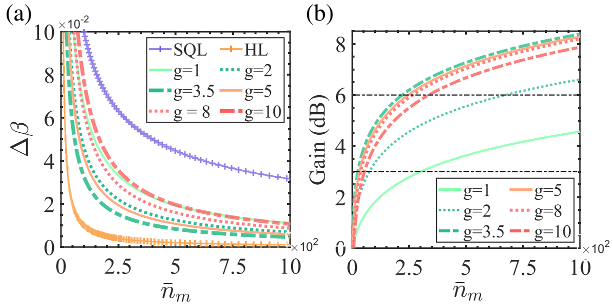

FIG. 3. (a) The sensitivity of displacement sensing $\Delta\beta$ and the corresponding SQL (scaling $1/\sqrt{\bar{n}_m}$) as well as HL (scaling $1/\bar{n}_m$) versus the mean phonon number $\bar{n}_m$ for different squeezing parameters g. (b)The metrological gains of the sensitivity over the SQL, i.e., G_{ms} and G_{ml} for different $g \leq 3.5\alpha$ and $g > 3.5\alpha$, respectively. Note that both the sensitivity and metrological gain above only depend on the ratio g/α, wherein α is set as $\alpha = 1$ for brevity.

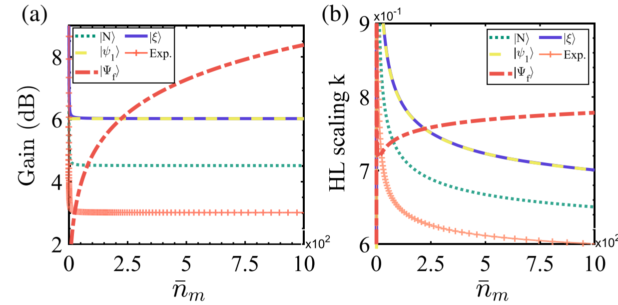

FIG. 4. (a) The metrological gains of the sensitivity over the SQL of the present protocol compared with other previous schemes for different input states as well as the specific experimental realization (i.e., Exp.) in Ref. [10]. (b) The HL scaling of the corresponding sensing protocol of figure (a). Here parameter g is set as $g = 3.5\alpha$.

where $\hat{D}(\alpha,\eta,g,\tau,s)=\hat{D}\left[s\left(i\eta(1-e^{-sg\tau})\pm\alpha\sigma_z(e^{sg\tau}-1)\right)/g\right]$, with $s = (-1)^{n(k)}$ and $\phi_0 = 2\eta\alpha[\sinh(g\tau) - g\tau]/g^2$. Then, after the whole evolution with the duration T, the final state of the interferometer reads

$$|\Psi_f\rangle = \hat{V}_{8+}(\tau)\hat{V}_{6-}(2\tau)\hat{V}_{5+}(\tau)\hat{V}_{4+}(\tau)\hat{V}_{2-}(2\tau)\hat{V}_{1+}(\tau)|\psi_0\rangle = e^{i\Phi\sigma_z}\hat{D}\left[\frac{8i\eta}{g}\sinh(g\tau)\right]|\psi_0\rangle, \tag{4}$$

where

$$\Phi = \frac{\eta\alpha T^2}{2}\frac{\sinh^2(gT/8)}{(gT/8)^2}, \tag{5}$$

is the accumulated phase, which can be directly measured by performing the projective measurement on the spin state. With the measured spin-state population $P_\downarrow = [1+\cos(\Phi)]/2$, the electric field parameter η can be estimated with the sensitivity

$$\Delta\eta_m = \frac{2}{\alpha T^2}\frac{(gT/8)^2}{\sinh^2(gT/8)}. \tag{6}$$

Obviously, such a sensitivity is amplified by a factor of $F = \sinh^2(gT/8)/(gT/8)^2$, compared with the one achieved by only using the SOE (i.e., $g = 0$) [8] for the same duration T.

