Experimental Virtual Distillation of Entanglement and Coherence

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Noise is, in general, inevitable and detrimental to practical and useful quantum communication and computation. Under the resource theory framework, resource distillation serves as a generic tool to overcome the effect of noise. Yet, conventional resource distillation protocols generally require operations on multiple copies of resource states, and strong limitations exist that restrict their practical utilities. Recently, by relaxing the setting of resource distillation to only approximating the measurement statistics instead of the quantum state, a resource-frugal protocol, "virtual resource distillation," is proposed, which allows more effective distillation of noisy resources. Here, we report its experimental implementation on a photonic quantum system for the distillation of quantum coherence (up to dimension four) and bipartite entanglement. We show the virtual distillation of the maximal superposed state of dimension four from the state of dimension two, an impossible task in conventional coherence distillation. Furthermore, we demonstrate the virtual distillation of entanglement with operations acting only on a single copy of the noisy Einstein-Podolsky-Rosen (EPR) pair and showcase the quantum teleportation task using the virtually distilled EPR pair with a significantly improved fidelity of the teleported state. These results illustrate the feasibility of the virtual resource distillation method and pave the way for accurate manipulation of quantum resources with noisy quantum hardware.

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Introduction.—A critical challenge of practical quantum information processing is to overcome the effect of noise originating from hardware imperfections. A systematic treatment of this problem is to exploit the language of resource theory and consider resource distillation that converts multiple copies of imperfect quantum states to the ideal one [1–5]. For example, one can apply local operations and classical communication to extract a more accurate Einstein-Podolsky-Rosen (EPR) pair from multiple copies of noisy EPR pairs [6], generally referred to as entanglement distillation [3,7,8], an important procedure in long-distance quantum communication [9–12]. However, several theoretical works have shown strong limitations of resource distillation, which either require many copies of the resource state or cause large distillation errors [13–16]. It thus remains an experimental challenge to faithfully implement accurate resource distillation for current and near-term quantum technology.

In response to this fundamental limitation, the virtual resource distillation (VRD) protocol [17,18] has been proposed as a promising solution. The key idea of VRD is to relax the conventional distillation condition to only reproduce the measurement effects (properties) of the target ideal resource state without actually synthesizing it. Remarkably, with proper distillation operations and classical postprocessing of the measurement outcomes, the VRD scheme is able to surpass the no-go theorem of conventional resource distillations. For example, we could virtually obtain the ideal EPR pair using operations acting on a single copy of noisy EPR states, which is impossible conventionally. In Refs. [17,18], universal theoretical bounds have been derived for the VRD rate for general resource theories, however, these results rely on the ideal (without implementation error) and theoretical (practically hard to implement) assumptions of the distillation operation. Whether VRD remains practical and advantageous for existing quantum technology is still an open question.

Here, we answer this question by presenting an experimental demonstration of virtual distillation of coherence and entanglement with an optical system. For quantum coherence [19–21], we demonstrate the virtual preparation of a four-dimensional maximally coherent single-photon state by applying virtual distillation operations collectively on a single copy of the two-dimensional maximally coherent state, a forbidden task in conventional coherence resource theories [14,15]. For entanglement [1–3], we demonstrate that the maximally entangled state (MES) can be distilled from a family of two-photon Werner states (ranging from the maximally mixed state to the MES). The distillation is confirmed by the fidelity between the virtually distilled state and the MES as well as the negativity of the virtually distilled state. Furthermore, we show that the virtually distilled state can be used for realizing quantum teleportation. We show the feasibility of VRD in the presence of experimental noise, which benefits quantum information processing using current and near-term quantum hardware.

