Gapless Neutron Superfluidity Can Explain the Late Time Cooling of Transiently Accreting Neutron Stars

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(Received 30 November 2022; revised 9 January 2024; accepted 5 March 2024; published 29 April 2024)

The current interpretation of the observed late time cooling of transiently accreting neutron stars in low-mass x-ray binaries during quiescence requires the suppression of neutron superfluidity in their crust at variance with recent *ab initio* many-body calculations of dense matter. Focusing on the two emblematic sources KS 1731-260 and MXB 1659-29, we show that their thermal evolution can be naturally explained by considering the existence of a neutron superflow driven by the pinning of quantized vortices. Under such circumstances, we find that the neutron superfluid can be in a gapless state in which the specific heat is dramatically increased compared to that in the classical BCS state assumed so far, thus delaying the thermal relaxation of the crust. We perform neutron-star cooling simulations taking into account gapless superfluidity, and we obtain excellent fits to the data, thus reconciling astrophysical observations with microscopic theories. The imprint of gapless superfluidity on other observable phenomena is briefly discussed.

DOI: 10.1103/PhysRevLett.132.181001

Introduction.—Although neutron stars (NSs) are formed in the furnace of gravitational-core collapse supernova explosions with initial temperatures as high as $\sim 10^{11}$–10^{12} K, they cool down very rapidly by releasing neutrinos so that their temperature drops down to $\sim 10^9$ K within a few days [1]. Their extremely dense interior is expected to become cold enough for the occurrence of quantum phase transitions not observed in any other celestial bodies. Similar to electrons in conventional terrestrial superconductors, free neutrons in the crust and possibly in the outer core of a neutron star form $^1S_0$ Cooper pairs, which condense at temperatures below $\sim 10^{10}$ K. Predicted before the discovery of pulsars and only two years after the publication of the Bardeen-Cooper-Schrieffer (BCS) theory [2], neutron superfluidity has since been corroborated by radio-timing observations of sudden spin-ups so called (frequency) glitches in numerous pulsars [3] interpreted as the manifestation of the catastrophic unpinning of neutron quantized vortices [4,5]. However, superfluidity in the crust has been recently challenged by observations of NSs in low-mass x-ray binaries. In these systems, matter is transferred from a low-mass stellar companion to an NS via an accretion disk. The hydrogen-rich material that accumulates on the surface of the NS burns steadily producing a thick helium layer. Once the critical conditions for helium ignition are reached, the overlying envelope is converted into heavier nuclides within seconds. These thermonuclear explosions are observed as x-ray bursts lasting a few tens of seconds and with a recurrence time of hours to days [6]. Less frequent but more energetic are superbursts lasting for a few hours with recurrence times of several years [7], presumably triggered by the unstable carbon burning [8,9].

In most x-ray binaries, accretion is not persistent but occurs sporadically [10]. In particular, soft x-ray transients (SXTs) exhibit active periods of weeks to months separated by quiescent periods of years to decades. So-called “quasipersistent” SXTs remain active for years to decades. As matter accumulates on the NS surface, ashes of x-ray bursts are buried and further processed due to electron captures, neutron captures, and emissions, and possibly pycnonuclear fusions [11] releasing some heat in different parts of the crust. In quasipersistent SXTs, the accretion can last long enough for the crust to be driven out of thermal equilibrium with the core. Over the past two decades, the thermal relaxation of a dozen SXTs has been monitored long enough after their outbursts ($\sim 10^3$–10^4 days) to probe all regions of the crust [12]. The interpretation of the observed decline in the temperature during the first few weeks and months requires some additional heating in the shallow layers of the crust [13] (see, e.g., Ref. [14] for a compilation of the inferred heat and references to proposed sources). The cooling at later times is dictated by the physics of the inner crust and neutron superfluidity [15,16]. Observations of some SXTs, especially KS 1731-260 and MXB 1659-29, can hardly be explained by the standard cooling theory.

