Universal Spin Superconducting Diode Effect from Spin-Orbit Coupling

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We propose a universal spin superconducting diode effect (SDE) induced by spin-orbit coupling (SOC) in systems with spin-triplet correlations, where the critical spin supercurrents in opposite directions are unequal. By analysis from both the Ginzburg-Landau theory and energy band analysis, we show that the spin-↑↑ and spin-↓↓ Cooper pairs possess opposite phase gradients and opposite momenta from the SOC, which leads to the spin SDE. Two superconductors with SOC, a p-wave superconductor as a toy model and a practical superconducting nanowire, are numerically studied and they both exhibit spin SDE. In addition, our theory also provides a unified picture for both spin and charge SDEs.

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Introduction.—Supercurrent in superconductors is an enduring research topic [1–5]. Recent experiments have reported an exotic phenomenon that the critical charge supercurrents in opposite directions are different [6–13], which is called superconducting diode effect (SDE) [6–31]. Such nonreciprocity has potential applications in superconducting logic circuits and sensors with robust rectification [6,7]. From a theoretical perspective, the charge SDE stems from spatially modulated order parameter Δe^{iqx} or finite Cooper pair momentum [14–20].

The Cooper pair, as the carrier of superconductors, has a fixed charge $2e$, while its spin can be singlet or triplet [32–35]. Some superconductors with spin-triplet components can be managed to realize a spin supercurrent [35–48]. Since charge and spin are two inseparable intrinsic degrees of Cooper pairs, the existence of charge SDE provides the possibility of spin SDE. Very recently, a spin SDE is theoretically suggested in a Fulde-Ferrell superconducting chain [49]. However, we wonder whether demanding the Fulde-Ferrell term is necessary, and we would like to seek for a more general derivation of spin SDE. Compared to charge, the manipulation of spin has faster reaction speed and higher energy efficiency, so that the spintronics has received widespread attention in the last two decades [35,36,50,51]. The spin SDE has broad potential applications in spintronics, such as the highly efficient rectification device for spin current, low-power quantum spin logic devices, and magnetic sensors.

In this Letter, we propose that the spin SDE universally appears in the spin-triplet superconductors in the presence of spin-orbit coupling (SOC). From both Ginzburg-Landau (GL) theory [52–54] and energy band analysis, we show that the spin-↑↑ and spin-↓↓ triplet Cooper pairs get opposite phase gradients and momenta from SOC, thus resulting in unequal critical spin supercurrents in positive and negative directions [$I_{s,c+} \neq |I_{s,c-}|$, see Fig. 1(a)], i.e., the spin SDE. Then, we numerically confirm the existence of spin SDE from both the p-wave superconductor as a toy model and the practical superconducting nanowire under magnetic field. Besides, we also give a unified physical picture for both spin SDE and charge SDE.

