Editors' Suggestion

Pseudomagic Quantum States

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Notions of nonstabilizerness, or "magic," quantify how nonclassical quantum states are in a precise sense: states exhibiting low nonstabilizerness preclude quantum advantage. We introduce "pseudomagic" ensembles of quantum states that, despite low nonstabilizerness, are computationally indistinguishable from those with high nonstabilizerness. Previously, such computational indistinguishability has been studied with respect to entanglement, introducing the concept of pseudoentanglement. However, we demonstrate that pseudomagic neither follows from pseudoentanglement nor implies it. In terms of applications, the study of pseudomagic offers fresh insights into the theory of quantum scrambling: it uncovers states that, even though they originate from nonscrambling unitaries, remain indistinguishable from scrambled states to any physical observer. Additional applications include new lower bounds on state synthesis problems, property testing protocols, and implications for quantum cryptography. Our Letter is driven by the observation that only quantities measurable by a *computationally bounded observer*—intrinsically limited by finite-time computational constraints—hold physical significance. Ultimately, our findings suggest that nonstabilizerness is a "hide-able" characteristic of quantum states: some states are much more magical than is apparent to a computationally bounded observer.

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The boundary between quantum and classical computation is a central question in current research, with a focus on identifying uniquely quantum resources that contribute to a quantum advantage. One such resource is nonstabilizerness ("magic"), which is a measure of the non-Clifford resources needed to prepare a quantum state [1–3]. Nonstabilizerness is directly connected to the hardness of classically simulating a quantum state [4–12], the yield of magic state distillation protocols [1,13–22], the overhead required for fault-tolerant quantum computation [23–26], and the degree of chaos in a system [27–30]. Given these connections, one might expect that quantum states with high nonstabilizerness are inherently different, and more nonclassical, than states with low values.

In this Letter, we challenge this intuition by constructing ensembles of states that are poor in magic resources but, nonetheless, are *computationally indistinguishable* from an ensemble of states that are rich in magic resources. Because they masquerade as highly magical ensembles, we call the former "pseudomagic" ensembles. Moreover, their nonstabilizerness can also be *tuned*: for any value of nonstabilizerness strictly greater than $\log(n)$ and up to n, there is a pseudomagic ensemble with that amount of nonstabilizerness. While the "pseudoentangled" ensembles introduced in Ref. [31] happen to also display the above pseudomagic properties, we prove that the amount of each resource in them (i.e., entanglement and magic) can be *tuned independently*: the existence of pseudoentangled ensembles does not imply their pseudomagic counterparts, nor vice versa. While we quantify nonstabilizerness in the rest of this Letter with the measure of stabilizer Rényi entropy [27], we explain how to generalize our construction to many other popular magic measures, such as robustness of magic [13], stabilizer fidelity and extent [8], and max relative entropy of magic [32].

As physically motivated applications, we discuss the implication of pseudomagic for quantum scrambling. Our results imply, counterintuitively, that some states generated by nonscrambling unitaries are computationally indistinguishable from states generated by scrambling unitaries. Furthermore, we show that the existence of pseudomagic states immediately implies the existence of a quantum cryptographic primitive known as EFI pairs [33]. Finally, we employ our findings to obtain lower bounds for black-box magic state distillation and property testing protocols.

