Observation of Diverse Asymmetric Structures in High-Dimensional Einstein-Podolsky-Rosen Steering

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Einstein-Podolsky-Rosen (EPR) steering, a distinctive quantum correlation, reveals a unique and inherent asymmetry. This research delves into the multifaceted asymmetry of EPR steering within high-dimensional quantum systems, exploring both theoretical frameworks and experimental validations. We introduce the concept of genuine high-dimensional one-way steering, wherein a high Schmidt number of bipartite quantum states is demonstrable in one steering direction but not reciprocally. Additionally, we explore two criteria to certify the lower and upper bounds of the Schmidt number within a one-sided device-independent context. These criteria serve as tools for identifying potential asymmetric dimensionality of EPR steering in both directions. By preparing two-qutrit mixed states with high fidelity, we experimentally observe asymmetric structures of EPR steering in the $\mathbb{C}^3 \otimes \mathbb{C}^3$ Hilbert space. Our Letter offers new perspectives to understand the asymmetric EPR steering beyond qubits and has potential applications in asymmetric high-dimensional quantum information tasks.

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Introduction.—Einstein-Podolsky-Rosen (EPR) steering, an intermediate quantum correlation situated between entanglement [1] and Bell nonlocality [2], describes the ability of one observer to affect the state of another remote observer via local measurements [3–7]. More importantly, its directional property indicates an extraordinary asymmetry, which further leads to so-called one-way steering (1WS), that is, steering could be possible in a single direction but is impossible in the reverse direction [8]. The effect of 1WS has been experimentally demonstrated in both continuous-variable systems [9] and discrete-variable systems [10–14] and also been recognized as a remarkable resource for one-sided device-independent (1SDI) quantum information processing [15–22].

Previous experimental works on one-way steering focused on two-qubit or qubit-qutrit systems. Employing high-dimensional (HD) quantum systems features increased channel capacity and significantly improved robustness to noise and loss beyond qubits [23]. Recently, HD steering has been observed in experiments [24–27]. Meanwhile, developing reliable tools to characterize EPR steering of HD quantum systems is a central challenge. Considerable interest has been seen in quantifying HD steering using the Schmidt number [28] in theories and experiments [29–32]. The notion of genuine high-dimensional steering (GHDS) was proposed to introduce a dimensional quantification of steering, as quantified by Schmidt number [29]. In a practical quantum network, the asymmetric channels are very likely to exist due to differences of sites and users. A maximally asymmetrical steering was theoretically studied by constructing states in an asymmetric dimension $d \times (d+1)$ [33]. That is, Bob cannot steer the state of Alice, while Alice can strongly steer Bob's state by demonstrating GHDS. Meanwhile, the asymmetry of steerability could exist even in the standard two-way steering [11,34]. Intuitively, rich asymmetric structures of HD steering could be characterized by quantifying the Schmidt number in both steering directions, which still lacks a full description. Considering complex HD quantum systems, characterizing various asymmetric steering therein is of fundamental interest and has potential applications in quantum communications with asymmetric channels and settings [35–38].

In this Letter, we theoretically formalize and experimentally observe the diverse asymmetric structures of HD steering. To do this, we first propose a notion of genuine high-dimensional one-way steering (GHD-1WS) to showcase the novel forms of asymmetric HD steering. By exploring the monotone of consistent steering robustness (CSR) [39] and the connection between GHDS and simulability of HD measurements [40,41], we develop two criteria to sufficiently certify the lower and upper bounds of the Schmidt number in the 1SDI setting. Our approach would expose the possible asymmetry of steering dimensionality in two directions, even for standard two-way steering. With the restriction of finite projective measurements, a class of two-qutrit mixed states is constructed to illustrate how the GHD-1WS manifests in the $\mathbb{C}^3 \otimes \mathbb{C}^3$ Hilbert space. By generating these states encoded in the photonic polarization and path degrees of freedom, we experimentally observe asymmetric structures of genuine three-dimensional 1WS.

