Stable Vortex Solitons Sustained by Localized Gain in a Cubic Medium

Chunyan Li[1,2,*] and Yaroslav V. Kartashov[2]
[1] *School of Physics, Xidian University, Xi'an 710071, China*
[2] *Institute of Spectroscopy, Russian Academy of Sciences, 108840 Troitsk, Moscow, Russia*

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We propose a simple dissipative system with purely cubic defocusing nonlinearity and nonuniform linear gain that can support stable localized dissipative vortex solitons with high topological charges without the utilization of competing nonlinearities and nonlinear gain or losses. Localization of such solitons is achieved due to an intriguing mechanism when defocusing nonlinearity stimulates energy flow from the ringlike region with linear gain to the periphery of the medium where energy is absorbed due to linear background losses. Vortex solitons bifurcate from linear gain-guided vortical modes with eigenvalues depending on topological charges that become purely real only at specific gain amplitudes. Increasing gain amplitude leads to transverse expansion of vortex solitons, but simultaneously it usually also leads to stability enhancement. Increasing background losses allows creation of stable vortex solitons with high topological charges that are usually prone to instabilities in conservative and dissipative systems. Propagation of the perturbed unstable vortex solitons in this system reveals unusual dynamical regimes, when instead of decay or breakup, the initial state transforms into stable vortex solitons with lower or sometimes even with higher topological charge. Our results suggest an efficient mechanism for the formation of nonlinear excited vortex-carrying states with suppressed destructive azimuthal modulational instabilities in a simple setting relevant to a wide class of systems, including polaritonic systems, structured microcavities, and lasers.

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Vortices are ubiquitous topological objects showing intriguing evolution and rich interactions when they are nested in nonlinear fields [1]. Vortices were observed in Bose-Einstein condensates [2,3], hybrid light-matter systems [4], polariton condensates [5–11], laser systems [12–15], plasmas, and in different optical materials [16–20]. Such states are interesting for practical applications ranging from information encoding, particle trapping, to controllable angular momentum transfer from light to matter. In nonlinear optical media one may observe the formation of vortex solitons. Vortex solitons are excited higher-order nonlinear states [21] that are usually prone to instabilities that may lead to their collapse, decay or splitting into sets of fundamental solitons [22]. Among the strategies allowing us to generate stable vortex solitons in conservative optical media is the utilization of materials with competing [23–27] or nonlocal nonlinearities [28–31], various optical potentials [32–40], and other approaches [16,18,22].

Vortex solitons can form not only in conservative, but also in dissipative optical systems, in which case the search of potential stabilization mechanisms becomes particularly important and challenging, since fundamental solitons in such systems are typically characterized by wider attractor basins leading to their predominant dynamical excitation. Nevertheless, dissipative vortex solitons may form in lasers with saturable absorption [12,41–43], in systems governed by complex Ginzburg-Landau equation [44–46], not only in two- [47–52], but also in three-dimensional settings [53–56], in mode-locked lasers [57], and in systems with localized gain and nonlinear absorption [58–66]; see reviews [67,68] and recent experimental realizations [69]. In all dissipative systems mentioned above the presence of competing nonlinearities, nonlinear gain or absorption are central for suppression of instabilities of vortex solitons.

In this Letter we propose a new simple mechanism of formation of the ringlike dissipative vortex solitons that does not require competing nonlinearities, nonlinear absorption or optical potentials. Instead, it employs a ringlike gain landscape created in a medium with uniform background linear losses and defocusing cubic nonlinearity that in this case prevents an uncontrollable growth of light intensity. Our solitons have localized ringlike shapes despite defocusing nonlinearity and absence of any potentials. They have negative propagation constants laying in the continuous part of the spectrum, where usually only delocalized linear waves exist, thereby illustrating principally different mechanisms of formation from that of bright solitons. They bifurcate from gain-guided linear vortex modes existing only for a specific set of gain amplitudes, thereby allowing observation of spatial localization over broad range of gain amplitudes due to defocusing nonlinearity. Because in our system stabilization of vortex solitons occurs with increase of gain amplitude, they appear as remarkably robust states that can be stable even for high topological charges, in contrast to vortices in previously considered conservative and dissipative systems, where high-charge solutions are typically unstable.