The computationally bounded observer.—The central claim of this Letter is that an idea from computer science—namely, computational indistinguishability—has significant ramifications for the ways in which we understand physics. Say we have two n-particle systems S_1 and S_2 that differ in some physical characteristic C, and we aim to distinguish between the two systems. What if the time needed for *any* distinguishing method is on par with the age of the universe? If so, we then have two systems which purportedly differ in some physical attribute C, yet nevertheless can never be distinguished in any reasonable amount of time—so in what sense can we say C is a genuine physical attribute? In the language of computer science, a distinguishing algorithm is efficient if its requisite *computational time* (i.e., number of elementary operations) scales polynomially with the number of particles, denoted by $\mathrm{poly}(n)$, and inefficient if it scales exponentially as $\exp(n)$. This (fuzzy) distinction delineates between a feasible distinguishing scheme and an impractical one. Indeed, if every possible distinguishing algorithm scales exponentially with the number of particles n, we say the two systems S_1 and S_2 are "computationally indistinguishable," as for even a modest system size $n \sim 120$, an exponential distinguisher's runtime will surpass the age of the Universe $\sim 10^{18}$ s, impossible to execute in practice. This idea motivates us to introduce the notion of the *computationally bounded observer* (CBO), an observer constrained to measurement schemes that operate within polynomial time. The results presented in this Letter demonstrate that the (many-body) physics observed by a CBO differs profoundly compared to an unconstrained observer. This introduces a unique perspective into quantum many-body systems, transforming the laboratory from a mere verifier of quantum theories into an integral part of the theory itself, where the limitations of observers assume a central role.

Pseudomagic.—First, we review some useful definitions associated with nonstabilizerness. Let $\mathbb{P}_n$ be the Pauli group on n qubits, $\mathcal{C}_n$ be the Clifford group, and let Σ be the set of pure stabilizer states. A stabilizer operation $\mathcal{S}$ is a quantum channel obeying $\mathcal{S}(\Sigma) = \Sigma$, which is to say that $\mathcal{S}$ preserves the set of stabilizer states [11]. There are many ways to quantify nonstabilizerness [8,13,27,32,34–43], but we limit our attention to the *magic measures* introduced in Table I [44]. The second key concept in this Letter is the notion of computational indistinguishability for two ensembles of states, which means that no CBO (i.e., polynomially bounded algorithm) can tell the difference between the two ensembles. We refer the reader to Ref. [57] for a detailed treatment of this concept and its relevance for quantum cryptography. Having established the two notions of magic and computational indistinguishability, we now introduce pseudomagic.

TABLE I. Magic measures and their definitions.

Magic measure	Definition
Stabilizer entropy [27]	$M_\alpha(\psi) = [1/(1-\alpha)] \log(1/d) \sum_P \mathrm{tr}^{2\alpha}(P\psi)$
Robustness of magic [13]	$\mathcal{R}(\psi) = \log \min\{\|c\|_1 \vert \psi = \sum_i c_i \vert\sigma_i\rangle\langle\sigma_i\vert\}$
Stabilizer fidelity [8]	$\mathcal{F}_{\mathrm{stab}}(\psi) = -\log \max_{\sigma\in\Sigma} \vert\langle\psi\vert\sigma\rangle\vert^2$
Stabilizer extent [8]	$\xi(\psi) = \log \min(\sum_\phi \vert c_\phi\vert)^2$
Max-relative entropy [32]	$D_{\max}(\psi) = \log \min\{\lambda \vert \lambda\sigma - \psi \geq 0\}$

Definition 1.—(Pseudomagic) Let $\mathcal{M}$ be a magic measure. A pseudomagic pair with gap $f(n)$ vs $g(n)$ [where $f(n) > g(n)$] consists of two state ensembles: (a) a "high magic" ensemble of n-qubit quantum states $\{|\psi_{k_1}\rangle\}$ such that $\mathcal{M}(\psi_{k_1}) = f(n)$ with high probability over k_1, and (b) a "low magic" ensemble of n-qubit quantum states $\{|\phi_{k_2}\rangle\}$ such that $\mathcal{M}(\phi_{k_2}) = g(n)$ with high probability over k_2,such that the two ensembles are computationally indistinguishable, even when given polynomially many copies.