Preliminaries.—Considering a bipartite state ρ_{AB}, Alice implements k-setting local measurements $\{M_{a|x}\}_a$ on her subsystem (labeled by $x = 1, \ldots, k$) and receives a result $a \in \{0, \ldots, d-1\}$, where $\{M_{a|x}\}_{a,x}$ denotes a set of positive operators $M_{a|x} \geq 0$ and satisfies $\sum_a M_{a|x} = \mathbb{I}$ for each x. Correspondingly, Bob's subsystem is reduced in a conditional (unnormalized) state

$$\sigma_{a|x} = \mathrm{Tr}_A[(M_{a|x} \otimes \mathbb{I}_B)\rho_{AB}]. \tag{1}$$

The set $\{\sigma_{a|x}\}_{a,x}$ is an assemblage and features standard steering from Alice to Bob if it admits no local hidden state (LHS) model [3].

We recall the concept of GHDS [29], which provides a dimensional measure of steering by Schmidt number. The Schmidt number of a bipartite mixed state ρ is the minimum n such that it can be decomposed as $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, where all pure states $|\psi_i\rangle$ have a Schmidt rank of at most n. Formally, that is $SN(\rho) \coloneqq \min_{\{p_i, |\psi_i\rangle\}} \max_{\{i\}} SR(|\psi_i\rangle)$, where the minimization is ergodic over all possible pure-state decompositions $\{p_i, |\psi_i\rangle\}$ of $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$.

Hence, an assemblage $\{\sigma_{a|x}\}$ is termed n-preparable when it is prepared by local measurements $\{M_{a|x}\}_{a,x}$ on any bipartite state ρ of Schmidt number at most n. We denote the convex set of all n-preparable assemblages as $\mathbb{A}_n$. In this framework, $\mathbb{A}_1$ simply means the collection of LHS assemblages. Any assemblage is defined to exhibit genuine n-dimensional steering when it is not $(n-1)$-preparable but n-preparable, denoted as $d_{AB} = n$ from Alice to Bob for convenience.

Genuine high-dimensional one-way steering.—Recent works have explored GHDS on the symmetric steerable states, namely, $d_{AB} = d_{BA}$ [29–32]. Building upon the GHDS concept, we introduce the notion of GHD-1WS, which characterizes asymmetric steering in scenarios where the dimension of steering would be different when the roles of Alice and Bob are exchanged. A bipartite state features genuine n-dimensional 1WS from Alice to Bob, when (i) Alice can strongly steer Bob and prepare an n-steerable assemblage for Bob via local operations, that is, $d_{AB} \geq n$; (ii) but, in turn, Bob could generate only assemblages exhibiting at most $(n-1)$-steerable for Alice via local operations, $d_{BA} \leq n-1$. More formally, this indicates

$$\{\sigma^B_{a|x}\} \notin \mathbb{A}_{n-1}; \qquad \{\sigma^A_{b|y}\} \in \mathbb{A}_{n-1}. \tag{2}$$

Thus, GHD-1WS is defined in the sense that GHDS can be demonstrated in a single steering direction but not reciprocally. Notably, there exist $(n-1)$ forms in genuine n-dimensional 1WS. In this way, only genuine two-dimensional 1WS can be characterized by conventional methods [42]. As depicted in Fig. 1, the GHD-1WS concept would enable the formalization of complete asymmetric steering structures within HD systems.

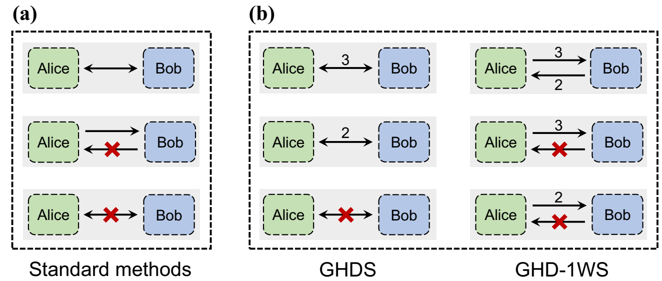

FIG. 1. Structures of steering in two-qutrit states. (a) Three standard forms of steerability can be observed in terms of the violation of the LHS model, among which the middle one is asymmetric. (b) The diverse steering structures are revealed by introducing the concepts of GHDS and GHD-1WS. The numbers above the arrows denote the dimension quantification of steerability. The symmetric forms in the left column correspond to GHDS [29]; the asymmetric forms in the right column correspond to GHD-1WS.