We consider the propagation of light beams along the z axis in a bulk medium with defocusing cubic nonlinearity and linear ringlike gain landscape $\mathcal{I}(x,y)$, in the presence of background linear losses characterized by the parameter α:

$$i\frac{\partial\psi}{\partial z}=-\frac{1}{2}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)-i[\alpha-\mathcal{I}(x,y)]\psi+|\psi|^2\psi, \quad (1)$$

where the coordinates x, y are normalized to the characteristic scale r_0, the propagation distance z is normalized to the diffraction length kr_0^2, $k=2\pi n_\mathrm{r}/\lambda$, intensity is normalized such that $I=n_\mathrm{r}|\psi|^2/k^2r_0^2|n_2|$, where n_2 is the nonlinear coefficient. The ring-shaped gain landscape is described by the function $\mathcal{I}(x,y)=\nu e^{-(|\mathbf{r}|-r_\mathrm{c})^2/d^2}$, where ν is the gain amplitude, r_c is the radius of the amplifying ring, d is its width, and $\mathbf{r}=(x,y)$. Spatial localization of gain ensures stability of the background at $r\to\infty$. The amplitude of gain or losses $|\alpha-\nu|\sim k^2r_0^2n_\mathrm{i}/n_\mathrm{r}$ is determined by small imaginary part n_i of the refractive index $n=n_\mathrm{r}+in_\mathrm{i}$, where $n_\mathrm{i}\ll n_\mathrm{r}$. Defocusing cubic nonlinearity is representative for semiconductors, such as AlGaAs or CdS, for photon energies above $0.7E_\mathrm{bandgap}$ [70–73]. Thus for CdS at $\lambda=0.61$ μm, where $n_2\approx-10^{-17}$ m^2/W and $n_\mathrm{r}\approx 2.5$, and for $r_0=10$ μm one gets diffraction length 2.58 mm, the amplitude of gain or losses $\alpha,\nu\sim0.4$ corresponds to 1.55 cm^{-1} (consistent with reported absorption coefficients) while $|\psi|^2=1$ corresponds to $I\sim 3.8\times10^{12}$ W/m^2. Different approaches to control of gain in semiconductors have been suggested, based, e.g., on electrical or optical pumping or creation of inhomogeneous concentrations of dopants [74]. Even though in this spectral range nonlinear absorption also comes into play, it should not affect our solitons at low powers. Effective defocusing cubic nonlinearity can also be produced by cascaded quadratic processes [75]. Models similar to Eq. (1) also supporting vortex solitons may arise in microcavity systems with a ringlike pump (see Refs. [69,76] and Supplemental Material [77], which includes Refs. [78–81]). Further we set $r_\mathrm{c}=5.25$ and $d=1.75$. Radially symmetric vortex soliton solutions of Eq. (1) can be obtained in the form $\psi=(w_\mathrm{r}+iw_\mathrm{i})e^{ibz+im\phi}$, where b is the real-valued propagation constant, m is the topological charge, $w_\mathrm{r,i}(r)$ are the real and imaginary parts of the field. The functions $w_\mathrm{r,i}$ satisfy the system

$$bw_{\mathrm{r},i}=\frac{1}{2}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{m^2}{r^2}\right)w_{\mathrm{r},i} + (|w_r|^2+|w_i|^2)w_{r,i}\mp[\alpha-I(r)]w_{\mathrm{i},r}. \quad (2)$$

In this dissipative system the propagation constant b is not an independent parameter and it depends on gain or loss amplitude α,ν. To find the profiles $w_\mathrm{r,i}$ and corresponding b values we used Newton method complemented with an energy flow balance condition that should hold for stationary states:

$$\frac{dU}{dz}=-2\pi\int[\alpha-\mathcal{I}(r)]|\psi|^2rdr=0, \quad (3)$$

where $U=2\pi\int|\psi|^2rdr$ is the energy flow of the vortex soliton. Constraint (3) produces an additional to (2) equation that is needed in the Newton method for definition of propagation constant b (see Ref. [77]). Solitons in this system are possible at $\nu>\alpha$, when gain inside the ring becomes sufficiently strong to overcome background losses. Increase of the field amplitude due to amplification within the ring leads to growing defocusing nonlinearity that expels light from the amplifying ring into a domain with losses, where energy is absorbed. As a result, a stable energy balance is possible even without nonlinear absorption, allowing us to obtain a rich variety of vortex solitons. Because of the mechanism of their formation, they are characterized by radial energy currents (besides azimuthal ones associated with vortical phase structure). It should be stressed that vortex solitons are stable only in ring-shaped gain landscapes while in bell-shaped landscapes they are unstable and usually transform into fundamental solitons.

