Editors' Suggestion

Landau Theory of Altermagnetism

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(Received 11 August 2023; revised 11 January 2024; accepted 1 April 2024; published 23 April 2024)

We formulate a Landau theory for altermagnets, a class of collinear compensated magnets with spin-split bands. Starting from the nonrelativistic limit, this Landau theory goes beyond a conventional analysis by including spin-space symmetries, providing a simple framework for understanding the key features of this family of materials. We find a set of multipolar secondary order parameters connecting existing ideas about the spin symmetries of these systems, their order parameters, and the effect of nonzero spin-orbit coupling. We account for several features of canonical altermagnets such as RuO_2, MnTe, and CuF_2 that go beyond symmetry alone, relating the order parameter to key observables such as magnetization, anomalous Hall conductivity, and magnetoelastic and magneto-optical probes. Finally, we comment on generalizations of our framework to a wider family of exotic magnetic systems derived from the zero spin-orbit coupled limit.

DOI: 10.1103/PhysRevLett.132.176702

Introduction.—Magnetism has long been a source of novel phases and phenomena of both fundamental and technological interest. Many thousands of magnetic materials are known with a wide variety of structures including simple collinear ferromagnets, ferrimagnets, and antiferromagnets, as well as more complex arrangements characterized by multiple incommensurate wave vectors [1].

The importance of spin-orbit coupling in magnetism is widely appreciated, through exotic transport phenomena such as the anomalous and spin Hall effects [2,3], as well as new physics arising from the interplay of topology and magnetism such as skyrmion physics [4,5], nontrivial magnon band topology [6], or Berry phases induced by spin chirality in the electronic band structures of itinerant magnets [2]. However, the zero spin-orbit coupled limit still holds surprises.

One phenomenon in this setting that has captured the attention of a broad cross section of the community [7–45] is "altermagnetism." Following the unexpected discovery of a *d*-wave spin splitting of the Fermi surface in RuO_2 based on *ab initio* calculations [8], it was realized that this is one instance of a large new class of magnets defined by spin symmetries [25]. This spin-split band structure combines aspects of simple metallic ferromagnets and antiferromagnets with its core features borne out by experiment [27–30]. Although spin-orbit coupling (SOC) is not negligible in this material, the altermagnetic spin splitting arising in the zero SOC limit greatly exceeds any SOC induced band gaps. Despite having zero net moment, these bands can support spin currents with polarization depending on the orientation of the applied voltage. Further, an anomalous Hall response has been measured in altermagnetic materials such as RuO_2 [27] and MnTe [46]. While research into these magnets is at an early stage, there is hope that they may complete the program of antiferromagnetic spintronics [47–49]: realizing terahertz switching devices with no stray fields and with low damping spin currents.

Despite their significant potential value in applications, there remain fundamental questions in situating these new phases of matter within the broader context of magnetism. From a practical standpoint, one can characterize most of the altermagnetic properties as originating from band structures with an anisotropic pattern of spin splitting in momentum space due to time-reversal symmetry breaking [25,26]. This is in contrast to simple Stoner ferromagnets with double sheeted Fermi surfaces for the different populations of up and down spins and those of simple antiferromagnets where the Fermi surfaces are perfectly spin compensated [1,50], as well as from frustrated isotropic antiferromagnets that can have complicated Fermi surfaces with electron and hole pockets, albeit with equal spin populations [2,51]. While appealing, this phenomenology does not delineate which properties of altermagnets are robust to small symmetry-allowed perturbations and which may depend on material specific details.

In this Letter, we argue that Landau theory adapted to the zero SOC limit captures the unique features of altermagnets. Starting from the definition of Šmejkal *et al.* [25], this Landau theory links spin symmetries to altermagnetic phenomenology, including their band structures, thermodynamics, and response functions, and reveals a deep connection to multipolar secondary order parameters [31]. The symmetries of these multipoles relate directly to the symmetries of the spin-split bands, with the anisotropy of the electronic kinetic terms manifesting the same quadrupolar or hexadecapolar spatial structure found in the secondary order parameters, reminiscent of electronic nematic or spin-nematic phases [52]. In addition, this Landau theory allows one to systematically address the effects of switching on SOC, identifying the leading coupling to the primary order parameter and how they relate to any multipolar secondary order parameters. As many of the features of altermagnets, such as the anomalous Hall conductivity, *only* appear when SOC is nonzero, by approaching from this limit, we can analyze in detail how the phenomenology of altermagnets is distinguished from generic spin-orbit coupled magnets. The zero SOC limit thus acts as the "parent" phase from which many of their principal features—features that are obscured within the standard symmetry analysis—can be understood in real materials.