A naturally raised question is how to characterize GHD-1WS. First, we use the steering monotone CSR as the quantifier. Standard steering robustness $\mathcal{S}_\mathrm{R}(\{\sigma_{a|x}\})$ quantifies the minimal amount t of arbitrary noise $\{\tau_{a|x}\}$ that can be added to the assemblage $\{\sigma_{a|x}\}$, such that the mixture $(\sigma_{a|x} + t\tau_{a|x})/(1+t)$ defines an LHS assemblage [43]. The notion of CSR is defined by a slight modification that demands the noise assemblage $\{\tau_{a|x}\}$ has the same reduced state, that is, $\sum_a \tau_{a|x} = \sum_a \sigma_{a|x} \ \forall \ x$ [39]. Thus, CSR is expressed as

$$\mathcal{S}_\mathrm{CSR}(\{\sigma_{a|x}\}) = \min_{t,\tau_{a|x}} \left\{ t \geq 0 \middle| \frac{\sigma_{a|x} + t\tau_{a|x}}{1+t} \text{ unsteerable}, \right.$$
$$\left. \text{and} \sum_a \tau_{a|x} = \sum_a \sigma_{a|x}, \ \forall \ x \right\}. \tag{3}$$

The above formulation can be numerically computed via the semidefinite program (SDP) [5]. Exploiting the property of CSR, the standard connection between steering and measurement incompatibility [44], we can obtain the necessary condition that any n-preparable assemblage $\{\sigma_{a|x}\}$ satisfies

$$\mathcal{S}_\mathrm{CSR}(\{\sigma_{a|x}\}) \leq \max_{\{M_{a|x}\}} \mathcal{I}_\mathrm{R}(\{M_{a|x}\}) \leq \beta_{k,n}, \tag{4}$$

where $\beta_{k,n}$ denotes the universal bounds of incompatibility robustness of all k-setting measurements with dimension n [30,45]. With the additional constraint in CSR, one can have that $\mathcal{S}_{\mathrm{CSR}}(\{\sigma_{a|x}\}) \geq \mathcal{S}_{\mathrm{R}}(\{\sigma_{a|x}\})$. Thus, the violation of inequality (4) provides a tighter lower bound of Schmidt number in the 1SDI setting. Moreover, we develop a resource-minimal steering inequality based on the dual objective function of CSR, which does not need the assemblage tomography. That is, $\mathrm{Tr}\sum_{a,x} F_{a|x}\sigma_{a|x} - 1 \leq \beta_{k,n}$, which is completely equivalent to formula (4). This is done by optimizing Bob's measurements $\{F_{a|x}\}$ via the dual program of the original SDP formulation of CSR to find a state-dependent optimal steering witness. A detailed derivation and the values of $\beta_{k,n}$ can be found in Supplemental Material [46].

To determine the upper bound of the dimension of steering, we exploit n-dimensional simulable measurements [40]. A set of measurements $\{M_{a|x}\}_{a,x}$, defined in the $\mathbb{C}^d$ Hilbert space, is considered n-simulable when the statistics of these measurements on any possible quantum state can be recovered via a form of information compression to a lower n-dimensional space. That is,

$$\sum_{\lambda} \mathrm{Tr}[N_{a|x,\lambda}\mathcal{E}_{\lambda}(\rho)] = \mathrm{Tr}(M_{a|x}\rho) \quad \forall\ \rho, \tag{5}$$

where $\{\mathcal{E}_{\lambda}\}_{\lambda}$ is a quantum instrument with each $\mathcal{E}_{\lambda}\colon B(\mathbb{C}^d) \to B(\mathbb{C}^n)$ being a completely positive map with classical output λ and $N_{a|x,\lambda}$ is a set of n-dimensional positive operator-valued measures. Crucially, it has been proven that the n-preparability of an assemblage $\{\sigma_{a|x}\}$ is equivalent to the n-simulability of its steering equivalent observables (SEOs) $E_{a|x} = \rho_B^{-1/2}\sigma_{a|x}\rho_B^{-1/2}$, where $\rho_B \coloneqq \sum_a \sigma_{a|x}$ [41]. Thus, the assemblage can be mapped to the corresponding SEOs, and its preparability can be determined. Using the SDP methods proposed in Ref. [40], one can certify the n-simulability of the steered party's SEOs. The quantum instrument $\{\mathcal{E}_{\lambda}\}_{\lambda}$ can be constructed by choosing one of the k mutually unbiased bases (MUBs) [47] randomly and perform a projection onto C_d^n n-dimensional subspaces. Since not all choices of the bases have been explored, it is a sufficient condition to ensure n-preparability, giving an upper bound of the Schmidt number in the steering scenario.

