Universally Robust Quantum Control

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We study the robustness of the evolution of a quantum system against small uncontrolled variations in parameters in the Hamiltonian. We show that the fidelity susceptibility, which quantifies the perturbative error to leading order, can be expressed in superoperator form and use this to derive control pulses that are robust to any class of systematic unknown errors. The proposed optimal control protocol is equivalent to searching for a sequence of unitaries that mimics the first-order moments of the Haar distribution, i.e., it constitutes a 1-design. We highlight the power of our results for error-resistant single- and two-qubit gates.

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Introduction.—Tremendous advances in the ability to manipulate states of light and matter are ushering in the new generation of quantum-enhanced devices. As recently remarked [1], it is precisely the ability to develop schemes to control a system that endows scientific knowledge with the potential to revolutionize technological landscapes [2,3]. However, while exquisite levels of control are now routinely applied in a variety of platforms [4–6], there will always be systematic errors due to imperfect fabrication and incomplete knowledge of the parameters, either in relation to the model itself or the ambient conditions under which it is operating. Thus, several strategies to explicitly mitigate such errors have been devised, e.g., shortcuts to adiabaticity [7–9], numerical optimization [1,10,11], geometric space curves [12–14], composite pulses [15,16], and dynamical decoupling [17].

When these systematic errors are important, typically the control problem is cast in such a way that two assumptions, sometimes implicit, are made regarding the source of the error: (i) that it arises from a weak perturbation, and (ii) that its mathematical structure is exactly known. While the former is a reasonable working condition to assume (if it were not then the fundamental description of the system would need to be adjusted), the latter is arguably less well justified. Indeed, concerted effort is currently invested in identifying the correct physical description of noisy intermediate-scale quantum devices, e.g., determining the most relevant noise sources that they are subject to in order to enhance their efficacy [18]. Ultimately, there will always be some level of uncertainty in our knowledge of the precise structure of the noise and therefore it is highly desirable to develop a framework that allows the coherent manipulation of quantum systems even in the presence of an unknown (even possibly unknowable) source of error.

In this Letter, we develop such a framework, one that accounts for this uncertainty, termed universally robust control (URC). It provides a straightforward cost function to be minimized to ensure generic robustness in quantum control problems. It can also be easily restricted to specific classes of errors to account for a limited but useful knowledge of the error type.

Fidelity in the presence of systematic error.—Consider the full system Hamiltonian $H_\lambda(t) = H_0(t) + \lambda V$ where $H_0(t)$ is the error-free control Hamiltonian, V is the error operator acting with unknown strength, λ. We assume a pure initial state, σ, with no λ dependence.

The time evolution operator of $H_\lambda(t)$ is given by $U_\lambda(t,0)$, which leads to the λ-dependent state $\rho_\lambda = U_\lambda(t_f,0)\sigma U_\lambda^\dagger(t_f,0)$ at the final time $t = t_f$. The fidelity between the perturbed and ideal evolution is $F(\lambda) = \mathrm{Tr}(\rho_\lambda \rho_0)$, which can be expanded for small λ as

$$F(\lambda) \approx F(0) + F'(0)\lambda + \frac{1}{2}F''(0)\lambda^2. \tag{1}$$

By definition $F(0) = 1$ and from this follows $F'(0) = 0$ [19].

The second derivative can be calculated by noting that, for pure states, $\partial_\lambda^2 \rho_\lambda = 2(\partial_\lambda \rho_\lambda)^2 + \rho_\lambda(\partial_\lambda^2 \rho_\lambda) + (\partial_\lambda^2 \rho_\lambda)\rho_\lambda$. Multiplying by ρ_0 and evaluating the trace at $\lambda = 0$ we get

$$F''(0) = -2\chi_S(\rho_\lambda), \tag{2}$$

where $\chi_S(\rho_\lambda) = \mathrm{Tr}\{\rho_0(\partial_\lambda\rho_\lambda)^2|_{\lambda=0}\}$ is the fidelity susceptibility [20–22], which quantifies how sensitive the evolution is with respect to small perturbations, i.e., $F(\lambda) \simeq 1 - \chi_S(\rho_\lambda)\lambda^2$. It is clear that $\chi_S(\rho_\lambda)$ is simply the quantum Fisher information (QFI) associated to the family of states $\{\rho_\lambda\}$ [19]. The QFI quantifies how much information about λ is encoded in the evolution of the state, thus minimizing the QFI at $\lambda = 0$ is equivalent to increasing the robustness of a control protocol.