Qualitatively, states from the ensemble $\{|\phi_{k_2}\rangle\}$ mimic much more "magical" states to all CBOs, even though they themselves are low magic. In the rest of this Letter we use stabilizer Rényi entropy as our magic measure, given by

$$M_\alpha(\psi) = \frac{1}{1-\alpha} \log \frac{1}{2^n} \sum_{P\in\mathbb{P}_n} \mathrm{tr}^{2\alpha}(P\psi), \tag{1}$$

which is a magic monotone for $\alpha \geq 2$ [58]; see Ref. [44] for a detailed discussion [59]. Combining the computational indistinguishability property with the properties of stabilizer Rényi entropy, we show that $g(n)$, the nonstabilizerness of the low-magic ensemble, can be no smaller than $\omega(\log n)$ [60]:

Lemma 1.—(Bound to stabilizer entropies) Let $\mathcal{E}$ be the low magic ensemble of a pseudomagic pair. Then the α-Rényi stabilizer entropies obey $M_\alpha(\psi) = \omega(\log n)$, with high probability over the choice of $|\psi\rangle \in \mathcal{E}$.

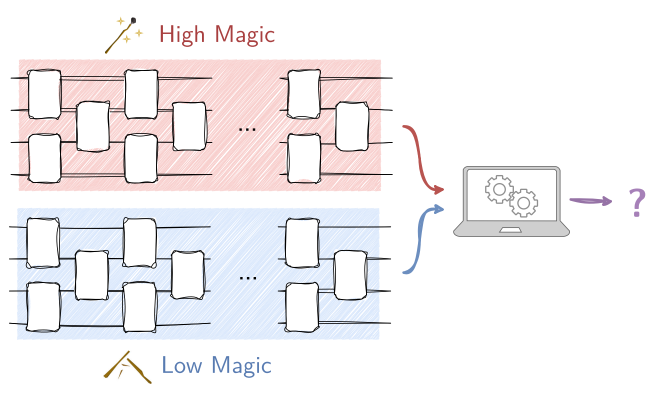

FIG. 1. The pseudomagic state ensembles discussed in this Letter are computationally indistinguishable from highly magical states produced by scrambling dynamics (e.g., Haar random states).

In the next section, we turn to the construction of pseudomagic pair with maximal gap, i.e., $\omega(\log n)$ vs $O(n)$.

Construction of pseudomagic ensembles.—For any function $f\colon \{0,1\}^n \to \{0,1\}$ and subset $S \subseteq \{0,1\}^n$, we define the associated subset phase state as [31]

$$|\psi_{f,S}\rangle = \frac{1}{\sqrt{|S|}}\sum_{x\in S}(-1)^{f(x)}|x\rangle. \tag{2}$$

In Ref. [31], it was shown the ensemble $\mathcal{E} = \{|\psi_{f,S}\rangle\}$ of subset phase states (for pseudorandom functions f and subsets S) is both efficiently preparable, and computationally indistinguishable from Haar random states. In this section, we show that $\mathcal{E}$ also saturates the nonstabilizerness lower bound in Lemma 1. Since the ensemble of Haar-random states $\mathcal{E}_{\text{Haar}}$ have stabilizer entropy $\Theta(n)$ with overwhelming probability [44], $(\mathcal{E}, \mathcal{E}_{\text{Haar}})$ form a pseudomagic pair with gap $\omega(\log n)$ vs $O(n)$. We then show that applying a polynomial depth quantum circuit U to $\mathcal{E}$ results in a high-magic ensemble $\mathcal{E}_U$ that, thanks to the transitivity of computational indistinguishability, is computationally indistinguishable from the low-magic ensemble $\mathcal{E}$, thus forming another pseudomagic pair $(\mathcal{E}, \mathcal{E}_U)$ (see Fig. 2).

Theorem 1.—(Subset phase states display pseudomagic) For any $k \in [\omega(\log n), n]$, there exists an ensemble $\mathcal{E} = \{|\psi_{f,S}\rangle\}$ for $|S| = 2^k$ that has $M_\alpha(\psi_{f,S}) = O(k)$. For $\alpha \le 2$, this bound is tight: $M_\alpha(\psi_{f,S}) = \Theta(k)$. Furthermore, (a) $\mathcal{E}$ is computationally indistinguishable the ensemble of Haar random states $\mathcal{E}_{\text{Haar}}$. Therefore, $(\mathcal{E}, \mathcal{E}_{\text{Haar}})$ forms a pseudomagic pair with gap $\omega(\log n)$ vs $O(n)$. (b) There exists a quantum circuit U composed solely of single-qubit gates (i.e., it has depth 1) such that $(\mathcal{E}, \mathcal{E}_U)$, where $\mathcal{E}_U = \{U|\psi_{f,S}\rangle\}$, also forms a pseudomagic pair with maximal gap $\omega(\log n)$ vs $O(n)$.