Framework.—We start by briefly reviewing the framework of resource theories and virtual resource distillation [17,18]. A resource theory of states consists of two basic parts: the set $\mathcal{T}$ of free states and the set $\mathcal{O}$ of free operations satisfying $\Gamma(\sigma) \in \mathcal{T},\ \forall\ \sigma \in \mathcal{T}$ for any $\Gamma \in \mathcal{O}$. Conventionally, given an imperfect resource state ρ, the aim of resource distillation is to generate as many m copies of the optimal unit pure resource state, denoted as ψ, using free operations $\Gamma \in \mathcal{O}$, such that

$$\frac{1}{2}\|\Gamma(\rho) - \psi^{\otimes m}\|_1 \leq \varepsilon \tag{1}$$

for given accuracy $\varepsilon \geq 0$. Here $\|A\|_1 = \mathrm{Tr}[\sqrt{A^\dagger A}]$ is the trace norm.

The idea of VRD is to first consider an equivalent form of Eq. (1), i.e., $|\mathrm{Tr}[O\Gamma(\rho)] - \mathrm{Tr}[O\psi^{\otimes m}]| \leq \varepsilon$, for any bounded Hermitian observable O. Then we relax the condition by considering linear combinations of free operations, i.e., $\tilde{\Gamma} = \gamma_+\Gamma_+ - \gamma_-\Gamma_-$ with $\gamma_+ - \gamma_- = 1$ and $\gamma_+, \gamma_- \geq 0$, and we have the virtual resource distillation condition

$$|\mathrm{Tr}[O\tilde{\Gamma}(\rho)] - \mathrm{Tr}[O\psi^{\otimes m}]| \leq \varepsilon,\ \ \forall\ \ O, \tag{2}$$

which is equivalent to

$$\frac{1}{2}\|\tilde{\Gamma}(\rho) - \psi^{\otimes m}\|_1 \leq \varepsilon. \tag{3}$$

Compared to conventional resource distillation where the state $\Gamma(\rho)$ is obtained, a fundamental difference here is that we only virtually obtain $\tilde{\Gamma}(\rho)$ in the sense that we can only obtain observable expectation values of $\tilde{\Gamma}(\rho)$. Nevertheless, we can still treat $\tilde{\Gamma}(\rho)$ as a virtual quantum state and further apply quantum operations (such as state tomography [22], quantum teleportation [23], and quantum computing) on $\tilde{\Gamma}(\rho)$ as long as the quantum state is finally measured.

The virtual operation $\tilde{\Gamma}$ could be realized in the probabilistic manner as

$$\mathrm{Tr}[O\tilde{\Gamma}(\rho)] = C(p_+\mathrm{Tr}[O\Gamma_+(\rho)] - p_-\mathrm{Tr}[O\Gamma_-(\rho)]), \tag{4}$$

where $p_\pm \coloneqq \gamma_\pm/C$ and $C = \gamma_+ + \gamma_-$ is the cost of VRD. Specifically, we randomly apply $\Gamma_\pm$ to ρ with probability $p_\pm$, measure the state with O, assign the minus sign when applying Γ_-, and finally multiply C to the measurement outcome. Then the average of the measurement outcomes gives an unbiased estimation of $\mathrm{Tr}[O\tilde{\Gamma}(\rho)]$. Since $C \geq 1$ increases the range of the measurement outcomes, it introduces a sampling overhead, which is a common price of virtual distillation. Nevertheless, as shown in Refs. [17,18], even when conventional distillation is not possible, i.e., Eq. (1) does not satisfy for any $m \geq 1$, we could, in general, still run virtual resource distillation such that Eq. (2) holds for nonzero m. We refer to Refs. [17,18] for a detailed discussion of the virtual distillation rate and its comparison to conventional resource distillation.