KS 1731-260 entered into a quiescent phase in 2001 after having accreted for 12.5 years. Its observation [17] provided the first direct evidence of the thermal relaxation of the NS crust [18]. Monitoring campaigns of this source with Chandra and XMM-Newton satellites confirmed this scenario [13,19–21]. However, later observations [22] revealed that this source had become colder than expected. MXB 1659-29 was monitored in quiescence after an accretion outburst of 2.5 years [20,23–25]. The data were modeled in Ref. [13]. Observations taken 11 years after outburst [26] showed an unexpected drop of luminosity. This could be explained by an increased hydrogen column density N_H on the line of sight due to precession of the accretion disk [26]. Alternatively, these observations suggested that the thermal equilibrium between the crust and the core had not been restored. Based on classical molecular dynamics simulations, it was proposed that the densest layers of the crust have a low thermal conductivity [27]. But quantum molecular dynamics simulations performed later did not support this possibility [28]. The data of both sources were also analyzed in Ref. [29], and the best fits were achieved by artificially suppressing superfluidity in most parts of the crust. In 2015, MXB 1659-29 went back into outburst [30], which lasted 1.7 years [31]. No significant variations of N_H that would confirm a hypothetical precession of the accretion disk were found. In 2016, Merritt *et al.* [32] reported observations of KS 1731-260 14.5 years after outburst and were able to fit the data (with a rather large χ^2) using the small neutron pairing gaps of Ref. [33]. Deibel *et al.* [34] obtained very good fits for both KS 1731-260 and MXB 1659-29 considering that neutrons remain normal in the deep crust and in the outer core based on extrapolations of quantum Monte Carlo calculations [35] (MXB 1659-29 was further studied in Refs. [31,36–38], but the observations reported in Ref. [26] were discarded). However, more recent calculations have ruled out this possibility [39], and results are now consistent with those from other approaches [40–42] (see [43]). Besides, superfluidity in both the crust and the outer core is independently required for the interpretation of pulsar glitches [49–52]. Such phenomena have been detected in accreting NSs as well [53,54].

In this Letter, we show how those apparently contradictory observations can be reconciled by considering the existence of a superflow in accreted NS crusts. In particular, we contemplate the possibility that the superfluid is gapless: The energy spectrum of quasiparticle excitations is continuous, whereas the (complex) order parameter (whose modulus coincides with the pairing gap in the absence of superflow) remains finite. The microscopic theory is presented in Ref. [55]. The absence of a gap translates into a neutron specific heat that is orders of magnitude larger than that predicted by the classical BCS theory. The impact of gapless superfluidity on the late time cooling of SXTs is studied, focusing on the emblematic sources MXB 1659-29 and KS 1731-260. After briefly describing our model, NS cooling simulations are presented and discussed. Finally, we mention other observational phenomena that could confirm the existence of gapless superfluidity in NSs.

NS cooling model.—The thermal evolution of SXTs is followed using the code CRUSTCOOL [56], which solves the heat diffusion equation in the NS crust assuming a constant gravity [13]. This code, based on the accreted-crust model of Ref. [57], was previously employed in Refs. [26,27,34] to analyze the same sources. Shallow heating is accounted for by adjusting the temperature $T_{\rm b}$ at the bottom of the envelope at the column depth of 10^{12} g cm^{-2} (see also Refs. [29,58,59]).

Gusakov and Chugunov [60,61] have recently shown that the diffusion of superfluid neutrons in accreting NS crusts changes the composition and the equation of state. Moreover, the nuclear heating is substantially reduced. We have modified the CRUSTCOOL code accordingly (see [43]). More importantly, we have implemented more realistic microscopic neutron pairing calculations and allowed for gapless superfluidity. In the normal phase at temperatures T much lower than the neutron Fermi temperature, the neutron specific heat is approximately given by (with k_B Boltzmann's constant and $\hbar$ the Planck-Dirac constant)

$$c_N^{(n)}(T) \approx \frac{1}{3}\frac{k_{Fn} m_n^\oplus}{\hbar^2} k_B^2 T, \qquad (1)$$

where $m_n^\oplus$ is the neutron effective mass, which can be approximated by the bare neutron mass m_n [40], and k_{Fn} is the neutron Fermi wave number. In the superfluid phase and in the absence of superflow, which we will refer to as the classical BCS state, the neutron specific heat is exponentially suppressed

$$c_S^{(n)}(T < T_{cn}^{(0)}) = R_{00}^{(\rm BCS)}(T/T_{cn}^{(0)}) c_N^{(n)}(T). \qquad (2)$$