More importantly, we show below that the sensitivity $\Delta\eta_m$ demonstrated above can further surpass the SQL [scaling $1/(T\sqrt{\bar{n}_m})$] to go beyond the 6 dB limit as shown in Figs. 3 and 4. Here, $\bar{n}_m$ denotes the maximum mean phonon number, which can be calculated as

$$\bar{n}_m \approx \max\left\{\left(\frac{4\alpha}{g}\right)^2\sinh^2(r_m), \left(\frac{4\alpha^2}{g^2}+1\right)\sinh^2(r_m)\right\}, \quad r_m = g\tau, \tag{7}$$

at $t = 4\tau$ for $g \leq 3.5\alpha$ or $t = \tau$ for $g > 3.5\alpha$. Thus, one can verify that the vibrational displacement uncertainty of the ion $\Delta\beta = \Delta\eta_m T$ satisfies the inequality:

$$\Delta\beta \leq \frac{1}{\sqrt{\bar{n}_m}}, \tag{8}$$

for arbitrary parameter g. Figure 3(a) shows how the value of $\Delta\beta$ changes with the $\bar{n}_m$, and indicates clearly that the SQL is really beaten. Specifically, for $g \leq 3.5\alpha$, we have $\bar{n}_m = (4\alpha/g)^2\sinh^2(r_m)$. In this case, the sensitivity, i.e., $\Delta\beta = r_m/[\sinh(r_m)\sqrt{\bar{n}_m}]$, increases with the squeezing strength g for the arbitrarily given $\bar{n}_m$. While, for $g > 3.5\alpha$, we get $\bar{n}_m \approx (4\alpha^2/g^2 + 1)\sinh^2(r_m)$ and $\Delta\beta = gr_m/[4\alpha\sinh(r_m)\sqrt{\bar{n}_m}]$, which decreases with g for the arbitrarily given $\bar{n}_m$. This indicates that, the optimal sensitivity of the displacement sensing is achieved at $g = 3.5\alpha$, wherein the sensitivity is farthest beyond the SQL. Physically, the beaten of the SQL demonstrated here is mainly originated from the simultaneous amplification of the electric field parameter η and the spin-oscillator entanglement parameter α [in Eq. (4)] and thus the phase accumulation is speeded up greatly. Furthermore, for $\eta \ll \alpha$ we have $[\eta\sinh(r_m)e^{r_m}/g]^2 \ll \bar{n}_m$. As a consequence, the η parameter can be significantly amplified with the mean phonon number $\bar{n}_m$ being almost unchanged. Noted that, the simultaneous amplifications of the α parameter and the η parameter can also be achieved by sequentially applying the squeezing and anti-squeezing operations [28–30] [as the sequential scheme of Fig. 2(a)], but the demonstrated sensitivity gain over the SQL is still limited, as the mean phonon number there is also amplified correspondingly.

It is valuable to emphasize that, although the beaten of the SQL has also been experimentally achieved with ion-trap interferometer [9–11] input, respectively, by the Fock state, squeezed state, and the entangled squeezed cat state, the demonstrated sensitivity gain over the relevant SQL cannot surpass 6 dB for $\bar{n}_m \gg 1$ theoretically [see Fig. 4(a)]. With the present protocol, the metrological gains of the sensitivity (6), over the SQL, can be obtained as

$$G_{ms} = 10\log\left(\frac{\sinh(gT/8)}{gT/8}\right)(\mathrm{dB}), \tag{9}$$

for $g \le 3.5\alpha$, and

$$G_{ml} = 10\log\left(\frac{4\alpha}{\sqrt{4\alpha^2+g^2}}\frac{\sinh(gT/8)}{(gT/8)}\right)(\mathrm{dB}), \tag{10}$$

for $g > 3.5\alpha$, respectively. Obviously, both of them increase with the value gT and thus can reach very high value, once gT is sufficiently large. Typically, they can approach 8 dB for $\bar{n}_m \approx 10^3$ and $3.5\alpha \le g \le 10\alpha$ [see Fig. 3(b)]. Of course, Fig. 4(b) indicates that, both of them can not reach the Heisenberg limit (HL) ($\propto 1/\bar{n}_m$) yet, although the larger gain can be realized for the larger phonon excitations. Anyway, the present protocol provides an effective approach to further surpass the SQL beyond the 6 dB gain of the sensitivity and thus more closely approach to the relevant HL theoretically.