Experiment.—We first generate the polarization-entangled photons with the setup shown in Fig. 1(a). A periodically poled potassium titanyl phosphate (PPKTP) crystal is set in a Sagnac interferometer, which is bidirectionally pumped by a 405 nm ultraviolet laser diode [24]. The generated two photons A and B are entangled in polarization degree of freedom (d.o.f.) with the ideal form of $|\Psi^+\rangle = (1/\sqrt{2})(|HV\rangle + |VH\rangle)$, where H and V represent horizontal and vertical polarization, respectively. The VRD of coherence is implemented with the setup shown in Fig. 1(b), where photon B is detected on state $|V\rangle$ triggering photon A on state $|H\rangle$. The path d.o.f. is manipulated by a beam displacer (BD) with the assistance of polarization d.o.f., i.e., the BD transmits vertical polarization to mode $|v\rangle$ and deviates horizontal polarization to mode $|h\rangle$. We encode a single-ququart (fourdimensional quantum system) state by $|0\rangle = |H\rangle|v\rangle$, $|1\rangle = |V\rangle|v\rangle$, $|2\rangle = |H\rangle|h\rangle$, and $|3\rangle = |V\rangle|h\rangle$. Thus, the state of photon A is $|H\rangle|v\rangle = |0\rangle$, which can be converted to $|\psi_+^1\rangle = (|H\rangle|v\rangle + |V\rangle|v\rangle)/\sqrt{2}$ by applying a half-wave plate (HWP) set at 22.5°. Our primary goal is to virtually obtain the maximally four-dimensional coherent state (MCS) $|\psi\rangle_{\mathrm{MCS}} = \frac{1}{2}\sum_{i=1}^{4}|i\rangle$ by applying virtual operations each on a single copy of the two-dimensional coherent state $|\psi_+^1\rangle$, which is impossible in conventional resource theory [14,15]. In Refs. [17,18], universal constructions of the VRD operations are given, however, the results assume general resource nongeneration operations, which are mathematically easier, yet could also be practically unphysical. Here, consider the case of $\varepsilon = 0$ [25]; we give an explicit and practical VRD protocol for input state of $|\psi_+^1\rangle$,

$$\rho_{\mathrm{MCS}} = C[p_+\Gamma_+(\rho_+^1) - p_-\Gamma_-(\rho_+^1)], \tag{5}$$

where $\rho_{\mathrm{MCS}} = |\psi\rangle_{\mathrm{MCS}}\langle\psi|_{\mathrm{MCS}}$, $\rho_+^1 = |\psi_+^1\rangle\langle\psi_+^1|$, $\Gamma_+ = \sum_{k=1}^{6}\Gamma_+^k$, $\Gamma_- = \sum_{k=1}^{6}\Gamma_-^k$, $p_+ = 2/3$, $p_- = 1/3$, and $C = 3$. The incoherent operations $\Gamma_\pm^k$ are given in Table I, which is optimal with a minimal cost (we refer to Supplemental Material [26] for details).

Experimentally, four HWPs [$\mathbf{H}_1 \sim \mathbf{H}_4$ in Fig. 1(b)] with angle setting of $\boldsymbol{\theta}_\pm^k = [\theta_{k_1}, \theta_{k_2}, \theta_{k_3}, \theta_{k_4}]$ are used to realize the 12 individual incoherent operations $\Gamma_\pm^k$ (see Supplemental Material [26] for details of $\boldsymbol{\theta}_\pm^k$). In our experiment, the 12 incoherent operations are applied on ρ_+^1 with equal probability, each of which leads the outcome state $\rho_\pm^k = |\psi_\pm^k\rangle\langle\psi_\pm^k| = \Gamma_\pm^k(\rho_+^1)$. For each $\rho_\pm^k$, we perform Pauli measurements on polarization d.o.f. and path d.o.f.,