The problem is computationally feasible when limiting the number of measurements and outputs to be finite. We apply a class of two-qutrit mixed states with restricted projective measurements of three-setting MUBs to showcase GHD-1WS in the $\mathbb{C}^3 \otimes \mathbb{C}^3$ Hilbert space. These states are constructed by introducing asymmetric noise into the partially entangled two-qutrit state:

$$\rho(p,\theta,\phi) = p|\psi(\theta,\phi)\rangle\langle\psi(\theta,\phi)| + (1-p)\frac{\mathbb{I}_A^3}{3} \otimes (|0\rangle\langle 0|)_B, \tag{6}$$

where $p \in [0,1]$ is the mixing parameter, $\theta \in (0, \pi/4]$, $\phi \in (0, \pi/2)$, and $|\psi(\theta,\phi)\rangle = \cos(\phi)|00\rangle + \sin(\theta)\sin(\phi)|11\rangle + \cos(\theta)\sin(\phi)|22\rangle$. When we set $\theta = \pi/4$, the distribution of other two parameters (p, ϕ) for various HD steering structures is depicted in Fig. 2. For all nontrivial states $\rho[p, (\pi/4), \phi]$, Alice can always steer Bob's subsystem, namely, $\{\sigma_{a|x}^B\} \notin \mathbb{A}_1$. The orange, blue, and red curves, obtained by steering inequalities (4), shows the critical value of p given certain ϕ for sufficiently certifying $\{\sigma_{a|x}^B\} \notin \mathbb{A}_2$, $\{\sigma_{b|y}^A\} \notin \mathbb{A}_1$, and $\{\sigma_{b|y}^A\} \notin \mathbb{A}_2$, respectively. The black curve is obtained by the criterion based on 2-simulability, below which one can certify $\{\sigma_{b|y}^A\} \in \mathbb{A}_2$. The uncertainty regarding the presence of either 2-steerable or 3-steerable assemblages in some steering directions arises from the absence of both sufficient and necessary criteria so far. The complete parameter spaces are presented in Supplemental Material [46].

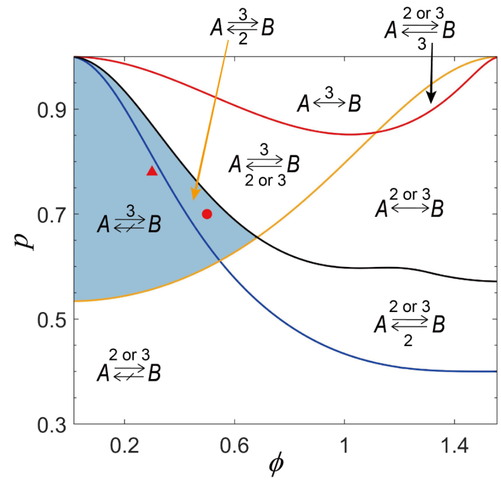

FIG. 2. Diverse HD steering structures are parametrized by (p, ϕ) for the two-qutrit states $\rho(p, \pi/4, \phi)$ in the case of three-measurement settings. The blue area represents the states exhibiting genuine three-dimensional 1WS. It is uncertain that the states feature either 2- or 3-steerable in some areas due to the lack of both sufficient and necessary criteria.

Experimental scheme.—Experimentally, we focus on the genuine three-dimensional 1WS regime, denoted by the blue area in Fig. 2. Two quantum sources were employed to generate three-dimensional pure state $\rho_s = |\psi(\theta,\phi)\rangle\langle\psi(\theta,\phi)|$ with probability p and asymmetric noise $\rho_n = (\mathbb{I}_A^3/3) \otimes |0\rangle\langle 0|_B$ with probability $1-p$.