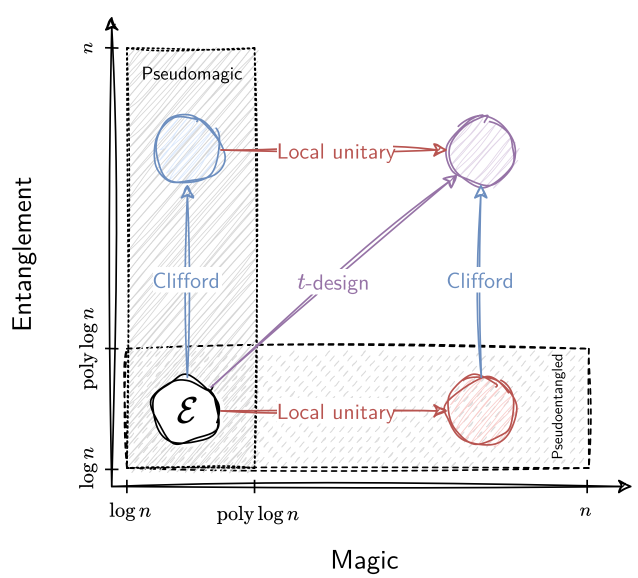

FIG. 2. The pseudorandom ensemble of states $\mathcal{E}$ (subset phase states) we consider in this Letter exhibit both pseudomagic and pseudoentanglement. However, we show that we can *independently* tune the entanglement and nonstabilizerness of these pseudorandom states via local and Clifford unitaries, respectively.

It is natural to ask if, besides the stabilizer Rényi entropy, other magic measures (see Table I) can be used to define pseudomagic. In [44], we show that both the α-stabilizer entropies $M_\alpha(\psi)$ and the log-robustness of magic $\mathcal{R}(\psi)$ are measures of pseudomagic with gap $\Theta(\text{poly}\log n)$ vs $\Theta(n)$. As a corollary, we have a sufficient condition for a magic measure to be a good measure for defining pseudomagic. For any magic monotone $\mathcal{M}$, if there exists an α such that $\Omega[M_\alpha(\psi)] \le \mathcal{M}(\psi) \le O[\mathcal{R}(\psi)]$ for pure states ψ, then $\mathcal{M}$ is also a measure of pseudomagic with gap $\Theta(n)$ vs $\Theta(\text{poly}\log n)$. In [44], we use this fact to show that the magic monotones $\mathcal{M} = \mathcal{F}_{\text{stab}}(\psi), \xi(\psi), D_{\max}(\psi)$, as defined in Table I, are also measures of pseudomagic with gap $\Theta(\text{poly}\log n)$ vs $\Theta(n)$. Notably, these measures serve as genuine magic monotones even in the context of mixed-state magic resource theory.

Implications to quantum scrambling.—Having defined the notion of pseudomagic, we now use it to address some important aspects of quantum information scrambling, and explore how CBOs challenge its foundational principles. One way of defining a scrambling unitary evolution U is to say that it is scrambling if it attains the Haar value for its $2k$-point *out-of-time-order correlators* (OTOCs) [61,62]. These OTOCs are denoted by C_{2k} and are defined as

$$C_{2k}(U) \coloneqq \frac{1}{d}\text{tr}\left(\tilde{P}_1 Q_1 \tilde{P}_2 Q_2 \cdots \tilde{P}_k Q_k\right), \tag{3}$$

where P_i and Q_i are nonidentity Pauli operators for $i = 1, \ldots, k$ and $\tilde{P}_i \coloneqq U^\dagger P_i U$. To be precise, U is scrambling if $C_{2k}(U) = \tilde{O}[C_{2k}(U_{\text{Haar}})]$, where $\tilde{O}$ stands for an irrelevant polynomial overhead. Indeed, typically $C_{2k}(U_{\text{Haar}}) = O[\exp(-\gamma n)]$ with γ depending on the particular choice of the correlator C_{2k} [61].