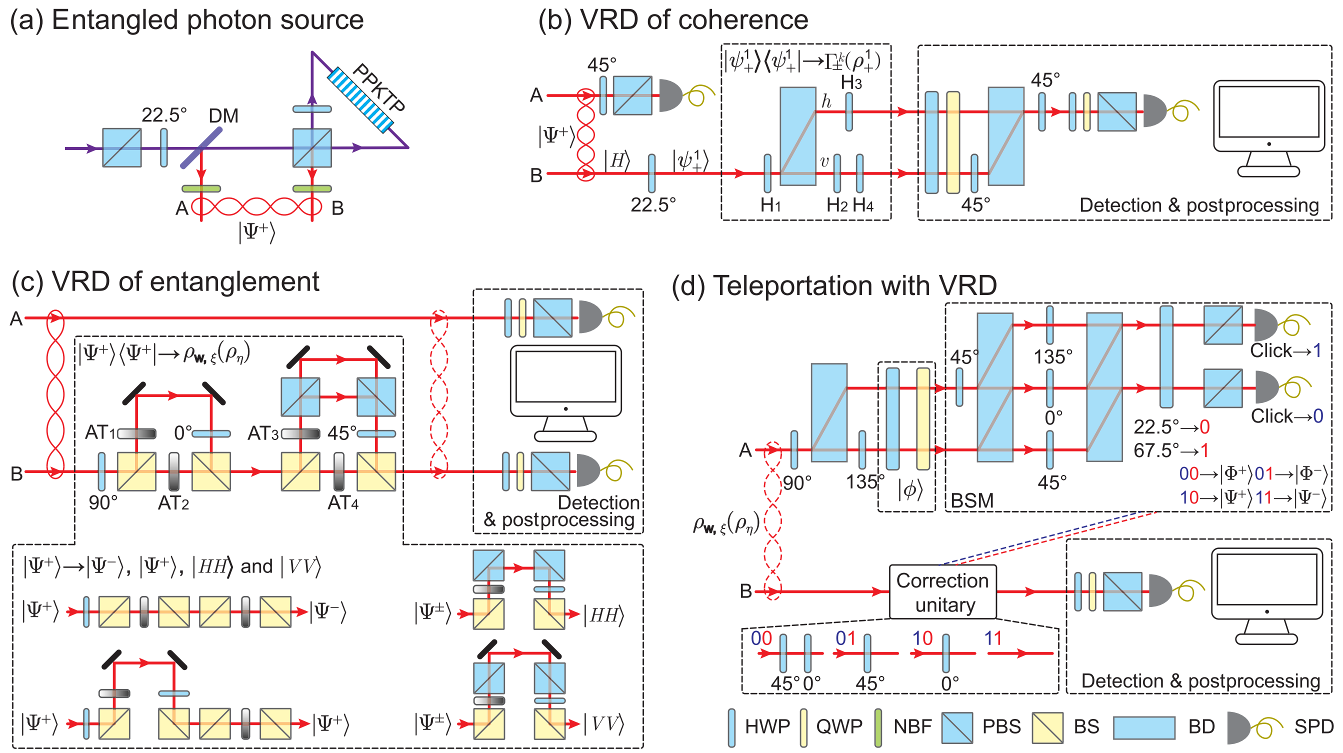

FIG. 1. Schematic illustration of the experimental setup. (a) The setup to generate polarization-entangled photon pair. (b) The setup to demonstrate VRD of coherence on single-quarter states. (c) Setup to generate Werner states and demonstrate VRD of entanglement on the prepared Werner states. (d) The setup to implement quantum teleportation with assistance of VRD. BSM, Bell-state measurement; QWP, quarter-wave plate; NBF, narrow band filter; SPD, single-photon detector; DM, dichroic mirror. Step-by-step descriptions of experimental realizations are provided in Supplemental Material [26].

respectively. Then, we estimate the density matrix $\hat{\rho}_\pm^{k,\mathrm{LIN}}$ using linear inversion (LIN) and calculate $\hat{\rho}_{\mathrm{MCS}}^{\mathrm{LIN}}$ according to Eq. (5). Note that $\hat{\rho}_{\mathrm{MCS}}^{\mathrm{LIN}}$ is not a physical state, as it generally has negative eigenvalues. This issue can be addressed by projecting $\hat{\rho}_{\mathrm{MCS}}^{\mathrm{LIN}}$ onto the set of physical states with respect to the Frobenius distance, which returns a physical state $\hat{\rho}_{\mathrm{MCS}}$. Hereafter, $\hat{\rho}^{\mathrm{LIN}}$ represents the estimation with LIN and $\hat{\rho}$ represents the corresponding physical state obtained from $\hat{\rho}^{\mathrm{LIN}}$ using the projective method.