Evaluating explicitly the QFI we find [19]

$$\chi_S(\rho_\lambda) = \frac{t_f^2}{\hbar^2}(\Delta\bar{V}_0)^2, \tag{3}$$

where

$$\bar{V}_0 = \frac{1}{t_f}\int_0^{t_f} ds\, U_0^\dagger(s,0)VU_0(s,0) \tag{4}$$

is the time average of V in the interaction picture with respect to the unperturbed evolution and the variance is taken with respect to the initial state, $(\Delta\bar{V}_0)^2 = \mathrm{Tr}[\sigma\bar{V}_0^2] - \mathrm{Tr}[\sigma\bar{V}_0]^2$.

A similar result can be derived for the case of the evolution of unitaries (instead of states). By defining the corresponding fidelity as $F_U(\lambda) = (1/d^2)|\mathrm{Tr}(U_0^\dagger U_\lambda)|^2$, we obtain that $F_U(\lambda) \simeq 1 - \chi_U(U_\lambda)\lambda^2$ [19]. The susceptibility is

$$\chi_U(U_\lambda) = \frac{t_f^2}{\hbar^2 d}\|\bar{V}_0\|^2, \tag{5}$$

where $\|\cdot\|$ is the norm associated with the Hilbert-Schmidt inner product $(A|B) = \mathrm{Tr}(A^\dagger B)$ and d is the Hilbert space dimension. Robust control protocols then correspond to finding a $H_0(t)$ such that $\rho_0 = \rho_{\mathrm{target}}$ or $U_0(t_f,0) = U_{\mathrm{target}}$ while concurrently minimizing χ_S for a known perturbation model V [23–25]. We now demonstrate that such robust control can be achieved even *without* knowledge of V.

Universally robust control.—Our construction is based on a superoperator picture where the operator

$$\mathcal{M}_0[V] \equiv \bar{V}_0 \tag{6}$$

can be seen as the action of a (linear) superoperator $\mathcal{M}_0$ acting on V and we assume that $\mathrm{Tr}V = 0$ [26]. To construct it more explicitly, we go to a doubled Hilbert space. If our original Hilbert space $\mathcal{H}$ is spanned by the orthonormal basis $\{|i\rangle\}$ where $i = 1, \ldots, d$, we take

$$A = \sum_{ij} A_{ij}|i\rangle\langle j| \to |A) = \sum_{ij} A_{ij}|i\rangle \otimes |j\rangle, \tag{7}$$

where $|A)$ lives in $\mathcal{H}\otimes\mathcal{H}$ [19]. From Eq. (6) we define

$$M_0 = \frac{1}{t_f}\int_0^{t_f} ds[U_0(s,0)\otimes U_0(s,0)^*]^\dagger. \tag{8}$$

such that $|\bar{V}_0) = M_0|V)$. The fidelity susceptibility of Eq. (5) can be expressed in terms of the superoperator M_0 as

$$\|\bar{V}_0\|^2 = (V|M_0^\dagger M_0|V). \tag{9}$$

By virtue of Eq. (5) we can increase the robustness of a unitary control protocol irrespective of V by choosing $H_0(t)$ to minimize the operator norm of M_0. Intuitively, this is because $\|M_0|V)\| \leq \|M_0\|\cdot\||V)\|$. This also holds for state control, c.f. Eq. (3), because $\Delta\bar{V}_0$ is upper bounded by $\|M_0\|$ [19].

The trace of any operator V is unitarily invariant. For the identity operator $\mathbb{I}$, $M_0|\mathbb{I}) = |\mathbb{I})$ so the norm of M_0 cannot be arbitrarily reduced. To sidestep this issue, we restrict to the set of traceless perturbation operators by defining the projector in the doubled Hilbert space $\mathbb{P}_0 = |\mathbb{I})(\mathbb{I}|/d$ such that $\mathbb{P}_0|A) = \mathrm{Tr}(A)|\mathbb{I})/d$, and redefine the relevant superoperator

$$\tilde{M}_0 = M_0(\mathbb{I} - \mathbb{P}_0). \tag{10}$$

For any operator V', this acts as

$$\tilde{M}_0|V') = M_0(\mathbb{I} - \mathbb{P}_0)|V') = M_0|V) = |\bar{V}_0), \tag{11}$$

where V is a traceless version of V'. We remark that any observable conserved under H_0 will also be an eigenvector of M_0 with eigenvalue 1, i.e., it cannot be counteracted due to the limited control terms of the Hamiltonian (see, e.g., the discussion in Ref. [27]). Similar limitations to robustness may apply in the case of other experimental constraints such as pulse intensity and bandwidth limits.