Magic resource theory is linked to quantum information scrambling [63–65]. Clifford unitaries can be scramblers, but they scramble information in a relatively simple way [66]. Any unitary exhibiting complex information scrambling for its OTOCs must contain $\Omega(n)$ non-Clifford gates [67,68]. Information scrambled by a unitary evolution with less than n non-Clifford gates can be *unscrambled* and reconstructed [66,69]. Consequently, the mere existence of pseudomagic states that are also pseudorandom (i.e., subset phase states), suggests the existence of noncomplex scramblers that nonetheless generate states indistinguishable from states generated by maximal scramblers. More precisely, we establish that such states must be generated by a unitary evolution that exhibits exponentially separated OTOCs from the typical Haar value.

Theorem 2.—(Hidden quantum scrambling) Let $\mathcal{E}$ be an ensemble of pseudomagic states that is also pseudorandom.

Let $|\psi\rangle \in \mathcal{E}$ and let U such that $|\psi\rangle = U|0\rangle^{\otimes n}$. The $2k$-point OTOCs of U (for $k \geq 4$) are exponentially separated from the Haar value,

$$C_{2k}(U) = \Omega\left[\exp(n) C_{2k}(U_{\mathrm{Haar}})\right]. \tag{4}$$

Therefore, although it generates a state that is on-average computationally indistinguishable from Haar random, U is not fully scrambling.

The proof of this can be found in [44]. The curious implication of Theorem 2 is this: any physical observer, inherently constrained by computational limits (i.e., a CBO), cannot differentiate between a maximally scrambling evolution and nonscrambling one solely based on the observed resultant state (see Fig. 1). As a result, a CBO interprets quantum scrambling evolutions differently from an unrestricted observer, exposing a significant conceptual challenge in characterizing quantum information scrambling. Additionally, our findings could potentially also lead to implications for the theory of quantum chaos. Indeed, by defining quantum chaotic evolution as maximally scrambling unitary operators, then Theorem 2 would imply the impossibility for a CBO to distinguish between a chaotic quantum evolution from a nonchaotic one. However, while some authors [61] are inclined to think about quantum chaotic evolution precisely as maximal scramblers [61], this concept has been challenged in Ref. [70], where it has been proven that scrambling (when restricted to $k = 2$) does not imply chaos. The question of whether maximal scrambling, as probed by higher order OTOCS, implies chaos remains an exciting venue for future research.

Implications to quantum cryptography.— An essential primitive for classical cryptography is the concept of a *one-way function* (OWF), which is a function that is efficient to evaluate but hard to invert. However, the story is much different in the *quantum* world: OWFs are unnecessary for some quantum cryptographic constructions to hold [71,72]. This leads naturally to a question of whether there is an indespensible primitive for quantum cryptography that serves a similar role to OWFs for classical cryptography. In Ref. [33], the authors introduce EFI pairs as this quantum analog and show that it is necessary for many secure quantum cryptographic schemes, including bit commitment [73,74], oblivious transfer [75,76], multiparty quantum computation [77], and zero knowledge proofs [78]. EFI pairs are state ensembles generated by efficient circuits that are statistically far but computationally indistinguishable. In light of the proposed significance of EFIs, we show the following.

Theorem 3.—(Cryptographic implications) Consider an ensemble of efficiently preparable pseudomagic states that have stabilizer entropy $M_1 = \Theta[g(n)]$ with high probability, where $g(n)$ is tunable in the range $\omega(\log n)$ and $O(n)$. Then, the pseudomagic ensemble, along with the high nonstabilizerness ensemble, forms an EFI pair.

For a proof, see Ref. [44]. Crucially, Theorem 3 holds even in a world without quantum-secure OWFs: it says that the bare existence of pseudomagic states with tunable stabilizer entropy implies the existence of EFI pairs and the world of cryptographic applications they unlock. This strengthens the case that EFI pairs are a more fundamental primitive for quantum cryptography than OWFs.