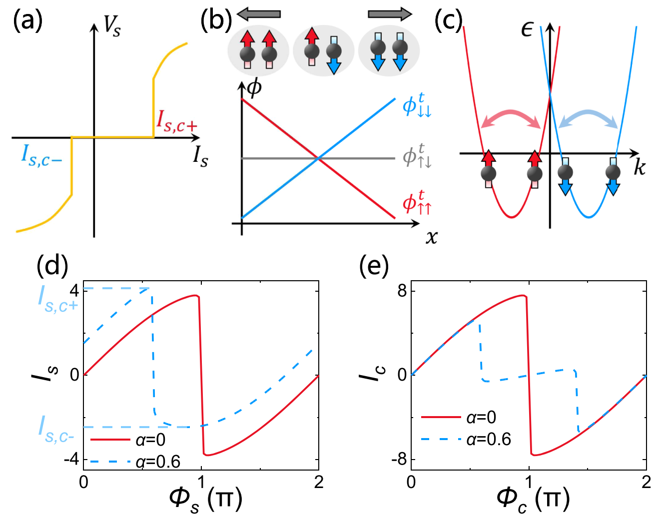

FIG. 1. (a) The schematic plot for the spin bias V_s versus the spin current I_s in a spin SDE device. (b) The SOC-induced different phase gradients on $s_z = 1, 0, -1$ spin-triplet Cooper pairs (lower part), which is equivalent to finite momenta and causes ↑↑ and ↓↓ Cooper pairs to move oppositely (upper part). (c) The electron energy band in the presence of SOC. The spin ↑↑ or ↓↓ Cooper pairs, if existent, will have nonzero momenta. (d) The spin CPR under the spin phase ϕ_s and (e) charge CPR under the charge phase ϕ_c in a p-wave superconductor Josephson junction. The critical spin supercurrents in positive and negative directions with $I_{s,c+} \neq |I_{s,c-}|$ are indicated in (d). Parameters: $e = \hbar = m = \Delta_p = 1$, $\mu = 10$.

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General theory of SOC-induced spin SDE.—Let us theoretically analyze the occurrence of spin SDE in spin-triplet superconductors while in the presence of SOC. To concisely demonstrate the spin SDE, we first concentrate on a one-dimensional system with Rashba SOC $H_{\rm SO} = \alpha\sigma_z p_x$ with α the SOC strength [55–57].

First, let us analyze the effect of SOC on spin transport from the GL theory: the GL equation writes [47,52–54]

$$b\mathbf{\Psi} + \frac{1}{2m}(-i\hbar\partial_x + 2m\alpha\hat{s}_z)^2\mathbf{\Psi} = 0, \tag{1}$$

where $\hat{s}_z = \mathrm{diag}(1, -1)$ is the z-direction spin operator, m is the effective mass, and $\mathbf{\Psi} = (\Psi_1, \Psi_{-1})^T$ denotes the order parameters of spin-triplet Cooper pair components with $s_z = 1$ and -1 [see Sec. SI of the Supplemental Material [58] for the spin-triplet order parameter Ψ_0 with $s_z = 0$]. When there is no SOC, we can set the order parameters to be $\Psi_{s_z}(0)$. Then one can find out that with SOC, they become $\Psi_{s_z}(\alpha) = \Psi_{s_z}(0)e^{-2is_z m\alpha x/\hbar}$. We define the phases of $s_z = 1, 0, -1$ Cooper pairs as $\phi^t_{\uparrow\uparrow}, \phi^t_{\uparrow\downarrow}, \phi^t_{\downarrow\downarrow}$, which are linearly dependent on the position x from the above expression of $\Psi_{s_z}(\alpha)$, as shown in Fig. 1(b). So, these Cooper pairs get different phase gradients from the SOC: $\nabla\phi^t_{\uparrow\uparrow} = -(2m\alpha/\hbar)$, $\nabla\phi^t_{\uparrow\downarrow} = 0$, $\nabla\phi^t_{\downarrow\downarrow} = (2m\alpha/\hbar)$. Remarkably, the phase gradients of spin-$\uparrow\uparrow$ and spin-$\downarrow\downarrow$ Cooper pairs are opposite, and they tend to move oppositely, see the upper part of Fig. 1(b). In fact, the phase gradient is equivalent to a Cooper pair momentum via a unitary transformation [17]. The charge SDE can be generally induced by the Cooper pair having nonzero total momentum [14–20]. In analogy, here the opposite phase gradients of spin-$\uparrow\uparrow$ and spin-$\downarrow\downarrow$ Cooper pair components correspond to opposite momenta $-2m\alpha$ and $2m\alpha$. This means that $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs have the same charge but opposite spin, opposite momenta, and opposite movement, so a spin SDE may emerge [59].