The goal of URC is to minimize the norm of the modified superoperator $\tilde{M}_0$, which is related to the previous norm as

$$\|\tilde{M}_0\|^2 = \|M_0\|^2 - \mathrm{Tr}(M_0^\dagger M_0\mathbb{P}_0) = \|M_0\|^2 - 1. \tag{12}$$

This allows us to find choices of U_0 that yield $\tilde{M}_0 \simeq 0$, thus achieving $|\bar{V}_0) \simeq 0$ for any V.

To understand how a single solution for $U_0(t)$ can be made robust to arbitrary perturbations, and when this is possible in principle, we note the following connection with unitary designs [28–30]. Discretizing the integral in Eq. (4) into $L \gg 1$ intervals, we find $\bar{V}_0 \sim (1/L)\sum_{k=1}^L U_0^{(k)\dagger}VU_0^{(k)}$, which has the form of an average of the operator V conjugated over a discrete set of unitaries, $U_0^{(k)}$. If the distribution of such unitaries is uniform according to the Haar measure [31], then the average,

$$\mathbb{E}_{\{U_0^{(k)}\}}[U^\dagger VU] = \frac{1}{d}\mathrm{Tr}(V), \tag{13}$$

is known to vanish for all traceless V [31]. A less stringent requirement is for the distribution to only match the first-order moment of the uniform distribution, i.e., to be a 1-design. In fact, since $\mathbb{P}_0[A] = \mathrm{Tr}(A)[\mathbb{I}]/d$, we see that the requirement $\tilde{M}_0 = 0$ immediately implies Eq. (13) for any operator, thus making the path traced by the unitary evolution operator $U_0(t)$ a 1-design. Given that 1-designs exist in $\mathrm{SU}(d)$ for any d, this connection serves as a formal proof of the existence of URC solutions, i.e., paths in unitary space that achieve perfect target fidelity while being robust to all possible perturbations to leading order [19].

Leveraging randomization to increase robustness in quantum processes is routinely done in the context of quantum computing, particularly by tools like dynamical decoupling [17,32], dynamically corrected gates [13,33,34], and randomized compiling [35]. Our work shows that, for general quantum systems, it is possible to translate this connection into a requirement on a single object, the superoperator $\tilde{M}_0$, leading to robustness to any perturbation to first order. As we show in the following, this allows us to set up a quantum optimal control problem to find evolutions that reach a predefined target while at the same time remain robust to arbitrary perturbations.

Optimal control.—We now demonstrate how URC can be naturally leveraged in numerical optimizations. A generic quantum optimal control (QOC) approach considers a series of control parameters, $\{\phi_k\}$, which determine the time dependence of $H_0(t)$ and aims to maximize the fidelity between a target process U_{target} and the actual (ideal) evolution operator $U_0(t_f, 0)$ by minimizing a cost functional $J_0 = 1 - F_U[U_{\mathrm{target}}, U_0(t_f, 0)]$ with respect to $\{\phi_k\}$. Additionally, robust QOC usually aims at achieving resilience to perturbations characterized by a known operator V. For this task, one can concurrently minimize the fidelity susceptibility given by the control functional $J_V = (1/d)\|\bar{V}_0\|^2$ (see for instance [25,34]). Our proposed approach of universally robust QOC instead aims at achieving robustness to an *unknown* error operator V. This can be achieved by instead minimizing the functional $J_{\mathrm{U}} = (1/d)\|\tilde{M}_0\|^2$ [36].