No efficient black-box magic-state distillation.—Several architectures for universal fault-tolerant quantum computing rely on applying stabilizer operations to carefully prepare resource states called magic states [25,79–81]. An example is the canonical magic state vector $|T\rangle = |0\rangle + e^{i\pi/4}|1\rangle$, which when provided as an input to auxiliary qubits, enables T-gate implementation using only stabilizer operations. However, not all nonstabilizer states are useful for implementing non-Clifford gates [82], especially noisy ones. This motivates the question, can we develop efficient (i.e., polynomial-sized circuit description) stabilizer protocols that can transform generic nonstabilizer states ρ into useful nonstabilizer states, such as $|T\rangle$? In line with the spirit of analogous tasks for entanglement resource theory [83,84], we term this task *black-box magic-state distillation*.

Theorem 4.—(Black-box magic state distillation) Given a magic monotone $\mathcal{M}$ such that $\Omega[M_\alpha(\psi)] \leq \mathcal{M}(\psi) \leq O[\mathcal{R}(\psi)]$ for all pure states ψ, any efficient stabilizer protocol that synthesizes a state vector $|B\rangle\langle B|$ from an arbitrary (potentially mixed) input state ρ requires

$$\Omega\left(\frac{\mathcal{M}(|B\rangle\langle B|)}{\log^{1+c}\mathcal{M}(\rho)}\right) \tag{5}$$

copies of ρ, for any constant $c > 0$. Remarkably the above is valid for $\mathcal{M} = \mathcal{F}_{\mathrm{stab}}, \xi, D_{\max}$ in Table I.

The proof of this can be found in [44]. This theorem shows how pseudomagic provides a complementary perspective to magic resource theory in determining the limits of magic state distillation protocols. If we have a target magic state $|B\rangle\langle B|$ and a generic input state ρ, naive lower bounds from resource theory say that we require $\Omega(\mathcal{M}(|B\rangle\langle B|)/\mathcal{M}(\rho))$ copies of ρ to synthesize $|B\rangle\langle B|$. This assumes that we can freely convert from nonstabilizerness in the input state to nonstabilizerness in the output state. However, once we take into account the computational efficiency of our synthesization protocol, Theorem 4 intuitively means that if we do not know what the input state is, the "value" of the nonstabilizerness in the input state is reduced logarithmically. For instance, assume we have a generic resource state vector $|\psi\rangle$ with $\mathcal{M}(\psi) = O(n)$. Naive bounds tell us that we can distill at most $r = O(n)$ canonical magic states $|T\rangle$, since $\mathcal{M}(|T\rangle\langle T|^{\otimes r}) \propto r$. However, Theorem 4 imposes a *much stricter* bound; if our synthesis protocol is efficient, it can synthesize at most $r = O(\log^{1+c} n)$ copies of $|T\rangle$.

Independence of pseudomagic and pseudoentanglement: Tighter distillation bounds.—The astute reader may notice that pseudomagic states are the *same* states that display pseudoentanglement [31]. Given the long history of resource theories centered on these two quantities, it is natural to wonder if the existence of pseudomagical ensembles is in some way implied by the existence of pseudoentangled ensembles. We answer this question in the negative by showing that the entanglement and magic of a pseudorandom ensemble can be independently tuned. These findings suggest that neither pseudomagic nor pseudoentanglement is a generic feature of pseudorandom ensembles.

Theorem 5.—(Pseudomagic and pseudoentanglement are independent properties) Given any two functions $f(n), g(n) \in [\omega(\log n), O(n)]$, there exists a pseudorandom ensemble whose states have entanglement $\Theta[f(n)]$ and magic $\Theta[g(n)]$ up to negligible failure probability. Moreover, there exists a pseudomagic pair with maximal gap and fixed entanglement $\Theta[f(n)]$. Conversely, there exists a pseudoentangled pair with maximal gap and fixed magic $\Theta[g(n)]$ (for any magic measure in Table I).