Typical dependencies of the energy flow U on gain amplitude ν for vortex solitons with different charges m are presented in Fig. 1 at various values of the background

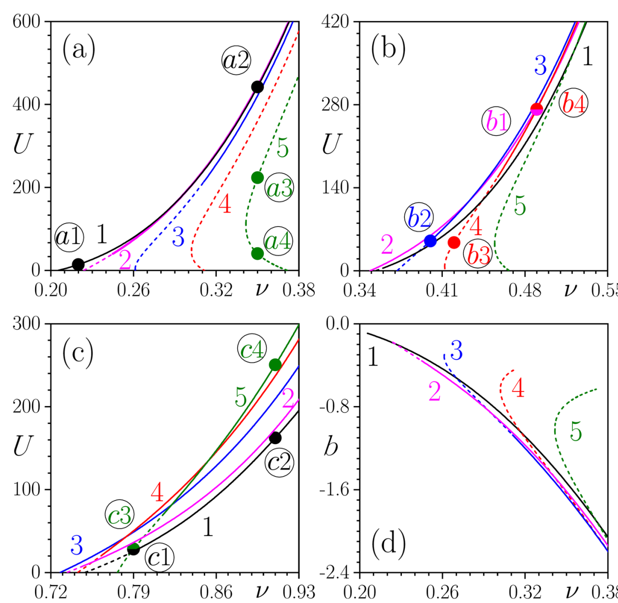

FIG. 1. Energy flow U vs gain amplitude ν at $\alpha=0.1$ (a), $\alpha=0.2$ (b), and $\alpha=0.5$ (c) for vortex solitons with charges $m=1,2,\ldots5$ indicated next to the curves. (d) Propagation constant b versus ν at $\alpha=0.1$. Stable (unstable) branches are shown with solid (dashed) lines. Colored circles and encircled labels correspond to solitons shown in Fig. 2.

losses α. Solid lines correspond to stable solitons while dashed lines correspond to unstable ones. Vortex solitons emerge when gain amplitude ν exceeds certain minimal value depending on topological charge m. While at small losses $\alpha = 0.1$ vortices with larger topological charges require larger gain levels for their appearance [Fig. 1(a)], the order of appearance of vortex solitons may change with increase of α, so that $m = 2$ [Fig. 1(b)] or $m = 3$ [Fig. 1(c)] vortices may acquire the lowest thresholds in ν. This order is determined by the overlap of the field of the vortex with charge m with gain landscape that determines its amplification efficiency. For a given m the threshold value of ν for appearance of the soliton increases with increase of losses α. Remarkably, with decrease of ν the energy flow of the vortex soliton vanishes exactly in the point where the imaginary part λ_i of the complex eigenvalue $\lambda = \lambda_r + i\lambda_i$ of linear eigenmode $\psi = (w_r + iw_i)e^{i\lambda z + im\phi}$ with topological charge m supported by the gain or loss landscape $i[\alpha - \mathcal{I}(r)]$ becomes zero. Such modes are obtained from Eq. (2) with an omitted nonlinear term that in this case transforms into a linear eigenproblem. Thus, exactly at this value of ν the corresponding linear vortex mode of the $i[\alpha - \mathcal{I}(r)]$ landscape evolves without net gain or attenuation. This means that vortex solitons bifurcate from linear gain-guided vortex modes. Notice that while the concept of gain guiding is well established [82], it was not applied to vortex states. The properties of gain-guided linear vortex modes are summarized in Fig. S1 of [77].