Second, the SOC-induced spin SDE can also be obtained by analyzing the electronic Hamiltonian with the SOC. In the electron basis $(\psi_{k,\uparrow}, \psi_{k,\downarrow})^T$, the one-dimensional electron system with SOC $H_{\rm SO} = \alpha\sigma_z p_x$ can be described by a simple Hamiltonian

$$H_1 = \frac{\hbar^2 k^2}{2m} - \mu + \alpha\sigma_z\hbar k = \frac{(\hbar k + m\alpha\sigma_z)^2}{2m} - \mu - \frac{m\alpha^2}{2}, \tag{2}$$

with the wave vector k. The SOC translates spin-$\uparrow$ band by momentum $-m\alpha$ and spin-$\downarrow$ band by momentum $m\alpha$, see Fig. 1(c). If we introduce s-wave pairing order parameter to Eq. (2), a spin-$\uparrow$ electron and a spin-$\downarrow$ electron can combine into a spin-singlet Cooper pair with zero momentum. However, as for spin-triplet Cooper pairs formulated by equal-spin electrons, i.e., $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs, they will get $-2m\alpha$ and $2m\alpha$ momenta, respectively [as shown in energy band Fig. 1(c)]. It coincides with the GL analysis and a spin SDE may emerge. In brief, above we propose that the SOC can naturally lead to a spin SDE in spin-triplet superconductors.

Models and method.—For the system with only SOC $\alpha\sigma_z p_x$ but no other spin-dependent terms, the $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs do not exist. Thus, based on the simple SOC system in Eq. (2), we add two kinds of spin-triplet superconductivity and show the spin SDE from numerical calculations. One is p-wave pairing as a fundamental toy model, which gives an intelligible example for spin SDE. The other is to apply magnetic field and proximity of s-wave superconductor, which is quite mature in the field of experiments [60–63].

In the presence of SOC, the definition of spin current is controversial [64], and it may not be a good choice to directly calculate the spin current in a superconductor with SOC. To avoid this, we insert a short normal metal (without SOC) into the superconductor to form a Josephson junction. The spin current is calculated via the normal region, where the spin current can be clearly defined. We emphasize that the inserted normal metal has almost no influence on the physical essence, and the calculation of Josephson junction can reflect the bulk spin SDE property [58]. The spin and charge current-phase relations (CPRs), i.e., $I_s - \phi_s$ and $I_c - \phi_c$, are computed using the nonequilibrium Green's function approach [65–67], with details provided in Secs. SIII and SIV of the Supplemental Material [58,68–76]. ϕ_s and ϕ_c are the spin and charge superconducting phase differences between the left and right superconductors. The charge phase ϕ_c corresponds to a transformation $\psi'_{\uparrow,\downarrow} = \psi_{\uparrow,\downarrow}e^{i\phi_c/2}$ that leads to $\phi^t_{\uparrow\uparrow} = \phi^t_{\uparrow\downarrow} = \phi^t_{\downarrow\downarrow} = \phi^s_{\uparrow\downarrow} = \phi_c$, with $\phi^s_{\uparrow\downarrow}$ the phase of spin-singlet Cooper pairs. This means all Cooper pairs have the same phase and are driven in the same direction [77–79]. There are various regulations for spin Josephson supercurrent [35,41–44,47], which can be regarded as effectively manipulating a spin phase ϕ_s with $\psi'_\uparrow = \psi_\uparrow e^{i\phi_s/2}$, $\psi'_\downarrow = \psi_\downarrow e^{-i\phi_s/2}$ [47]. This spin phase corresponds to $\phi^t_{\uparrow\uparrow} = -\phi^t_{\downarrow\downarrow} = \phi_s, \phi^t_{\uparrow\downarrow} = \phi^s_{\uparrow\downarrow} = 0$ [47,80], driving spin-$\uparrow\uparrow$ and spin-$\downarrow\downarrow$ Cooper pairs to move oppositely and induces a spin supercurrent. After obtaining the CPRs $I_s - \phi_s$ and $I_c - \phi_c$, the critical supercurrent in the positive (negative) direction $I_{s/c,c+}$ ($I_{s/c,c-}$) corresponds to the maximum (minimum) current of the CPR. The unequal critical supercurrents in opposite directions reveal the SDE [14–20].