As shown in Fig. 3, a continuous-wave laser with a wavelength of 775 nm is divided into two paths ("a" and "b") with a 4 mm separation between the horizontally and vertically polarized photons in the beam displacer (BD1) to ensure the phase stability [48]. By pumping the PPKTP1 crystal, the path-polarization entangled state

$$|\Psi(\theta,\phi)\rangle = \cos(\phi)|a_H a_H\rangle + \sin(\phi)\cos(\theta)|a_V a_V\rangle + \sin(\phi)\sin(\theta)|b_H b_H\rangle \tag{7}$$

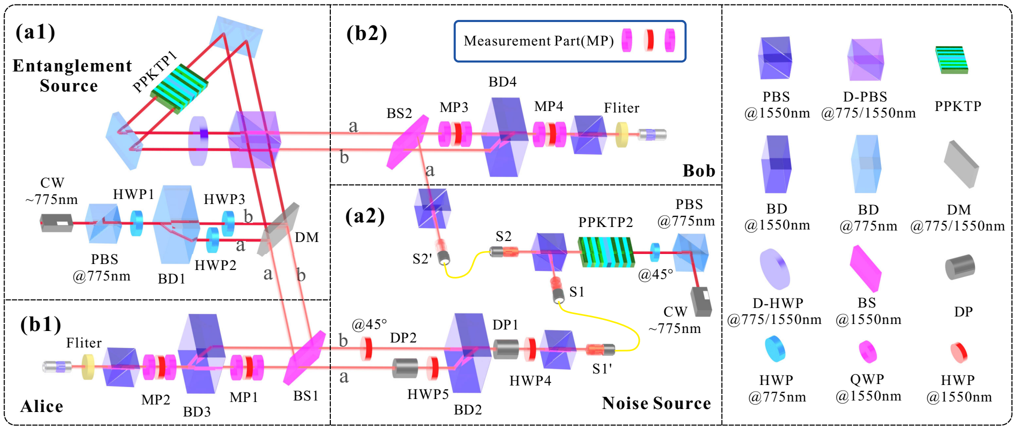

FIG. 3. Experiment setup. (a1) Preparation of the three-dimensional entangled state $|\psi\rangle = \cos(\phi)|00\rangle + \sin(\phi)\cos(\theta)|11\rangle + \sin(\phi)\sin(\theta)|22\rangle$. A continuous-wave laser with wavelength 775 nm pumps the PPKTP1 crystal in the Sagnac structure, resulting in correlated photon pairs with a wavelength of 1550 nm. (a2) Prepare asymmetric noise state $\mathbb{I}_3/3 \otimes |0\rangle\langle 0|$. We prepared in state $(1/\sqrt{3})(|0\rangle + |1\rangle + |2\rangle)$ and inserted YVO_4 to induce decoherence between each subspace. (b1),(b2) Measurement device of Alice and Bob. We combine QHQ (QWP, HWP, QWP) as a measurement part (MP) to achieve arbitrary operations in a two-dimensional subspace. The projection measurement between subspaces $|0\rangle$ and $|1\rangle$ is constructed by MP1 (MP3) and BD3 (BD4), and the projection measurement between subspaces $|0\rangle$, $|1\rangle$, and $|2\rangle$ is constructed by MP2 (MP4) and PBS for Alice (Bob). PBS, polarizing beam splitter. D-PBS, dichroic polarizing beam splitter. BD, beam displacer. DM, dichroic mirror. D-HWP, dichroic half wave plate. DP, decoherent phase birefringent crystal. HWP, half wave plate. QWP, quarter wave plate. BS, beam splitter. PPKTP, periodically poled potassium titanium phosphate.

is obtained with signal and idle photons at 1550 nm, via a type-II phase-matched spontaneous parametric down-conversion process in a Sagnac structure, where $|a(b)_{H(V)}\rangle$ represents horizontal (vertical) polarization on path a (or b). The parameters θ and ϕ can be adjusted by HWP1 and 2. By coding $|a_H\rangle$, $|a_V\rangle$, and $|b_H\rangle$ as $|0\rangle$, $|1\rangle$, and $|2\rangle$, we obtain the pure state. Then, we pump the PPKTP2 crystal with the same laser and collect correlated photon pairs into couplers $S1$ and $S2$. Single photons emitted from coupler $S1'$ are prepared in the state $(1/\sqrt{3})(|0\rangle + |1\rangle + |2\rangle)$ through the expansion of its subspace using HWPs and BD2. Birefringent crystals (DP1 and DP2) are inserted to induce decoherence between the three-dimensional subspaces [49], resulting in the noisy part $\rho_n = \frac{1}{3}(|0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|)_A \otimes |0\rangle\langle 0|_B$. Subsequently, BS1 and BS2 were used to combine ρ_s and ρ_n to complete the preparation of quantum state (6), where the parameter p can be adjusted by changing the intensity of the pump of the two sources.