Representative dependencies of propagation constants b of vortex solitons with various topological charges on gain amplitude ν are shown in Fig. 1(d). These dependencies start in the points where soliton's propagation constant b is equal to the eigenvalue $\lambda = \lambda_r$ of the associated gain-guided mode (that is purely real for this ν). Propagation constants of solitons are negative due to defocusing nonlinearity. The dependence $U(\nu)$ may be two-valued, i.e., two different solutions can coexist for the same ν. This usually happens for high topological charges [see $m = 4$, 5 branches in Fig. 1(a)]. With increase of α the dependencies $U(\nu)$ become single-valued for higher and higher m [Fig. 1(c)].

Representative profiles of dissipative vortex solitons corresponding to the dots near encircled labels in Figs. 1(a)–1(c) are presented in Fig. 2 in panels with the same labels. At fixed α with increase of ν vortex solitons expand in the radial direction far beyond the amplifying ring [compare solitons with $m = 1$ in Figs. 2(a1) and 2(a2) at $\alpha = 0.1$ or $m = 5$ solitons in Figs. 2(c3) and 2(c4) at $\alpha = 0.5$]. The soliton's width is minimal in the bifurcation point from the gain-guided mode. This expansion is accompanied by an increase of the soliton's amplitude. For high background losses $\alpha \sim 0.5$ the solitons with low charges m may have unusual field modulus distributions $u = (w_r^2 + w_i^2)^{1/2}$ with local radial minima [Figs. 2(c1) and 2(c2)]. The presence of radial currents outwards amplifying ring is obvious from oscillations of the tails of the real and imaginary parts $w_{r,i}$ at $r \to \infty$. In the parameter range where the dependence $U(\nu)$ is two valued, the solitons from lower and upper branches at fixed ν differ in peak amplitudes and in radial oscillation frequencies of tails of $w_{r,i}$ while widths of the field modulus distributions u may be close [Figs. 2(a3) and 2(a4)]. Increasing topological charge m at fixed α, ν results in increase of the radius of the