SDE in p-wave superconductors.—We first consider a simple toy model with direct spin-$\uparrow\uparrow$ and spin-$\downarrow\downarrow$ p-wave pairing. Adding this superconductivity to Eq. (2), the Hamiltonian becomes [81–83]

$$H_2 = (\psi^\dagger_{k,\uparrow}, \psi^\dagger_{k,\downarrow})\left(\frac{\hbar^2 k^2}{2m} - \mu + \alpha\sigma_z\hbar k\right)(\psi_{k,\uparrow}, \psi_{k,\downarrow})^T$$
$$+ k(\Delta_{\uparrow\uparrow}\psi^\dagger_{k,\uparrow}\psi^\dagger_{-k,\uparrow} + \Delta_{\downarrow\downarrow}\psi^\dagger_{k,\downarrow}\psi^\dagger_{-k,\downarrow} + \mathrm{H.c.}). \tag{3}$$

Based on the above general theory between SOC and spin SDE, as long as one of the ↑↑ and ↓↓ components exists, i.e., $\Delta_{\uparrow\uparrow} \neq 0$ or $\Delta_{\downarrow\downarrow} \neq 0$, the spin SDE will appear.

The SOC breaks the spatial inversion symmetry, as a necessary element to realize both charge and spin SDEs. For comparison, to realize charge SDE, the time-reversal symmetry $\mathcal{T}$ should also be broken, because the charge current is reversed by the $\mathcal{T}$ operation [19]. But a pure spin current is invariant under the $\mathcal{T}$ operation, because equivalent spin-up and -down components move oppositely. Therefore, the broken $\mathcal{T}$ symmetry is unnecessary for spin SDE. To show this, we specially calculate a $\mathcal{T}$ invariant order parameter $\Delta_{\uparrow\uparrow} = -\Delta_{\downarrow\downarrow} = \Delta_p$.

We present the spin SDE by the spin Josephson CPR, I_s-ϕ_s, of the p-wave superconductors in Fig. 1(d). When the SOC is absent, there exhibits a normal CPR with $I_s(-\phi_s) = -I_s(\phi_s)$ and $I_{s,c+} = |I_{s,c-}|$, i.e., the spin SDE is nonexistent. The SOC $\alpha = 0.6$ brings the spin SDE, with nonreciprocal critical supercurrent $I_{s,c+} \neq |I_{s,c-}|$, see the blue curve in Fig. 1(d). The ϕ_s-driven spin current comes from current flow of ↑↑ and ↓↓ Cooper pairs $I_s(\phi_s) = \hbar[j_{\uparrow\uparrow}(\phi_s) - j_{\downarrow\downarrow}(-\phi_s)]$. The $\mathcal{T}$ symmetry leads to the relation $j_{\uparrow\uparrow}(\phi) = -j_{\downarrow\downarrow}(-\phi)$. Therefore, $I_s(\phi_s) = 2\hbar j_{\uparrow\uparrow}(\phi_s) = -2\hbar j_{\downarrow\downarrow}(-\phi_s)$, and $I_{s,c\pm} = 2\hbar j_{\uparrow\uparrow,c\pm} = -2\hbar j_{\downarrow\downarrow,c\mp}$, with $j_{\uparrow\uparrow(\downarrow\downarrow),c\pm}$ the critical supercurrents of ↑↑ (↓↓) Cooper pairs. The SOC-induced nonzero momenta lead to unequal critical supercurrents $j_{\uparrow\uparrow(\downarrow\downarrow),c+} \neq |j_{\uparrow\uparrow(\downarrow\downarrow),c-}|$, and then $I_{s,c+} \neq |I_{s,c-}|$. This indicates that the spin SDE is closely related to nonreciprocity of ↑↑ and ↓↓ Cooper pair components and exists whether the $\mathcal{T}$ symmetry is broken or not.