The factor $R_{00}^{(\rm BCS)}(T/T_{cn}^{(0)})$ is given in Ref. [62]. The critical temperature $T_{cn}^{(0)}$ is determined by the order parameter $\Delta_n^{(0)}$ at $T = 0$ through the BCS relation $k_B T_{cn}^{(0)} = \exp(\gamma)\Delta_n^{(0)}/\pi \simeq 0.566\,93\Delta_n^{(0)}$ ($\gamma \simeq 0.577\,22$ being the Euler-Mascheroni constant). In the presence of superflow, the order parameter becomes complex and no longer represents the gap in the quasiparticle energy spectrum [55]. The gradient of its phase ϕ_n defines the superfluid velocity as $\boldsymbol{V_n} = \hbar/(2m_n)\boldsymbol{\nabla}\phi_n$. The effects of the superflow on $c_S^{(n)}$ are governed by some effective neutron superfluid velocity $\mathbb{V}_n$. At densities prevailing in the crust of NSs, $\mathbb{V}_n \approx V_n$ [63]. For $\mathbb{V}_n < \mathbb{V}_{Ln} \approx \Delta_n^{(0)}/(\hbar k_{Fn})$, no quasiparticles are present, and $c_S^{(n)}$ remains exponentially suppressed as in the BCS limit corresponding to $\mathbb{V}_n = 0$. Expressions can be found in Ref. [55]. For $\mathbb{V}_{Ln} \le \mathbb{V}_n \le \mathbb{V}_{cn}^{(0)} \approx \exp(1)\mathbb{V}_{Ln}/2$, the neutron superfluid is gapless (the modulus Δ_n of the order parameter remaining finite), and $c_S^{(n)}$ is only moderately reduced compared to that in the normal phase. At $T \ll T_{cn}^{(0)}$, the

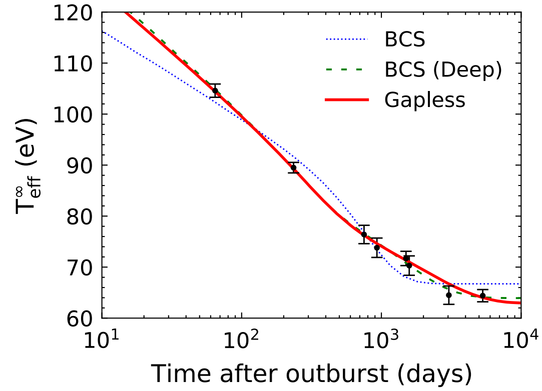

FIG. 1. Evolution of the effective surface temperature in electronvolts of KS 1731-260 (as seen by an observer at infinity) as a function of the time in days after a 12.5 year outburst. Symbols represent observational data with error bars. The dotted and solid lines are models considering BCS and gapless superfluidity, respectively, using the realistic pairing calculations of Ref. [39]. The dashed line was obtained assuming BCS superfluidity with the fine-tuned "Deep" gap of Ref. [29].

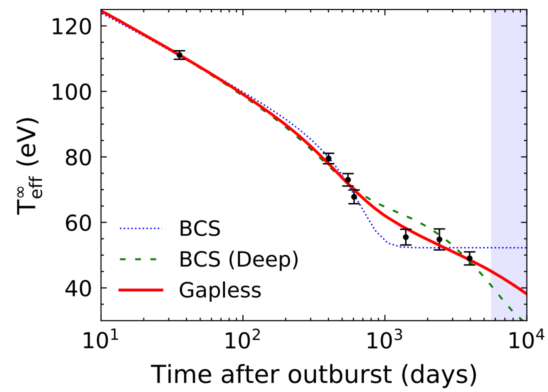

FIG. 2. Same as Fig. 1 for MXB 1659-29 after the first outburst. The shaded area corresponds to the second accretion phase (which occurred in 2015) and its subsequent cooling phase: The cooling curves within this region depict the expected behavior had this outburst not occurred.