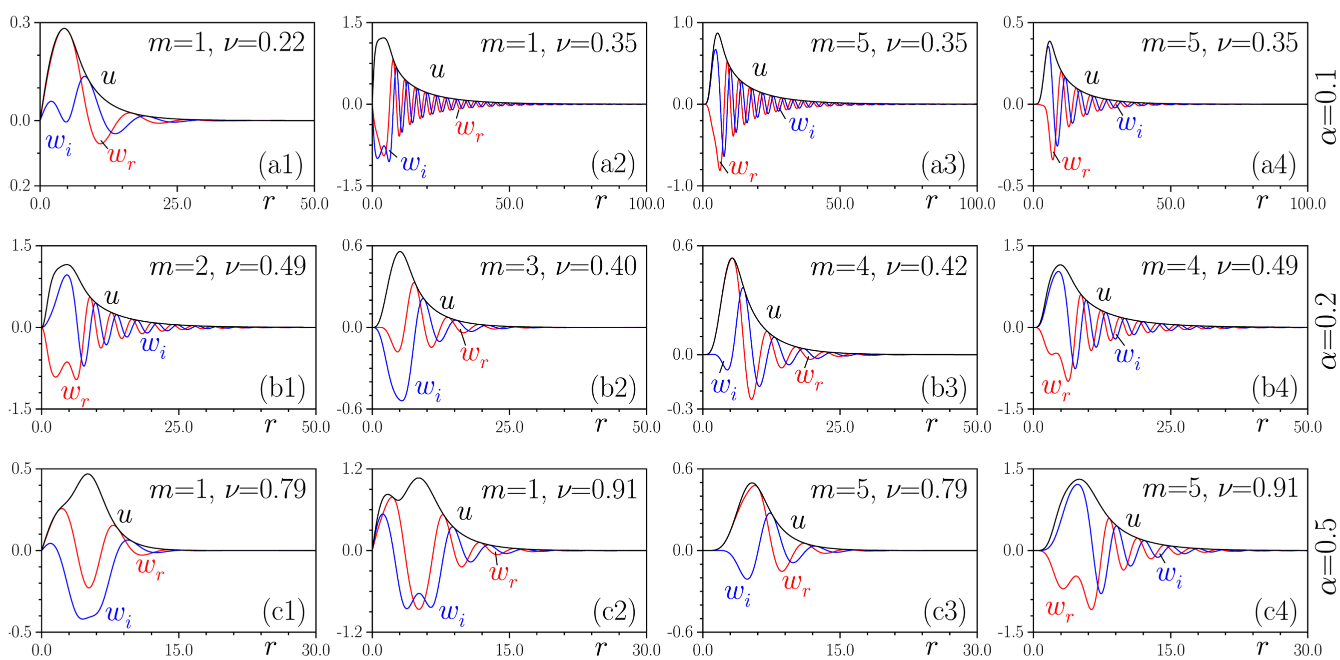

FIG. 2. Vortex solitons with $m = 1$ (a1), (a2) and $m = 5$ (a3), (a4) at $\alpha = 0.1$; $m = 2$ (b1), $m = 3$ (b2), and $m = 4$ (b3),(b4) at $\alpha = 0.2$; $m = 1$ (c1),(c2) and $m = 5$ (c3),(c4) at $\alpha = 0.5$, corresponding to labels in Figs. 1(a)–1(c).

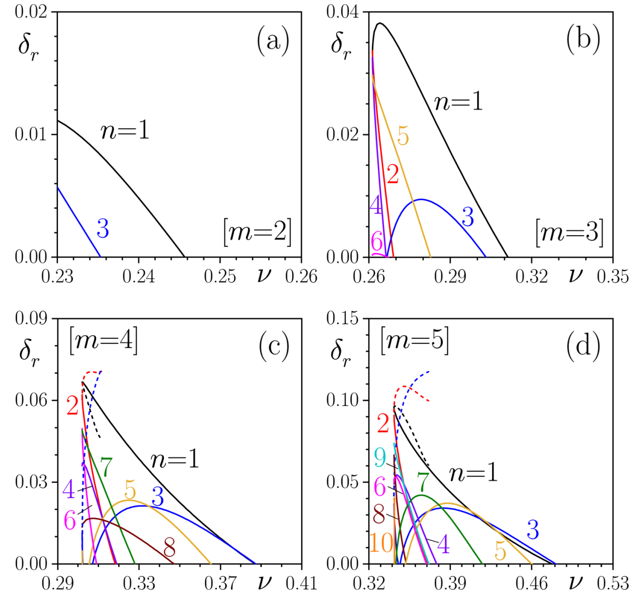

FIG. 3. Real part δ_r of perturbation growth rate for different azimuthal perturbation indices n versus gain amplitude ν at $\alpha = 0.1$ for solitons with $m = 2$ (a), 3 (b), 4 (c), and 5 (d). Dashed curves in (c),(d) correspond to lower branches of the two-valued $U(\nu)$ curves.

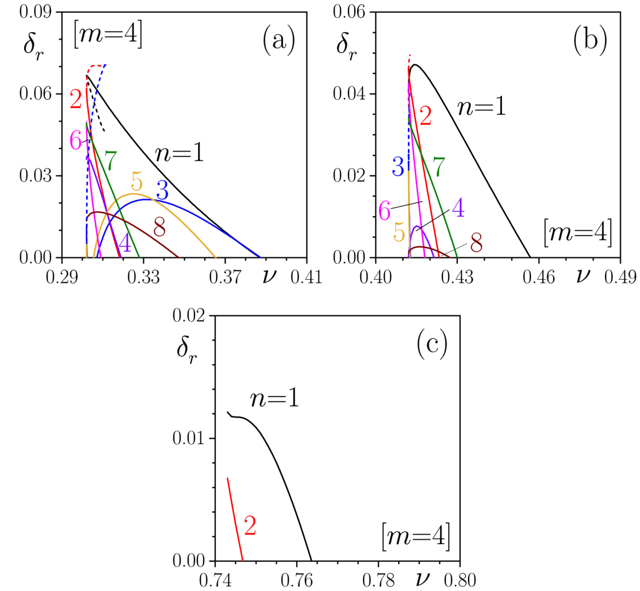

FIG. 4. δ_r vs ν dependencies for different azimuthal indices n and vortex soliton with $m = 4$ at different values of the background losses $\alpha = 0.1$ (a), $\alpha = 0.2$ (b), and $\alpha = 0.5$ (c).

vortex-ring [compare states in Figs. 2(b1) and 2(b4) or states in Figs. 2(c2) and 2(c4)]. Higher-order vortex solitons with radial nodes where field modulus u vanishes, were found too (not shown), but all such states are unstable. Qualitatively similar results were obtained for other ring-like $\mathcal{I}(r)$ profiles, for example with steplike gain variation (see Fig. S2 in [77]).