We study the implications of this theorem in the context of distillation protocols. Theorem 4 and Proposition 3.1 of Ref. [31] are similar in spirit: they demonstrate that resource-theoretic distillability limits for EPR pairs or magic states are a gross overestimate of what is achievable by any computationally efficient algorithm *that is agnostic to its input state*. However, many magic state or entanglement distillation protocols are handcrafted to work on particular *classes* of input states [1,85]. Do our lower bounds still hold up, then, if we knew something about the state we started with? For example, Ref. [86] asks whether one can distill magic from highly entangled states. We use Theorem 5 to show that prior knowledge of the nonstabilizerness of input states does not lead to increased efficiency in entanglement distillation.

Theorem 6.—(Prior knowledge of magic does not help entanglement distillation) Consider an entanglement distillation protocol that distills EPR pairs from states drawn from an ensemble $\{\psi_k\}$. Even if we are guaranteed that the states ψ_k have magic $\Theta[g(n)]$ for $g(n) \in [\omega(\log n), O(n)]$ with high probability, the protocol can distill at most $O[\log^{1+c} S(\rho)]$ Bell pairs with high probability, where $S(\rho)$ is the von Neumann entanglement entropy of an input state ρ and $c > 0$.

An identical strengthening of the black-box magic state distillation protocol in Theorem 4 is also possible, using a similar proof technique (see the Supplemental Material [44]). Our results say that for states with superlogarithmic entanglement (i.e., beyond MPS), magic distillation must be highly inefficient. Beyond strengthening distillation results, Theorem 5 allows us to significantly strengthen a wide array of no-go results for any entanglement (nonstabilizerness) manipulation or detection task to cases where we even have *a priori* knowledge about states' nonstabilizerness (entanglement).

Conclusions and outlook.—In this Letter, we introduced the concept of pseudomagic states, providing an insightful expansion of the magic resource theory. We first established the theoretical foundation for pseudomagic states. The core of this framework is the stabilizer entropy, which is unique among various magic measures (see Table I) as it does not require an impractical minimization procedure to be evaluated either theoretically or experimentally, and lower bounds all the other magic measures. These features allowed us to demonstrate the existence of pseudomagic ensembles with a large magic gap $\log n$ vs n, with respect to all the genuine magic monotones in Table I. We refer to the Supplemental Material for a technical discussion regarding the role of the stabilizer entropy in pseudomagic [44].

We investigated the implications of pseudomagic for quantum scramblers, quantum cryptography, and magic state distillation. We proved the existence of states, preparable with a nonscrambling unitary, that appear to have been generated by a scrambling one, concluding that a computationally bound observer cannot determine whether a process is scrambling solely by the generated states. More general questions concerning the existence of *pseudorandom unitaries* [87] and its consequence for quantum chaos theory will be the subject of future research. Another exciting avenue to explore is the implication that pseudomagic has for many-body physics, particularly in light of the recent studies of magic-state resource theory [88–95] and quantum complexity in many-body systems [96]. Finally, our exploration of computationally bounded observers has revealed the subtleties of quantum phenomena under computational constraints. This perspective, guided by computational limitations, deepens our understanding of quantum systems and underscores the necessity of considering computational constraints in quantum research, opening avenues for future investigations at the intersection of quantum information and computation.

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[59] Moreover, the stabilizer entropy possesses two desirable traits: it is experimentally measurable, and serves as a lower bound for all other magic monotones listed in Table I. These features enable us to demonstrate the existence of states with a significant and precise pseudomagic gap of $\log n$ versus n. See Supplemental Material [44] for an in-depth discussion about the role of M_α in defining pseudomagic.
[60] To remind the reader, $f = o[g(n)] \Rightarrow \lim f(n)/g(n) = 0$, $f = \omega[g(n)] \Rightarrow \lim f(n)/g(n) = \infty$, and finally $f = \Theta[g(n)] \Rightarrow \exists C_1, C_2 > 0$, such that $C_1 \leq \lim f(n)/g(n) \leq C_2$.
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