Feasibility of the proposal.—The interferometry proposed here can be implemented with a series of experimental platforms, typically such as the trapped ion [9,31], the trapped electron [32,33]and the NV center [5,34]. Generically, let us consider how to implement the proposed interferometric protocol, shown in Fig. 2(b), with an experimental trapped ion system. The desired spin-oscillator entanglement and vibrational squeezing operations can be realized by using the laser-ion interactions and the parametric drivings, respectively. Take the ^{9}Be+ trapped ion system, for example, the relevant parameters can be set, respectively, as $\alpha/(2\pi) \approx 4.28$ and $g/(2\pi) \approx 15.0$ kHz by adjusting the driving power and phase of the applied lasers. Correspondingly, the metrological gain can be estimated as; $G_{ml} \approx 6.7$ and $G_{ml} \approx 8.3$ (dB), for the typical evolution times: $T = 0.297$ and $T = 0.34$ ms, respectively. Noted that these evolution times are still much less than the total duration of the experimental trial, such as $T_{\mathrm{shot}} = 8.73$ ms in Ref. [8]. Therefore, the proposed interferometric protocol for the electric field sensing with sensitivity go beyond the 6 dB is entirely feasible, in principle.

The robustness of the protocol can be discussed by analyzing how the noises influence the feasible sensitivity. Physically, the noises in the experimental trapped ion system mainly originate from the fluctuation of trapped frequency, spin decoherence, and also the unperfect control of the operations [8,9]. Typically, the frequency fluctuation induce the detuning between the system and the control signals. Thus, after reverse evolution, the residual SOE $\Delta_{\alpha_f} \approx 4\alpha\sigma\tau\sinh(g\tau)/g$, as well as the phase fluctuation $\Delta_{\phi_f} \approx 4\sigma\tau\eta\alpha\sinh(g\tau)^2/g^2$, will be induced (see the Supplemental Material [22]), with σ denoting the frequency fluctuation. Similarly, the unperfect control of the applied entangled and squeezed operations might also yield the extra residual SOE and accumulated phase fluctuation, which can be roundly estimated as $\Delta_{\alpha_u} \sim 8\alpha\Delta t\sinh(g\tau)$ and $\Delta_{\phi_u} \sim 32\eta\alpha\Delta t\sinh(2g\tau)/g$, by simply adding a fluctuation of duration Δt in each evolution duration of the state of Eq. (4). These residual SOEs and accumulated phase fluctuations might reduce the contrast of the measurement signal and thus decrease the measurement sensitivity.

Consequently, by considering the influences from these noises and also the spin decoherence, the spin projective measurement reads

$$P_{\downarrow} = \frac{1}{2}\left(1 + e^{-\Gamma T - |\Delta_{\alpha_{\mathrm{tot}}}|^2}\cos(\Phi + \Delta_{\phi_{\mathrm{tot}}})\right), \tag{11}$$

where Γ is the spin-decoherence rate and $\Delta\alpha_{\mathrm{tot}} = \Delta_{\alpha_f} + \Delta_{\alpha_u}$, $\Delta_{\phi_{\mathrm{tot}}} = \Delta_{\phi_f} + \Delta_{\phi_u}$. As a consequence, the measurement sensitivity is modified as

$$\Delta\beta_L \approx \frac{2e^{\Gamma T + |\Delta_{\alpha_{\mathrm{tot}}}|^2}\sqrt{1 - e^{-\Gamma T - |\Delta_{\alpha_{\mathrm{tot}}}|^2}\cos^2(\Phi + \Delta_{\phi_{\mathrm{tot}}})}}{\alpha T\sin(\Phi + \Delta\phi)\sinh^2(g\tau)/g^2}. \tag{12}$$

If $\Delta\beta_L$ is expanded up to the first order of σ^2, one can obtain the similar result shown in Ref. [9].