The central result of this Letter is that in this simple system, where collapse is absent and azimuthal modulational instabilities are suppressed because material is defocusing, dissipative vortex solitons can be stable even for large topological charges. To illustrate this, we have performed linear stability analysis searching for perturbed states of the form $\psi(r,\phi,z) = (w_\mathrm{r} + iw_\mathrm{i} + fe^{\delta z + in\phi} + g^* e^{\delta^* z - in\phi})e^{ibz + im\phi}$, where $\delta = \delta_\mathrm{r} + i\delta_\mathrm{i}$ is the perturbation growth rate, $f(r)$ and $g(r)$ are the radial profiles of perturbation modes, n is the azimuthal perturbation index, and asterisks stand for complex conjugation. The substitution of this ansatz in Eq. (1) and its linearization around $w_\mathrm{r} + iw_\mathrm{i}$ yields the linear eigenvalue problem for δ (see Ref. [77]), that was solved numerically. Vortex soliton with a given m is stable if $\delta_\mathrm{r} \leq 0$ for all n and is unstable otherwise.

The results of stability analysis are summarized in Fig. 1 with vortex soliton families $U(\nu)$ where stable (unstable) families within depicted range of ν values are plotted with solid (dashed) curves. Representative dependencies of the real part of perturbation growth rate on gain amplitude ν for different topological charges m or different losses α are shown in Figs. 3 and 4, respectively. We show $\delta_\mathrm{r}(\nu)$ dependencies only for azimuthal indices n that can lead to instability within depicted range of ν values. When instability is present, it usually occurs for low-amplitude solitons near bifurcation point from gain-guided modes [when the dependence $U(\nu)$ is two-valued, its lower branch is usually always unstable, see dashed curves in Figs. 3 and 4 corresponding to such branches]. One of the unusual properties of this system is that at fixed α vortex solitons typically become stable with increase of gain amplitude ν, since growth rates for all n tend to vanish after certain maximal value of ν. Higher-charge vortex solitons usually require larger gain amplitudes ν for stabilization (see Fig. 3 that shows that the width of the instability domain broadens with increase of m), but this picture may change if vortex states with higher charge bifurcate at smaller values of ν in comparison with $m = 1$ states, in which case the former families can be completely stable (Fig. 1). Increasing background losses result in suppression of instabilities associated with large azimuthal perturbation indices n (Fig. 4) leading to narrowing of the instability domain for solitons with sufficiently high topological charges. Thus, at $\alpha = 0.9$ even $m = 10$ soliton can be stable close to the bifurcation point from linear gain-guided state.

The existence of stable dissipative vortex solitons was confirmed by direct simulations of their evolution dynamics in the frames of Eq. (1). While stable perturbed vortex solitons do not show any appreciable distortions over huge propagation distances and their peak amplitude max $|\psi|$ (defined over the entire transverse plane) remains nearly constant indicating on the absence of unstable perturbations [see dynamics of perturbed stable $m = 4$ soliton in Fig. 5(c)], the unstable solitons instead of decaying may