Alice's and Bob's measurement devices can achieve arbitrary three-dimensional projection measurements by decomposing them into two-dimensional projection measurements [50]. Finally, the measured photons pass through bandpass filters (1550 ± 15 nm) and long pass filters (1200 nm), are gathered in couplers, and are detected by the superconducting detectors.

Experimental results.—The experimental brightness of the target states is approximately 2000 Hz, and the integration time for each measurement is 200 s. We assume that the state preparation during the experimental process is independent and identically distributed [51,52] and investigate the following two cases. The experimental results are shown in Fig. 4.

Case 1 (▲).—It is 3-steerable from Alice to Bob, i.e., $d_{AB} = 3$, while Bob cannot steer Alice, i.e., $d_{BA} = 1$.

We experimentally prepare state ρ_1 with $p = 0.78$, $\theta = 0.3$, and $\phi = \pi/4$. We obtained the density matrix $\rho^{\exp 1}$ of the prepared quantum state by performing tomography [53,54]. To verify d_{AB} by using steering inequality (4), $\rho^{\exp 1}$ was substituted into the SDP program to obtain Bob's measurements $\{F_{a|x}\}_{a,x}^{\exp 1}$, while Alice's measurements are three sets of MUBs [47]. Corresponding measurements are performed on $\rho^{\exp 1}$, and the inequality value obtained was $\mathcal{S}_{\mathrm{CSR}}^{AB\,\exp 1} = 0.3316 \pm 0.0040$, which exceeds the bound $\beta_{3,2} = 0.2679$ with 15.9σ violation. This proves that the lower bound of $d_{AB}^{\exp 1}$ is 3. To verify d_{BA}, Alice performed tomography [53] on her assemblage, while Bob performed a three-setting MUB measurement $\{M_{b|y}\}_{b,y}$ on his part, and the corresponding experimental result was $\{\sigma_{b|y}\}^{\exp 1}$ [46]. Its SEOs $\{E_{b|y}\}^{\exp 1}$ are certified as joint measurability via the SDP proposed in Ref. [44], which indicates that Bob cannot steer Alice, i.e., $d_{BA}^{\exp 1} = 1$.

Case 2 (•).—It is 3-steerable from Alice to Bob, i.e., $d_{AB} = 3$; conversely, $d_{BA} = 2$.

We experimentally prepare state ρ_2 with $p = 0.7$, $\theta = 0.5$, and $\phi = \pi/4$. Similar to case 1, we obtained the density matrix $\rho^{\exp 2}$ experimentally. To verify d_{AB}, we obtain Bob's measurement $\{F_{a|x}\}_{a,x}^{\exp 2}$ and then obtain the