reduction factor is essentially independent of the temperature and is given by [55]

$$R_{00}^{(\rm gapless)}(\mathbb{V}_{Ln} < \mathbb{V}_n \leq \mathbb{V}_{cn}^{(0)}) = \sqrt{1 - \left(\frac{2}{\exp(1)} \frac{\Delta_n}{\Delta_n^{(0)}} \frac{\mathbb{V}_{cn}^{(0)}}{\mathbb{V}_n}\right)^2}, \tag{3}$$

with [63]

$$\Delta_n(\mathbb{V}_{Ln} < \mathbb{V}_n \leq \mathbb{V}_{cn}^{(0)}) = 0.5081 \Delta_n^{(0)} \sqrt{1 - \frac{\mathbb{V}_n}{\mathbb{V}_{cn}^{(0)}}} \times \left(3.312 \frac{\mathbb{V}_n}{\mathbb{V}_{cn}^{(0)}} - 3.811 \sqrt{\frac{\mathbb{V}_{cn}^{(0)}}{\mathbb{V}_n}} + 5.842\right). \tag{4}$$

In the gapless state, the neutron specific heat is a universal function of $\mathbb{V}_n/\mathbb{V}_{cn}^{(0)}$ or equivalently of $\mathbb{V}_n/\mathbb{V}_{Ln}$; i.e., it is independent of the adopted results for $\Delta_n^{(0)}$. Neutron superfluidity is destroyed (i.e., $\Delta_n = 0$) when $\mathbb{V}_n \geq \mathbb{V}_{cn}^{(0)}$, and the neutron specific heat then reduces to Eq. (1). The actual value of $\mathbb{V}_n$ depends on the dynamical evolution of the specific SXT under consideration. In the following, we will treat $\mathbb{V}_n/\mathbb{V}_{Ln}$ as a free parameter.

Results and discussion.—We now discuss the cooling of KS 1731-260 and MXB 1659-29. Observational data and full numerical results are given in [43]. As in previous studies [13,27,32,34,64], we assume in both cases a constant accretion rate 10^{17} g s^{-1} consistent with the time-averaged accretion rate found in Ref. [65]. A variable accretion rate can affect the cooling, but not in the late stage of interest here [66]. We set the NS mass to $1.62 M_\odot$ and the radius to 11.2 km, as in Refs. [13,27,34,64]. Unless stated otherwise, we adopt the realistic pairing calculations of Ref. [39]. To estimate the uncertainties in the parameters $T_{\rm b}$, $T_{\rm core}$, $Q_{\rm imp}$, and $\mathbb{V}_n$, we have run Markov chain Monte Carlo simulations [43].

Results for the thermal evolution of KS 1731-260 after the 12.5 years of outburst are displayed in Fig. 1. Ignoring the presence of superflow as in previous studies, this model (dotted curve) fails to explain the late time cooling after 10^3 days and leads to a rather poor fit of the earlier observations. The optimum parameters with uncertainties at the 68% level are $Q_{\rm imp} = 10.56^{+2.15}_{-1.97}$, $T_{\rm b} = 2.45^{+0.19}_{-0.18} \times 10^8$ K, and $T_{\rm core} = (4.69 \pm 0.14) \times 10^7$ K. The last four data points can only be reproduced by artificially fine-tuning the pairing gap (dashed curve), as in Ref. [29].

In contrast, allowing for gapless superfluidity (solid curve) yields an excellent fit to the full dataset. Our best model is obtained for $\mathbb{V}_n = 1.21^{+0.10}_{-0.11} \mathbb{V}_{Ln}$. However, the distribution of $\mathbb{V}_n$ is rather broad, and the 95% credibility interval extends down to about $\mathbb{V}_{Ln}$. The other parameters are $Q_{\rm imp} = 5.80^{+1.68}_{-1.25}$, $T_{\rm b} = 3.13^{+0.19}_{-0.20} \times 10^8$ K, and $T_{\rm core} = 3.99^{+0.26}_{-0.34} \times 10^7$ K.