We begin with the simple case of a single qubit with restricted controls with Hamiltonian

$$H_0(t) = \Omega\big[\cos[\phi(t)]\sigma_x + \sin[\phi(t)]\sigma_y\big], \qquad (14)$$

where σ_α are the Pauli operators, and we consider the control field $\phi(t)$ to be piecewise constant with time steps Δt and values $\{\phi_k\}$, $k = 1, \ldots, N_P$ [37]. The model in Eq. (14) is fully controllable [38,39]. We set the target transformation to be $U_{\mathrm{target}} = \exp(-i\sigma_z\pi/2)$ and numerically seek the QOC parameters that minimize either only $\mathcal{J}_{\mathrm{target}} = J_0$, $\mathcal{J}^{(z)}_{\mathrm{robust}} = (J_0 + wJ_{V=\sigma_z})/(1+w)$, or $\mathcal{J}_{\mathrm{univ}} = (J_0 + wJ_{\mathrm{U}})/(1+w)$, where w is a non-negative weight that can be changed to improve the resulting balance between the terms. Note that evaluating these functionals requires only computing the error-free evolution given by

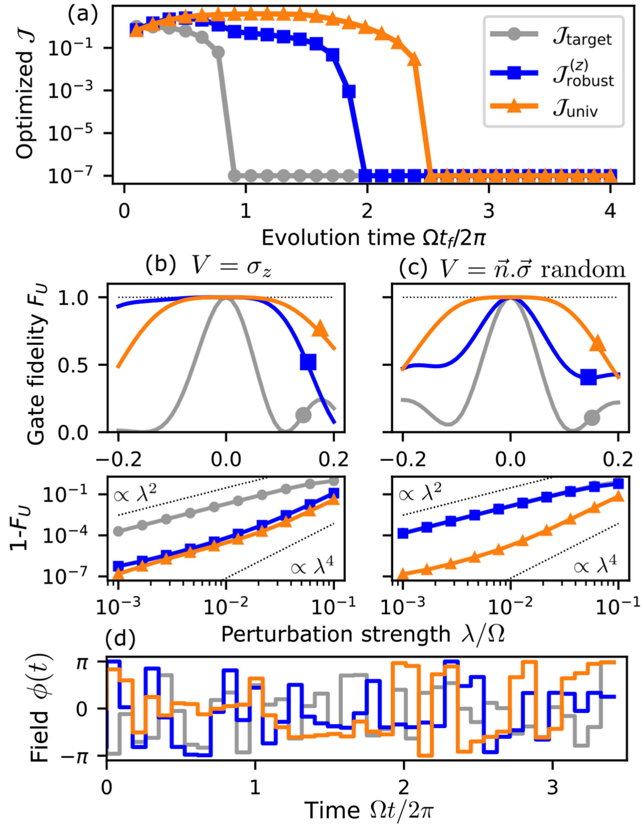

FIG. 1. Universally robust control for single-qubit gates. (a) Optimized control functionals as a function of the total evolution time t_f for target-only control (gray, circles), target and robustness to a known V (blue, squares), and target and robustness to an unknown V (orange, triangles). (b),(c) Gate fidelity as a function of perturbation strength λ for the cases where $V = \sigma_z$ and $V = \vec{n} \cdot \vec{\sigma}$ with $\vec{n}$ a random unit vector (results shown correspond to the average fidelity over 20 realizations). Lower panels shows an enlargement of the data of the infidelity $1 - F$ in log-log scale. (d) Optimal control fields $\phi(t)$ obtained for each case. We choose a target $U_{\mathrm{target}} = \exp(-i\sigma_z\pi/2)$, $N_P = 40$ control parameters, a balanced functional $w = 1$, and an operation time $\Omega t_f/(2\pi) = 3.5$ for (b), (c), and (d).

$H_0(t)$, and so no numerical simulations of the perturbed dynamics are required at any stage. In Fig. 1(a) we plot the optimized functional for each case against the evolution time t_f. The curves display behavior reminiscent of Pareto fronts [40,41], indicative of the fact that optimization succeeds for sufficiently large t_f but fails when the evolution time becomes too constrained. A minimum control time, t_{MCT}, can be assigned to each process by identifying the minimum value of t_f such that the optimization succeeds (which in this case we take as yielding functional values below 10^{-7}). For target-only and robust control optimizations, we find $t^{\mathrm{T}}_{MCT} = 2\pi/\Omega$ and $t^{\mathrm{R}}_{MCT} = 4\pi/\Omega$, which are consistent with previous analytical and numerical studies [38,39]. In contrast, universally robust control demands $t^{\mathrm{U}}_{MCT} = 5\pi/\Omega$ (see also [42]).