The distilled state $\hat{\rho}_{\mathrm{MCS}}$ is first characterized by calculating its fidelity with respect to ideal ρ_{MCS}, i.e., $\mathcal{F}(\hat{\rho}_{\mathrm{MCS}},\rho_{\mathrm{MCS}})=\mathrm{Tr}(\hat{\rho}_{\mathrm{MCS}}\rho_{\mathrm{MCS}})$ [40]. The result is shown in Fig. 2, and we observe $\mathcal{F}(\hat{\rho}_{\mathrm{MCS}},\rho_{\mathrm{MCS}})=0.932\pm0.004$. We also calculate the fidelity between input state $\hat{\rho}_+^1$ and ρ_{MCS}, and observe $\mathcal{F}(\hat{\rho}_+^1,\rho_{\mathrm{MCS}})=0.426\pm0.007$. The enhancement of fidelity evidently convinces the successful distillation of $|\psi_{\mathrm{MCS}}\rangle$. The distillation is further confirmed by calculating the relative entropy of coherence $\mathcal{C}(\rho)=\mathrm{Tr}(\rho\log_2\rho)-\mathrm{Tr}(\rho_d\log_2\rho_d)$ of $\hat{\rho}_+^1$ and $\hat{\rho}_{\mathrm{MCS}}$, respectively, where ρ_d is the diagonal matrix of ρ [41]. We observe that $\mathcal{C}(\hat{\rho}_{\mathrm{MCS}})=1.769\pm0.029$, while $\mathcal{C}(\hat{\rho}_+^1)=0.958\pm0.024$ as shown in Fig. 2. Both the fidelity $\mathcal{F}$ and the relative entropy of coherence $\mathcal{C}$ are significantly enhanced with the VRD protocol.

TABLE I. The VRD operations for quantum coherence. Here $X_{jk}=|j\rangle\langle k|+|k\rangle\langle j|$ and $Z_{jk}=|j\rangle\langle j|-|k\rangle\langle k|$, which are all incoherent operations [20]. The operations $\Gamma_\pm^k$ correspond to the unitary in the table.

k	1	2	3	4	5	6
Γ_+^k	I	X_{12}	X_{13}	X_{02}	X_{03}	$X_{02}\oplus X_{13}$
Γ_-^k	Z_{01}	$Z_{02}X_{12}$	$Z_{03}X_{13}$	$Z_{12}X_{02}$	$Z_{13}X_{03}$	$Z_{23}(X_{02}\oplus X_{13})$

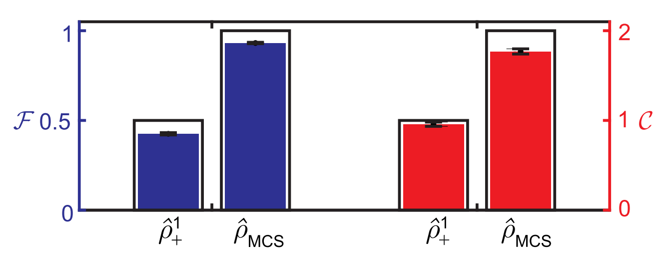

FIG. 2. The experimental results of VRD of quantum coherence. Two figures of merit are adopted to indicate the improvements of distilled state $\hat{\rho}_{\mathrm{MCS}}$, i.e., the fidelity $\mathcal{F}$ (blue bars) and relative entropy of coherence $\mathcal{C}$ (red bars). The black boxes represent the theoretical values. The statistic error is calculated from Poissonian counting statistics of the raw detection events.