Figure 2 shows results for the first outburst of MXB 1659-29. Restricting to BCS superfluidity with realistic pairing, the best model (dotted curve) is obtained for $Q_{\rm imp} = 8.05^{+0.71}_{-0.61}$, $T_{\rm b} = (3.11 \pm 0.13) \times 10^8$ K, and $T_{\rm core} = (2.69 \pm 0.15) \times 10^7$ K. With this model, the NS is cooling so rapidly that thermal equilibrium is restored about 10^3 days after the end of the outburst, thus failing to reproduce the last data point. This puzzle can be naturally solved by taking into account the superflow. Our best model (solid curve) is found for $\mathbb{V}_n = 1.23^{+0.09}_{-0.11} \mathbb{V}_{Ln}$, $Q_{\rm imp} = 17.16^{+3.49}_{-3.70}$, $T_{\rm b} = 3.14^{+0.16}_{-0.15} \times 10^8$ K, and $T_{\rm core} = 0.85^{+0.59}_{-0.54} \times 10^7$ K. To check the consistency of our model, we have analyzed the second outburst keeping fixed the core temperature. As discussed in Ref. [31], variations of $T_{\rm core}$ between the two outbursts and during the subsequent crust cooling are expected to lie within the observational uncertainties. Our model reproduces the observations very well [43]. We find no significant change of $Q_{\rm imp}$ (consistent with the analysis of Ref. [31] in the standard paradigm) and $\mathbb{V}_n$ at the 95% level, contrary to $T_{\rm b}$. However, $T_{\rm b}$ (related to shallow heating) needs not remain the same between outbursts. For completeness, we have also shown the best-fit model with the "Deep" gap of Ref. [29]. However, this gap, which was empirically adjusted to fit the cooling data of KS 1731-260 within the traditional model of accreted NSs of Haensel and Zdunik [57], does not provide satisfactory results for the first outburst of MXB 1659-29.

For both sources, by allowing for gapless superfluidity, we have obtained excellent fits to the data without having to introduce a highly disordered layer in the deep crust, in agreement with quantum molecular dynamics simulations [28]. Running simulations within the traditional model of accreted NSs [57], we have found that the diffusion of superfluid neutrons introduced in Refs. [60,61] does not solve in itself the puzzle of the late time cooling [43]. At the time of this writing, KS 1731-260 and MXB 1659-29 are still in quiescence [67]. According to our best models, the crust of the former has finally reached thermal equilibrium, whereas the crust of the latter is further cooling.

The presence of a finite superflow in NS crusts, as suggested by our best cooling models of MXB 1659-29 and KS 1731-260, is not unexpected. During accretion episodes, the crust and all particles strongly coupled to it (constituting most of the star) are expected to be spun up due to the transfer of angular momentum from the infalling material. This so-called "recycling" scenario proposed to explain the existence of millisecond pulsars [68,69] has been recently confirmed by the discovery of accreting millisecond x-ray pulsars [70,71] and transitional millisecond pulsars [72]. Evidence for the fact that both KS 1731-260 and MXB 1659-29 have been recycled during their history comes from observations of x-ray burst oscillations at high frequency, respectively, ~524 and 567 Hz [17,23,65,73], likely related to the NS spin frequency. Because of pinning of quantized vortices, the neutron superfluid velocity is locked, so that in the crust frame V_n, therefore also $\mathbb{V}_n$, both increase. At the end of an outburst and during the quiescent period that follows, $\mathbb{V}_n$ is likely to remain essentially constant unless vortices unpin; unlike isolated pulsars, NSs in low-mass x-ray binaries have typically very weak magnetic fields $\sim 10^8$–10^9 G so that the spin-down caused by electromagnetic braking is not expected to be very effective [74]. This justifies our assumption of a constant superflow throughout the thermal relaxation.