To characterize the robustness of these control processes, we study how well the evolution under the perturbed Hamiltonian $H_\lambda(t) = H_0(t) + \lambda V$ is able to achieve the target transformation. Figure 1 shows the cases for (b) $V = \sigma_z$ and (c) $V = \vec{n}\cdot\vec{\sigma}$ with $\vec{n}$ a randomly chosen unit vector. The gate fidelity is plotted against the uncertainty parameter λ for the three types of optimal controls found. All cases yield high fidelities if $\lambda = 0$, but the target-only optimization results (gray) deviate substantially from the ideal value once $\lambda \neq 0$. In (b), we see that the robust control optimization (blue) is insensitive to perturbations in $V = \sigma_z$, as expected. But (c) reveals that the same control is sensitive to generic perturbations. Remarkably, the URC solution (orange) is insensitive to perturbations along *any* direction. This holds true even accounting for the faster minimal control times required for the other protocols [19]. We also highlight that the increase in robustness does not require the use of a more complex control waveform, as can be seen from Fig. 1(d).

Generalized robustness.—Building upon the superoperator in Eq. (10) we can generalize this framework to optimize for robustness to any desired subset of operators. This is particularly relevant for systems beyond a single qubit where the nature of the noise or inhomogeneity is partially known instead of being completely arbitrary. Thus, rather than making a control protocol robust to all possible operators V, we can instead focus on achieving robustness to a particular set of perturbations, for instance, those generated by local operators. In this case, we are interested in the action of the superoperator, M_0, only on this reduced set. The advantage of imposing these generalized robustness requirements is that the optimization is less constrained, as effectively less matrix elements are being minimized. Therefore, it is easier to find good solutions even with restricted control time. For example, the total number of operators for N qubits is 4^N, while for the set of local operators is only $3N$.

Consider a quantum system with Hilbert space dimension, d, and an orthonormal operator basis $\{\Lambda_j\}$, $j = 0, 1, \ldots d^2 - 1$. We introduce a covering of this basis set, $\{C_k\}$, such that $\{\Lambda_j\} = \cup_{k=1}^{K} C_k$. The projector onto C_k is $\mathcal{P}_k(A) = \sum_{\Lambda_j \in C_k} \mathrm{Tr}(\Lambda_j^\dagger A)\Lambda_j$. In the superoperator picture, this is equivalent to defining $\mathbb{P}_k = \sum_{\Lambda_j \in C_k} |\Lambda_j)(\Lambda_j|$. These superoperators are clearly projectors, as $\mathbb{P}_k^2 = \mathbb{P}_k$ and $\sum_{k=0}^{K} \mathbb{P}_k = \mathbb{I}$. By construction, we take $\Lambda_0 = \mathbb{I}/\sqrt{d}$ so that $\mathbb{P}_0$ is defined as before. In order to look for controls that are insensitive to any operator within a given subset, we seek to minimize the norm of

$$\tilde{M}_0 = M_0\left(1 - \sum_{k \in \eta} \mathbb{P}_k\right), \quad (15)$$

where the sum runs over all relevant operator subsets η (typically including Λ_0). Note that $\mathbb{P}_k$ corresponds to the operators to which our system's dynamics need not be robust. To illustrate the procedure of imposing generalized robustness requirements into a QOC problem, consider a model of two-qubits with symmetric controls,

$$H_0(t) = \Omega_x(t) S_x + \Omega_y(t) S_y + \beta S_z^2, \quad (16)$$

where $S_\alpha = (\sigma_\alpha^{(1)} + \sigma_\alpha^{(2)})/2$ are collective spin operators and the interaction strength $\beta > 0$ is fixed. The perturbation operator, V, can be a combination of single-body (C_1) or two-body (C_2) operators. We thus have a variety of possible optimization functionals depending on the level of robustness desired. Here, we compare three cases: robustness to a single $V = S_x$, robustness to all single-body operators ($V = V_{1-\mathrm{body}} \in C_1$) and universal robustness ($V = V_{\mathrm{arb}} \in C_1 \cup C_2$). Here, $V_{1-\mathrm{body}}$ and V_{arb} are chosen randomly within the corresponding subspaces. We set the target as a randomly chosen symmetric two-qubit unitary U_{random} [19]. For this system we find that a good balance between fidelity at zero perturbation and robustness can be achieved by performing a two-stage optimization. First, we minimize the target alone until a certain threshold $J_0 < \varepsilon$ is met. The resulting optimized field is then seeded to the robustness optimization that minimizes J_V or J_U alone, with the added constraint that J_0 never exceeds ε. [43]. In Fig. 2, we showcase the performance of the optimization using the