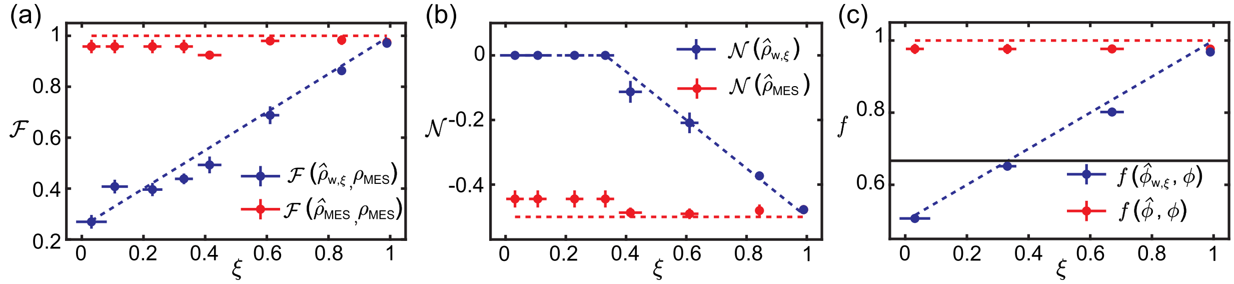

FIG. 3. Experimental results of VRD of entanglement. (a) Fidelity of $\mathcal{F}(\hat{\rho}_{\mathrm{MES}},\rho_{\mathrm{MES}})$ and $\mathcal{F}(\hat{\rho}_{\mathrm{w},\xi},\rho_{\mathrm{MES}})$. (b) Negativity of $\mathcal{N}(\hat{\rho}_{\mathrm{MES}})$ and $\mathcal{N}(\hat{\rho}_{\mathrm{w},\xi})$. (c) The average fidelity f of quantum teleportation with $\hat{\phi}_{\mathrm{w},\xi}$ or $\hat{\phi}$. Dashed lines represent the theoretical predictions with the ideal states, and dots represent the experimental results. The blue color represents results before VRD and the red color represents the results after VRD. The black line in (c) represents classical threshold $f_c = 2/3$ in quantum teleportation.

Next, we implement the VRD of MES $|\Psi^-\rangle = (1/\sqrt{2})(|01\rangle - |10\rangle)$ from a family of 2-qubit Werner states in the form of [42]

$$\rho_{\mathrm{w},\xi} = \xi\Psi^- + (1-\xi)\mathbb{1}_4/4, \qquad 0 \le \xi \le 1, \tag{6}$$

with $\Psi^- = |\Psi^-\rangle\langle\Psi^-|$. Note that the 2-qubit Werner states are separable for $\xi < 1/3$ and entangled for $\xi \ge 1/3$ according to the positive partial transpose criterion [43,44]. The VRD protocol for Werner states is given by

$$\rho_{\mathrm{MES}} = \Psi^- = C_\xi[p_+\Gamma_+(\rho_{\mathrm{w},\xi}) - p_-\Gamma_-(\rho_{\mathrm{w},\xi})], \tag{7}$$

where Γ_+ is the identity operation, $\Gamma_-(\cdot) = \rho_\eta = (\mathbb{1}_4 - \Psi^-)/3$ is a replacement channel, $p_+ = [4/(7-3\xi)]$, $p_- = [(3-3\xi)/(7-3\xi)]$, and $C_\xi = [(7-3\xi)/(1+3\xi)]$. In the cases $\xi < 1/3$, the VRD is performed with input state $\rho_{\mathrm{w},1/3}$ regardless of ξ. (See Supplemental Material [26] for the proof of optimality of this VRD protocol.)