Conclusions.—Gapless neutron superfluidity in the inner crust of NSs provides a natural explanation for the observed late time cooling of quasipersistent SXTs due to the huge enhancement of the neutron specific heat compared to that in the classical BCS case. The neutron specific heat can thus be comparable to that in the normal phase without requiring the unrealistic suppression of superfluidity as previously proposed. Focusing on the emblematic sources KS 1731-260 and MXB 1659-29 for which the interpretation via the standard cooling theory has been challenged, we have obtained excellent fits to the observational data using realistic neutron pairing calculations [39] and without introducing a highly disordered layer at the crust bottom in agreement with quantum Monte Carlo calculations [28]. We have also checked the consistency of our model between the two outbursts of MXB 1659-29. According to our simulations, the crust of KS 1731-260 is now in thermal equilibrium with the core, whereas MXB 1659-29 is still cooling (at variance with the standard paradigm [43]). These predictions could be tested by future observations and could provide more stringent constraints on $\mathbb{V}_n$.

Gapless superfluidity is driven by the presence of a superflow in the crust, as expected to arise from the pinning of quantized vortices. The lag between the superfluid and the rest of the star is limited by the strength of pinning forces acting on individual vortices. The maximum superfluid velocity can be estimated as $V_{\rm cr} \sim 10^7 (f_p/10^{18}\ {\rm dyn/cm})\ {\rm cm/s}$ [75], where f_p denotes the average (on the appropriate scale) pinning force per unit length. Systematic fully microscopic calculations of the force on a single pinning site remain computationally challenging [76]. Averaging over many pinning sites could lead to much stronger forces [77]. Current estimates for f_p are at most of order 10^{18} dyn/cm leading to $V_{\rm cr} \sim 10^7$ cm/s; $\mathbb{V}_{Ln}$ is found to be somehow higher, of the order of 10^8 cm/s. However, experiments using cold atoms [78] suggest that Landau's velocity could be significantly suppressed by the presence of clusters, which we have ignored here. Moreover, vortices extend to the core where they can pin to proton fluxoids, thus further increasing f_p, hence also $V_{\rm cr}$. It is therefore not inconceivable that $\mathbb{V}_{Ln} \lesssim V_{\rm cr}$. The excellent fit of the cooling data from SXTs brings support to this hypothesis and calls for further studies of the vortex dynamics in NSs.

The existence of a superflow in the crust could potentially have other observational consequences. At some point, most likely during outburst, vortices may be unpinned (e.g., due to thermal activation [79]) leading to a sudden spin-up of the superfluid accompanied by a spin-down of the star [80]. This will be manifested by an antiglitch, i.e., a decrease of the spin frequency, or possibly a glitch under certain circumstances [81]. Whether such an event occurred in MXB 1659-29 is difficult to assess due to the comparatively large uncertainties in the spin frequency measured from x-ray burst oscillations. The same difficulty will arise when KS 1731-260 will return to outburst. These sources are nuclear-powered x-ray pulsars exhibiting x-ray bursts due to thermonuclear explosions. More accurate measurements of the spin frequency are made in accretion-powered x-ray pulsars undergoing channeled accretion due to magnetic fields. Among them, Aql X-1, whose accurately measured spin frequency is 550.2744 Hz [82], has a prolific activity with 23 outbursts from 1996 to 2015, and the thermal emission during quiescence has been observed [83]. However, the periods between outbursts only last for weeks and are too short to probe the deep crust and superfluidity. The analysis of the observational data is further complicated by low-level residual accretion during quiescence [84]. A more promising source to test our scenario is HETE J1900.1-2455 with a spin frequency of 377.296 171 971(5) Hz [85]. This pulsar has recently been observed after the end of a ten-year-long accretion outburst and appears unusually cold [86,87]. Assuming that the crust has fully relaxed, the standard cooling theory requires the suppression of nucleon superfluidity in the core [87] at variance with theoretical expectations. The analysis of this source is left for future studies.

This work was financially supported by the Fonds de la Recherche Scientifique (Belgium) under Grants No. PDR T.004320 and No. IISN 4.4502.19. We thank Professor A. Sedrakian, N. Shchechilin, and L. Planquart for discussions.

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