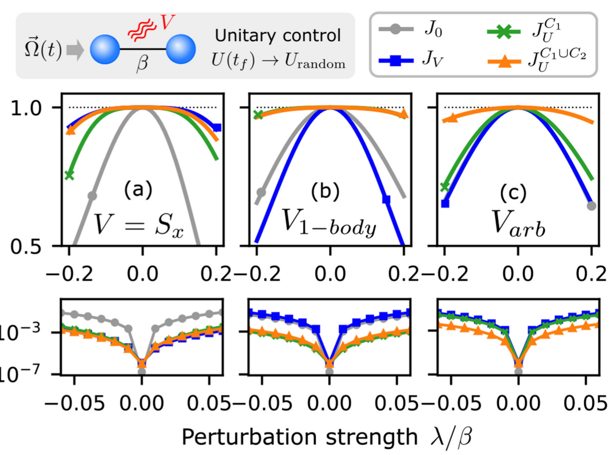

FIG. 2. Universally robust control for two-qubit gates. Plots show the gate fidelity of the perturbed evolution $H_0(t) + \lambda V$, where $H_0(t)$ is the control Hamiltonian of Eq. (16). Different curves correspond to different types of optimization procedures: target only (nonrobust, gray circles), target and robustness to a fixed $V = S_x$ (blue squares), target and robustness to all single-body operators (green crosses), target and universal robustness (orange triangles). The lower row shows the infidelity $1 - F_U$ for each case. Each column shows the performance of each of the four optimized solutions under the evolution $H = H_0(t) + \lambda V$; (a) $V = S_x$, (b) V is random 1-body operator, (c) V is a random arbitrary operator. Evolution time in all cases is $\beta t_f/(2\pi) = 5$, and $N_P = 50$ control parameters are used. Results shown are averages over 20 instances.

different variants introduced thus far, in the presence of various perturbations. As expected, the optimal control procedure is able to find fields that are robust to arbitrary single-body perturbations (green curve), but are not necessarily robust to completely arbitrary perturbations. In contrast, the URC solution (orange curve) results in evolutions that are markedly more robust to any type of perturbation, including two-body operators, when compared to the other methods.

The approach outlined above for designing generalized robustness requirements can be readily carried over to more complex systems. In the Supplemental Material [19] we show additional results that illustrate how this framework can be used to robustly generate entangled states in many-body systems.

Conclusion.—We have introduced a versatile method, universally robust control, to mitigate the effects of unknown sources of error. By recasting the impact of an arbitrary perturbation to the systems in terms of a single object, here captured by the superoperator in Eq. (8), we showed that since this superoperator has no explicit dependence on the precise operator form of the error, it can be efficiently minimized to provide the necessary, highly robust, control pulses. This goes beyond previous approaches [33,44,45] since it provides a unifying framework for achieving universal robustness for *arbitrary* finite-dimensional quantum systems, while concurrently defining a concise methodology to implement numerical optimization to achieve robust controls in practice. We demonstrated the effectiveness of our approach for the realization of single- and two-qubit quantum gates, and have shown that it can be generalized to tackle state control problems or to the case of classical fluctuations [46]. Furthermore, we have demonstrated that the URC formalism can exploit partial information about the source of errors to build arbitrary robustness requirements into the optimal control problem. When combined with powerful numerical optimization techniques, we expect this flexible approach to be able to tackle a broad class of questions in quantum control. For instance exploring the fundamental trade-off between robustness and experimental constraints (such as bandwidth or evolution time), or determining what control resources are required to achieve various levels of robustness in a quantum device. Finally, as our protocol introduces control pulses that dynamically implement 1-designs, this could be generalized to other t-designs that can be readily exploited for quantum computing protocols such as randomized benchmarking [47].

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