Experimentally, the Werner state is encoded in polarization d.o.f., i.e., $|H\rangle = |0\rangle$ and $|V\rangle = |1\rangle$. The Werner states in Eq. (6) can be rewritten by a mixture of four pure states,

$$\rho_{\mathrm{w},\xi} = \frac{1+3\xi}{4}\Psi^- + \frac{1-\xi}{4}(\Psi^+ + \rho_{HH} + \rho_{VV}), \tag{8}$$

where $\rho_{HH} = |HH\rangle\langle HH|$ and $\rho_{VV} = |VV\rangle\langle VV|$. As shown in Fig. 1(c), ρ_ξ^{w} can be prepared with an array of beam splitters (BSs), polarizing beam splitters (PBSs), HWPs, and attenuators (ATs). Specifically, the setup is able to covert input state $|\Psi^+\rangle$ to a mixture of Ψ^-, Ψ^+, ρ_{HH}, and ρ_{VV}, where the probability of each component is determined by the transmittance of ATs. Thus, the parameter ξ is determined by $\boldsymbol{t}_\xi = \{t_{\xi_1} = [(1-\xi)/(2+2\xi)], t_{\xi_2} = [(1+3\xi)/(2+2\xi)], t_{\xi_3} = [(1-\xi)/2], t_{\xi_4} = [(1+\xi)/2]\}$ with t_{ξ_i} being the transmittance of the ith AT (see Supplemental Material [26] for more details). In our experiment, we prepare eight $\rho_{\mathrm{w},\xi}$ with parameter $\xi \in [0, 0.1, 0.2, 1/3, 0.4, 0.6, 0.8, 1]$ for distillation. For each ξ, the VRD requires the preparation of ρ_η, which can be decomposed into $\rho_\eta = \frac{1}{3}(\Psi^+ + \rho_{HH} + \rho_{VV})$ and can be prepared with the same setup by setting $\boldsymbol{t}_\eta = [t_{\eta_1} = 1, t_{\eta_2} = 0, t_{\eta_3} = \frac{2}{3}, t_{\eta_4} = \frac{1}{3}]$. We set the parameters of four ATs being $\boldsymbol{t}_\xi$ and $\boldsymbol{t}_\eta$ with equal probability and estimate the corresponding density matrix $\hat{\rho}_{\mathrm{w},\xi}^{\mathrm{LIN}}$ and $\hat{\rho}_\eta^{\mathrm{LIN}}$. Then, $\hat{\rho}_{\mathrm{MES}}^{\mathrm{LIN}}$ is calculated according to Eq. (7), and $\hat{\rho}_{\mathrm{MES}}$ is consequently obtained.

We calculate the fidelity $\mathcal{F}(\hat{\rho}_{\mathrm{MES}},\rho_{\mathrm{MES}})$ and $\mathcal{F}(\hat{\rho}_{\mathrm{w},\xi},\rho_{\mathrm{MES}})$. As shown in Fig. 3(a), we observe that $\mathcal{F}(\hat{\rho}_{\mathrm{MES}},\rho_{\mathrm{MES}})$ is close to 1 regardless of the value of ξ, while $\mathcal{F}(\hat{\rho}_{\mathrm{w},\xi},\rho_{\mathrm{MES}})$ is linearly dependent on the value of ξ. To further investigate the quantity of entanglement, we calculate the entanglement negativity of a quantum state [45]

$$\mathcal{N}(\rho) = \frac{\|\rho^{T_{\mathrm{A}}}\|_1 - 1}{2}, \tag{9}$$

where T_{A} represents partial transpose of the density matrix ρ with respect to its subsystem A and $\|\cdot\|_1$ is the trace norm. Note that $\mathcal{N} = 0$ for separable states and $\mathcal{N} = -0.5$ for the MES. The negativity of $\hat{\rho}_{\mathrm{w},\xi}$ and $\hat{\rho}_{\mathrm{MES}}$ are shown in Fig. 3(b), in which we observe that the amount of entanglement is significantly enhanced after distillation. In particular, $\hat{\rho}_{\mathrm{w},\xi}$ with $\xi = 0$, 0.1, 0.2, and 1/3 are separable states as their negativities are 0, which indicates no entanglement exists in these states. However, the corresponding virtually distilled states admit a large amount of entanglement as their negativities are close to -0.5 [46].