Different from the spin transport, the charge SDE does not appear in Fig. 1(e) while under the drive of the charge phase ϕ_c. The ϕ_c-driven charge current $I_c(\phi_c) = 2e[j_{\uparrow\uparrow}(\phi_c) + j_{\downarrow\downarrow}(\phi_c)]$. Indeed, the relation from $\mathcal{T}$ symmetry $j_{\uparrow\uparrow}(\phi) = -j_{\downarrow\downarrow}(-\phi)$ gives $I_c(-\phi_c) = -I_c(\phi_c)$, and the critical charge supercurrents in opposite directions are equal. Therefore, the charge SDE is forbidden by the $\mathcal{T}$ symmetry. For another understanding, the ↑↑ and ↓↓ Cooper pair numbers are equal due to the $\mathcal{T}$ symmetry. Their opposite momenta lead to opposite SDEs, which cancel out and cause the absence of charge SDE. Only when the $\mathcal{T}$ symmetry is broken, the numbers of the ↑↑ and ↓↓ Cooper pairs can be unequal and the charge SDE can appear. Because spin SDE has lower requirements for breaking symmetry, spin SDE may appear regardless of whether charge SDE exists or not, and hence may be considered as a more universal phenomenon.

SDE of artificial spin-triplet superconductors.—Next, we study a practical system, the superconducting nanowire under magnetic field. Notably, mature experimental techniques [60–63] are available for fabricating superconducting nanowires and detecting their spin-dependent transport.

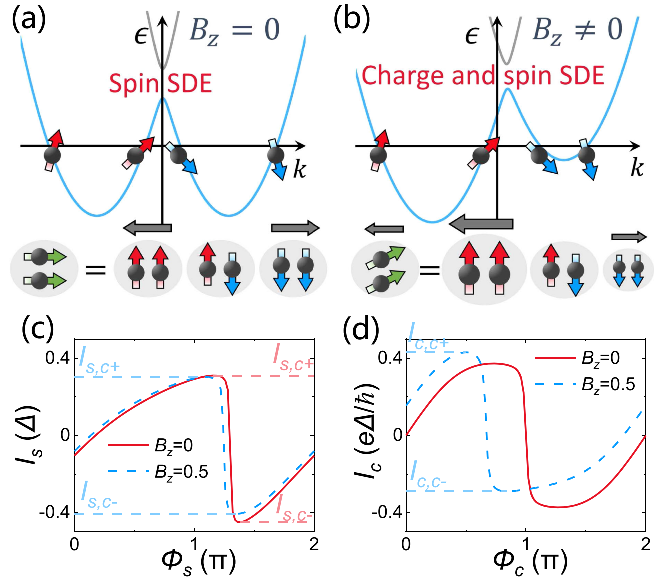

FIG. 2. (a),(b) The schematic normal-state ($\Delta = 0$) energy bands of the superconducting nanowires (upper part) and decompositions of the dominated Cooper pairs (lower part) with $B_x \neq 0$, and (a) $B_z = 0$ and (b) $B_z \neq 0$. (c) The CPR of I_s-ϕ_s and (d) the CPR of I_c-ϕ_c for the superconducting nanowires with the SOC strength $\alpha = 0.6$. The unequal critical supercurrents $I_{s(c),c+} \neq |I_{s(c),c-}|$ are indicated in (c),(d). The other parameters: $\hbar = m = \Delta = 1$, $B_x = 1.5$, $\mu = 0$.

The Hamiltonian of the superconducting nanowires with SOC writes as [84,85]

$$H_3 = (\psi^\dagger_{k,\uparrow}, \psi^\dagger_{k,\downarrow})\left(\frac{\hbar^2 k^2}{2m} - \mu + \alpha\sigma_z\hbar k - B_x\sigma_x - B_z\sigma_z\right) \times (\psi_{k,\uparrow}, \psi_{k,\downarrow})^T + (\Delta\psi^\dagger_{k,\uparrow}\psi^\dagger_{-k,\downarrow} + \text{H.c.}). \quad (4)$$

Without loss of generality, here we set magnetic field components B_x and B_z in the x and z directions. We consider an s-wave pairing potential Δ. Although SOC and magnetic field may influence Δ in self-consistent calculations, generally the result is rarely affected [58,86–89]. The combination of magnetic field and superconductivity can generate a dominated effective spin-triplet p-wave pairing [47,90]. For a strong magnetic field, the gap is dominated by the magnetic field, where the spin-triplet Cooper pairs are *spin polarized almost parallel to the magnetic field*, as shown in Figs. 2(a), 2(b) [47,85,90,91].