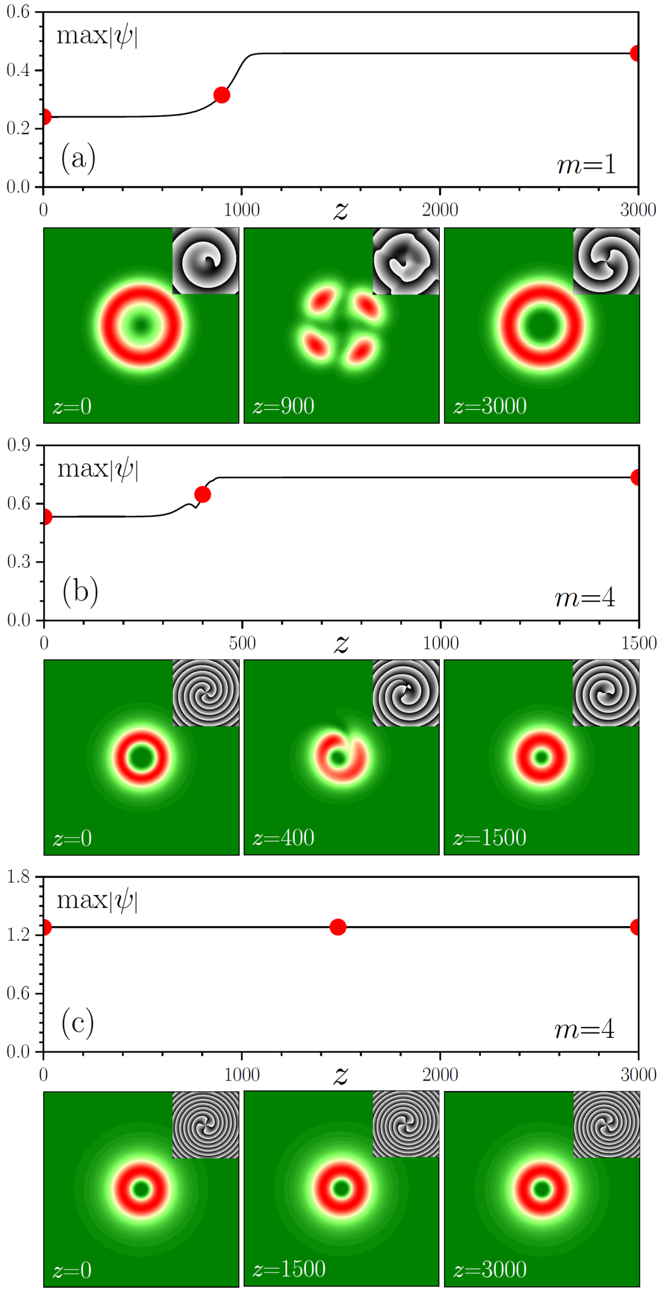

FIG. 5. Evolution dynamics of unstable vortex solitons with (a) $m = 1$ at $\alpha = 0.5$, $\nu = 0.76$ and (b) $m = 4$ at $\alpha = 0.2$, $\nu = 0.42$. Stable evolution of vortex soliton with $m = 4$ at $\alpha = 0.2$, $\nu = 0.51$ (c). Peak amplitude $\max|\psi|$ versus distance z is shown along with field modulus $|\psi|$ and phase $\arg(\psi)$ (insets) distributions at different distances. $(x, y) \in [-15, 15]$ in panel (a), $(x, y) \in [-25, 25]$ in panels (b) and (c).

transform into other stable vortex states not only with lower, but also with higher topological charges. This usually happens when higher-charge states have lower threshold in ν and are stable while lower-charge states appearing at larger ν values are unstable [see $m = 3$ and $m = 1$ branches in Fig. 1(c)]. The example of transformation of unstable $m = 1$ vortex into stable $m = 3$ state is shown in Fig. 5(a). Another scenario of instability development accompanied by the reduction of topological charge from $m = 4$ to $m = 2$ is illustrated in Fig. 5(b). In all cases, the vortex solitons emerging as a result of instability development are stable.

In conclusion, we proposed a simple model allowing to obtain remarkably robust dissipative vortex solitons with high topological charges in structured gain or loss landscape combined with defocusing cubic nonlinearity. Being stable attractors of the system such vortex-carrying states, can be easily excited from noisy inputs [77]. Our results pave the way to experimental realization of stable higher-charge vortex solitons in dissipative physical systems, including polaritonic ones, nonlinear structured microcavities, and various laser systems [77]. Vortices are important for practical applications connected with communications, where they can serve as carriers of information encoded in the magnitude or sign of topological charge [83], digital spiral imaging [84], particle manipulation and high-resolution imaging [85,86], and for development of vortex lasers [87]. Robustness of high-charge vortex solitons illustrated in our system and the possibility of transformation between states with different charges may be beneficial for such applications.

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[*]Corresponding author: chunyanli@xidian.edu.cn

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