Finally, we demonstrate quantum teleportation with assistance of VRD. $\phi = |\phi\rangle\langle\phi|$ is the quantum state to be teleported, and the teleportation is performed with $\rho_{\mathrm{w},\xi}$ and its corresponding ρ_η, respectively. After teleportation, the output state is denoted by $\phi_{\mathrm{w},\xi}$ and ϕ_η, respectively, and the VRD for teleportation is

$$\phi = \frac{4}{1+3\xi}\phi_{\mathrm{w},\xi} - \frac{3-3\xi}{1+3\xi}\phi_\eta. \tag{10}$$

Experimentally, we implement the two-photon teleportation scheme [47–50] as illustrated in Fig. 1(d). We prepare

$\rho_{\mathrm{w},\xi}$ and ρ_η with $\xi \in [0, 1/3, 2/3, 1]$ and select four states encoded in polarization d.o.f. for teleportation, i.e., $|\phi\rangle \in [|H\rangle, |V\rangle, |+\rangle = (1/\sqrt{2})(|H\rangle + |V\rangle), |R\rangle = (1/\sqrt{2})(|H\rangle + i|V\rangle]$. We calculate the average fidelity $f(\hat{\phi}, \phi) = \frac{1}{4}\sum_\phi \mathcal{F}(\hat{\phi}, \phi)$ and $f(\hat{\phi}_{\mathrm{w},\xi}, \phi) = \frac{1}{4}\sum_\phi \mathcal{F}(\hat{\phi}_{\mathrm{w},\xi}, \phi)$, where $\hat{\phi}$ and $\hat{\phi}_{\mathrm{w},\xi}$ are the reconstructed output states with VRD protocol and $\rho_{\mathrm{w},\xi}$, respectively. As shown in Fig. 3(c), $f(\hat{\phi}, \phi)$ is close to the ideal value of 1 (teleportation with maximal entanglement), while $f(\hat{\phi}_{\mathrm{w},\xi}, \phi)$ is strongly related to ξ, which is similar to the results in Fig. 3(a). In particular, $f(\hat{\phi}_{\mathrm{w},\xi}, \phi)$ is below $f_c = 2/3$ when $\xi = 0$, where $f_c = 2/3$ is the classical limit in quantum teleportation [51]. The experimental results indicate the practical usefulness of the VRD method is able to circumvent the theoretical barrier [17,18] of conventional distillation methods where the distillation may not be feasible for $\xi \neq 1$.

Conclusion.—In this Letter, we experimentally study the performance of the VRD protocol for coherence and entanglement. In addition, we demonstrate quantum teleportation with VRD. In all three experiments, we see that, despite imperfect quantum operations involved in the distillation process, our results show significant improvements after the distillation. Moreover, we demonstrate the distinct features of the VRD protocol for circumventing obstacles for conventional distillation schemes. We thus expect the VRD protocol to find more utilities in practical situations such as quantum communication [9–12] and quantum networks [52–54].

Regarding future work, it is crucial to extend the VRD method to a more realistic scenario such that the noisy resource state is unknown beforehand. Furthermore, the performance of the VRD protocol is affected by the knowledge of the noise channel. As a proof of principle, we manufacture the imperfect state in this work. Yet, as the noisy state is not easily accessible in a realistic scenario, it is not obvious how the VRD protocol can be applied in a resource-efficient way at first glance. We remark that the unknown noisy state can be approximated through the estimation of the noise channel using statistical machine learning frameworks [55–59]. It is worth noting that several experiments have adopted the noisy-channel learning methods as a subroutine for implementing quantum error mitigation algorithms [57,58]. The combination of these learning techniques with the VRD protocol may be fruitful in extending the utility of the VRD method. Finally, although virtual distillation outperforms conventional resource distillation, it is not the optimal way of processing information, see Supplemental Material [26] for details. Thus, it is interesting to study other protocols to more efficiently retrieve information from noisy resources.

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[26] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.132.180201 for the detailed framework, theoretical analysis, experimental implementations, and results, which includes Refs. [27–39].

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