When the magnetic field is along direction x, the dominated Cooper pairs can be decomposed into the spin-z basis $|xx\rangle = \frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ (for detailed discussions, see Sec. SVI of the Supplemental Material [58]). Therefore, the ↑↑ and ↓↓ components have the same weight [Fig. 2(a)]. The opposite momenta of ↑↑ and ↓↓ Cooper pairs lead to spin SDE, while the charge SDE is offset. Figure 2(c) shows the spin supercurrent I_s versus the spin phase ϕ_s, and it clearly shows the spin SDE, i.e., $I_{s,c+} \neq |I_{s,c-}|$, see the red curve. But $I_c(-\phi_c) = -I_c(\phi_c)$ and there is no charge SDE, see the red I_c-ϕ_c curve in Fig. 2(d). The disappearance of charge SDE corresponds to a symmetric normal-state energy band in Fig. 2(a), and this is consistent with some theoretical statements that the charge SDE relates to an asymmetric band [16].

Specially, when a z-direction magnetic component exists, for example, $B_z > 0$, the spin polarization is deviated from the x direction towards the z direction. Then the $\uparrow\uparrow$ component exceeds the $\downarrow\downarrow$ component, as shown in the lower part in Fig. 2(b). As these two components have opposite momenta, the system still exhibits spin SDE [see the blue curve in Fig. 2(c)]. Moreover, the Cooper pairs have a nonzero momentum in total, as the charge transport is dominated by the $\uparrow\uparrow$ Cooper pairs. As a result, the charge SDE also exists as shown by the blue curve in Fig. 2(d). Correspondingly, the normal-state energy band becomes asymmetric about $k = 0$ [see the upper part in Fig. 2(b)].

For comparison, the spin and charge CPRs with SOC $\alpha = 0$ are shown in Sec. SVII of Supplemental Material [58]. There is no SDE in neither spin nor charge transport. This means that the SOC is a key factor causing the spin and charge SDE.

From the spin SDE perspective, our theory provides an explanation for the charge SDE studied previously. These charge SDEs emerge in superconductors with both SOC and magnetic term [6,10,14–16,20], and strongly depend on the direction of the magnetic term [6,10,16,20]. We here give an understanding that in these systems the SOC induces opposite momenta and opposite nonreciprocities on $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs. The magnetic term regulates the numbers of $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs. When their numbers are unequal, the charge SDE emerges.

We next investigate the efficiency of spin SDE and concentrate on the practical superconducting nanowire with $B_z = 0$. We define the spin SDE efficiency as

$$\eta = \frac{I_{s,c+} - |I_{s,c-}|}{I_{s,c+} + |I_{s,c-}|}. \tag{5}$$

A nonzero η relates to the appearance of spin SDE.

We find that the spin SDE efficiency η can be effectively regulated by the strength of SOC α. Figure 3(a) shows $I_{s,c\pm}$ versus α. When $\alpha < 0$, $I_{s,c+} > |I_{s,c-}|$, while for $\alpha > 0$, $I_{s,c+} < |I_{s,c-}|$. The Hamiltonian H_3 has the relation $UH_3(\alpha)U^\dagger = H_3(-\alpha)$ with $U = e^{-i(\pi/2)\sigma_x}$. Meanwhile, the spin $\uparrow$ and $\downarrow$ exchange under the U transformation, thus $j_{\uparrow\uparrow}(-\alpha, \phi) = j_{\downarrow\downarrow}(\alpha, \phi)$. Therefore, the spin CPR satisfies $I_s(-\alpha, \phi_s) = \hbar[j_{\uparrow\uparrow}(-\alpha, \phi_s) - j_{\downarrow\downarrow}(-\alpha, -\phi_s)] = \hbar[j_{\downarrow\downarrow}(\alpha, \phi_s) - j_{\uparrow\uparrow}(\alpha, -\phi_s)] = -I_s(\alpha, -\phi_s)$, and $I_{s,c+}(\alpha) = |I_{s,c-}(-\alpha)|$ as shown in Fig. 3(a). Correspondingly, the efficiency η is an odd function of α, i.e., $\eta(-\alpha) = -\eta(\alpha)$, see Fig. 3(a). The efficiency can reach 40% at $\alpha = -0.06$, which corresponds to a remarkable spin SDE with the ratio $I_{s,c+}/|I_{s,c-}| \approx 2.3$.