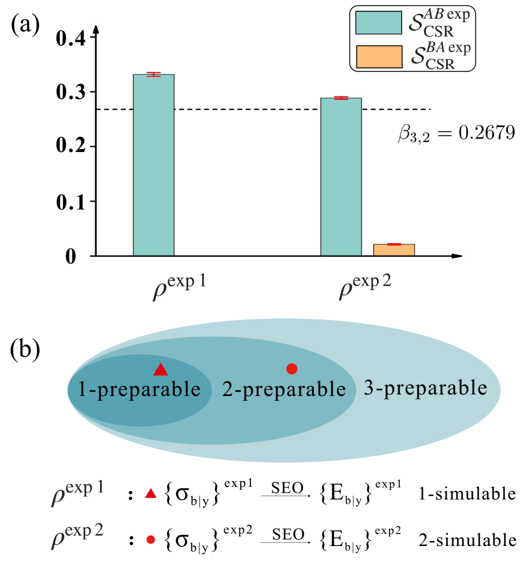

FIG. 4. (a) Experimental values of $\mathcal{S}_{\rm CSR}$. The green (yellow) column represents the obtained $\mathcal{S}_{\rm CSR}^{AB\,\rm exp}$ ($\mathcal{S}_{\rm CSR}^{BA\,\rm exp}$). In terms of three-setting measurements in steering party, the corresponding bounds are $\beta_{3,1} = 0$ for $d = 1$ and $\beta_{3,2} = 0.2679$ for $d = 2$. For three-dimensional quantum states, it can be determined that $d_{AB}^{\rm exp\,1} = 3$ for $\rho^{\rm exp\,1}$ and $d_{AB}^{\rm exp\,2} = 3$, $d_{BA}^{\rm exp\,2} \geq 2$ for $\rho^{\rm exp\,2}$. (b) Determine d_{BA} according to the assemblage $\{\sigma_{b|y}\}$ obtained by Alice. We map Alice's experimental assemblages $\{\sigma_{b|y}\}^{\rm exp\,1}$ (▲) and $\{\sigma_{b|y}\}^{\rm exp\,2}$ (●) to their SEOs $\{E_{b|y}\}^{\rm exp\,1}$ and $\{E_{b|y}\}^{\rm exp\,2}$ and determine their simulability. According to experimental results, we obtained that $\{E_{b|y}\}^{\rm exp\,1(2)}$ is 1(2)-simulable via SDP, which means $\{\sigma_{b|y}\}^{\rm exp\,1(2)}$ is 1(2)-preparable and $d_{BA}^{\rm exp\,1} = 1$ ($d_{BA}^{\rm exp\,2} \leq 2$).

value of the inequality as $\mathcal{S}_{\rm CSR}^{AB\,\rm exp\,2} = 0.2895 \pm 0.0031$, which exceeds the bound $\beta_{3,2} = 0.2679$ with 7.0σ violation. This proves that the lower bound of $d_{AB}^{\rm exp\,2}$ is 3. We employ both criteria simultaneously to establish both the upper and lower bounds for d_{BA}. While Bob preforms three-setting MUB measurement $\{M_{b|y}\}_{b,y}$, we determine the SEOs $\{E_{b|y}\}^{\rm exp\,2}$ of Alice's assemblage $\{\sigma_{b|y}\}^{\rm exp\,2}$ as 2-simulability via the SDP proposed in Ref. [40], which indicates $\{\sigma_{b|y}\}^{\rm exp\,2}$ is 2-preparable and demonstrates that the upper bound of $d_{BA}^{\rm exp\,2}$ is 2 [46]. Next, we measure $\mathcal{S}_{\rm CSR}^{BA}$ to further determine the lower bound of d_{BA}. We obtain Alice's measurements $\{F_{b|y}\}_{b,y}^{\rm exp\,2}$, while Bob's measurements $\{M_{b|y}\}_{b,y}$ are three sets of MUBs. It results in $\mathcal{S}_{\rm CSR}^{BA\,\rm exp\,2} = 0.0215 \pm 0.0020$, which exceeds the bound $\beta_{3,1} = 0$ with 10.7σ violation. This proves that the lower bound of $d_{BA}^{\rm exp\,2}$ is 2. In summary, $d_{BA}^{\rm exp\,2} = 2$.

Hence, we experimentally reveal two asymmetric structures of three-dimensional EPR steering.

Discussion.—We have introduced the concept of GHD-1WS, providing an effective method to explore the intricate structures of asymmetric steering in HD systems. Moreover, we have formulated corresponding criteria and conducted a photonic experiment to prepare two-qutrit mixed states with asymmetric noise, enabling the observation of various asymmetric steering structures.

Compared to theoretical investigations [32,33,42], our Letter showcases the novel structures of HD steering in both theory and experiment. Particularly, we reveal the asymmetry in standard two-way steering within HD systems under the limitation of projective measurements, as illustrated by instances such as $d_{AB} = 3$ and $d_{BA} = 2$. These cases deviate from symmetric steering and standard one-way steering, highlighting the complexity and richness of high-dimensional quantum systems. An interesting question is if the similar cases can be found by applying more general measurements and asymmetric-loss channels. Furthermore, it could be of significant interest in quantum cryptography for realistic quantum networks with asymmetric channels and settings [35–38].

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