Figure 5 shows that, under the noise influences, the measurement sensitivity $\Delta\beta_L$ oscillates with the duration T and reduces dramatically for $\Phi + \Delta_{\phi_{\mathrm{tot}}} \approx k\pi$. However, the sensitivity can still revive and approach the one without the noise influence for the relatively small T, Γ and $\Delta_{\alpha_{\mathrm{tot}}}$, and finally approach to its optimal value which is mainly determined by $\Delta_{\alpha_{\mathrm{tot}}}$. Obviously, this behavior is different from that shown in Ref. [9]. Such as, for the typical frequency fluctuation, e.g., $\sigma \approx 250$ and $\Gamma \approx 500$ Hz, the measurement sensitivity can reach $\Delta\beta_L \approx 0.02$, 0.01 for $T \approx 0.28$ and $T \approx 0.32$ ms, respectively. Furthermore, one

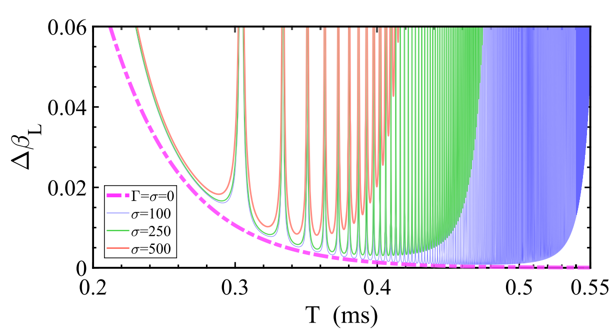

FIG. 5. The sensitivity of the displacement parameter sensing, i.e., $\Delta\beta_L$, changes with the total evolution duration T for different detuning $\sigma = 0.1$, 0.25, 0.5 kHz, respectively, with $\Gamma = 500$ Hz. Here the squeezing strength g and spin-oscillator entangled strength α are typically set as $g = 2\pi \times 15$ kHz, $\alpha = 2\pi \times 4.28$ kHz, and $\eta = 100$ Hz, respectively.

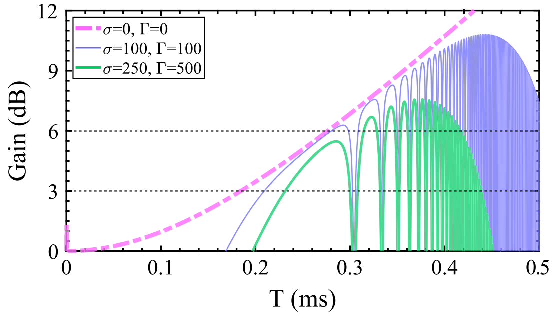

FIG. 6. The metrological gain under different frequency fluctuation σ and spin decoherence rate Γ.

can see from Fig. 6 that, the achievable metrological gain of the proposed protocol can still go beyond the 6 dB limit for certain T, even the influence from the noises is considered.

Conclusions.—In summary, we propose an effective quantum sensing protocol, by using the hybrid spin-oscillator interferometer, to further improve the measurement sensitivity of the weak electric field. By simultaneously utilizing the quantum entanglement and squeezing resources, we showed that the strength of the oscillator coupling with electric field and the spin can be amplified simultaneously. As a consequence, the phase accumulation can be speeded up significantly and thus the sensitivity gain of the electric field sensing can be manifestly enhanced to go beyond the theoretically 6 dB (over the relevant SQL) bound, even in the presence of the noises. Therefore, the present protocol provides an effective approach to implement the sensitive electric field sensing at the sub-SQL regime. Our result demonstrate clear the possibility and advantages of the simultaneous using of various quantum technologies in hybrid quantum system for quantum sensings.

In principle, the present scheme might be also applied to implement the precise measurements of the other physical quantities, such as the rotation [35–37], the spin-dependent interaction effects [38–40] (e.g., the gravitationally induced quantum entanglement), etc., by using the limited quantum resources.

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[21] In the present protocol, the SQL is defined naturally in terms of the maximum mean phonon number, which is excited during the whole evolution procedure. In this case, the theoretically reachable sensitivity gain is 6 dB, which is held for arbitrary input states, typically, such as the previous demonstrated Fock state, squeezed state, and spin-dependent squeezed cat states. However, the experimental demonstration of the measurement sensitivity, e.g., by using the squeezing state resource, is just a 3 dB gain over the defined SQL. The relevant proof is also given in the Supplemental Material [22].

[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.132.220801 for details of the results shown in the main text. We derive the sensitivity of electric filed sensing for the various involved sequential sensing schemes and prove that they are limited by 6 dB. Furthermore, the detail calculation of the dynamical evolution operators for the proposed simultaneous sensing scheme with and without considering the noise is also given.

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