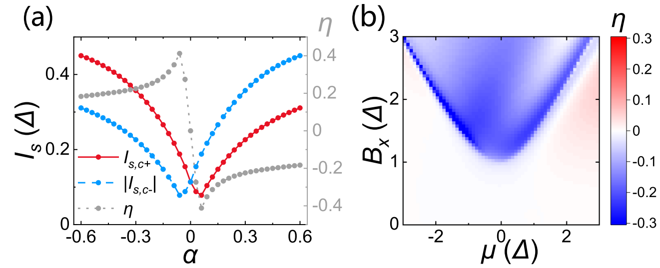

FIG. 3. (a) The critical supercurrents $I_{s,c+}$, $|I_{s,c-}|$ and spin SDE efficiency η as functions of SOC. (b) The efficiency η as a function of B_x and μ. Here $B_z = 0$ and $\alpha = 0.6$, the other parameters are the same as those in Fig. 2.

As the necessary factor of spin SDE, the strength of SOC is feasible. In Fig. 2, we choose a dimensionless $\alpha = 0.6$. When the superconducting gap is 250 μeV, $\alpha = 0.6$ corresponds to 20 meV · nm, a typical value in InSb semiconductors [60,62]. Importantly, we emphasize that a large SOC is not a requirement for the spin SDE. Even when α is as low as 0.06 (equivalent to 2 meV · nm), the spin SDE is apparent with $\eta \approx -40\%$. Therefore, the range of materials that can demonstrate spin SDE is expanded to include those with weak SOC. Additionally, even zero-SOC systems can be regulated to exhibit spin SDE, as SOC can be added through an applied electric field [92–95].

We also study the dependence of η on magnetic field B_x and chemical potential μ, as shown in Fig. 3(b). As the magnetic field increases, the superconducting gap changes from a spin-singlet type (dominated by Δ) to a spin-triplet type (dominated by B_x) [47,85], and the transition line is $B_x^2 = \Delta^2 + \mu^2$. Because the spin-triplet superconductivity is the key element for spin SDE, the spin SDE is quite noticeable when the gap is B_x dominated with $B_x^2 > \Delta^2 + \mu^2$ [see Fig. 3(b)].

Discussion and conclusion.—In summary, we propose that the SOC leads to opposite phase gradients and opposite momenta on $\uparrow\uparrow$ and $\downarrow\downarrow$ spin-triplet Cooper pairs, and a universal spin SDE is caused. For both a p-wave superconductor and an artificial superconducting nanowire system, the spin Josephson CPRs with $I_{s,c+} \neq |I_{s,c-}|$ verify the existence of spin SDE. The spin SDE appears in a wide parameter space. Our theory also provides a new perspective to view the previously studied charge SDE.

In addition, we analyze and calculate the spin SDEs for arbitrary spin orientation and from Rashba SOC and linear Dresselhaus SOC [95,96] in two-dimensional systems (see Sec. SVIII of the Supplemental Material [58]). We can conclude that (i) in principle, in the spin-triplet superconductor system, the existence of SOC destroys the spin degeneracy and breaks inversion symmetry (except for the special spatial inversion symmetry-preserving SOC, such as the Kane-Mele SOC [97]). This gives rise to finite momenta of spin-triplet Cooper pairs, and a spin SDE is generated. (ii) The type of the SOC just influences the spin orientation of spin SDE.

The spin SDE can be detected from various methods, including spin-polarized transport [37–39] and nonlocal spin transport [98–100]. The spin current can be injected and detected via ferromagnet [37–39,98,101,102] or spin Hall system [103]. The detailed detecting methods are shown in Sec. SIX of the Supplemental Material [58,104].

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