Landau theory.—We adopt the essential definition put forth in Šmejkal *et al.* [25]: an "ideal" altermagnet is a spin-orbit free magnet with collinear antiferromagnetic order where the two sublattices are symmetry related by something other than translation or inversion symmetry. Since without SOC spatial and spin operations can act separately, we can frame this as a statement about the spatial transformation properties of the Néel order parameter $\boldsymbol{N}$. To rephrase in this new language: in an altermagnet, $\boldsymbol{N}$ transforms as an inversion even nontrivial one-dimensional irreducible representation (IR) under the action of the crystal point group [25,26].

To be concrete, we assume that we have a system in which we can define a uniform magnetization $\boldsymbol{M}$ and staggered magnetization $\boldsymbol{N}$ (both inversion even). In the absence of SOC, the uniform magnetization transforms as $\Gamma_1 \otimes \Gamma^S_A$ where Γ_1 is the trivial IR of the point group and Γ^S_A is the (axial) vector IR of the spin-rotation group. We assume that $\boldsymbol{N}$ instead transforms as $\Gamma_N \otimes \Gamma^S_A$ where Γ_N is a nontrivial one-dimensional IR of the point group. The condition that $\Gamma_N \neq \Gamma_1$ encodes the assumption of altermagnetism [25,26].

An immediate consequence is that a net magnetization is not necessarily induced in the Néel phase. To see this, we consider direct linear couplings between $\boldsymbol{N}$ and $\boldsymbol{M}$ that transform as the product $(\Gamma_1 \otimes \Gamma^S_A) \otimes (\Gamma_N \otimes \Gamma^S_A) = \Gamma_N \otimes (\Gamma^S_1 \oplus \Gamma^S_A \oplus \Gamma^S_Q)$ where Γ^S_1 and Γ^S_Q are the scalar ($\ell = 0$) and quadrupolar ($\ell = 2$) IRs of the spin-rotation group. Since Γ_N is a nontrivial IR, these couplings are forbidden in the absence of SOC.

We now connect this to higher multipoles [53]. Going beyond $\boldsymbol{N}$ or $\boldsymbol{M}$, we can define a time odd, inversion even octupole, transforming like an axial vector in spin space, but a quadrupole spatially. Tracking spin and spatial indices separately, we can define [31]

$$\boldsymbol{O}_{\mu\nu} = \int d^3 r r_\mu r_\nu \boldsymbol{m}(\boldsymbol{r}), \tag{1}$$

where $\boldsymbol{m}(\boldsymbol{r})$ is the microscopic magnetization density. Note that $\boldsymbol{O}_{\mu\nu}$ transforms under spin-space symmetries as $O^\alpha_{\mu\nu} \to \sum_{\rho\tau\beta} S_{\alpha\beta} R_{\mu\rho} R_{\nu\tau} O^\beta_{\rho\tau}$ where $\boldsymbol{S}$ is a rotation in spin space and $\boldsymbol{R}$ is a rotation in real space. Other multipoles can be constructed analogously. This octupole transforms as $\boldsymbol{O}_{\mu\nu} \sim \Gamma_Q \otimes \Gamma^S_A$, where Γ_Q is the (generally reducible) representation of a spatial quadrupole. A linear coupling between $\boldsymbol{N}$ and $\boldsymbol{O}_{\mu\nu}$ then transforms as

$$(\Gamma_N \otimes \Gamma^S_A) \otimes (\Gamma_Q \otimes \Gamma^S_A) = (\Gamma_N \otimes \Gamma_Q) \otimes (\Gamma^S_1 \oplus \Gamma^S_A \oplus \Gamma^S_Q).$$

Thus if Γ_Q contains Γ_N then $\boldsymbol{N}$ and $\boldsymbol{O}_{\mu\nu}$ can couple linearly in the absence of SOC, and the octupole will appear as a secondary order parameter in the Néel phase. In the language of Šmejkal *et al.* [25], this would define a d-wave altermagnet.

We expect these multipolar secondary order parameters to be *generic*; for a given symmetry there should exist a high enough rank multipole such that its spatial part contains Γ_N. How do these secondary order parameters relate to the altermagnetic phenomenology? We first consider implications for bulk thermodynamic and transport probes, but as we will see in our discussion of the rutiles, these multipoles also connect to the symmetry of the spin-split bands.

Consider whether $\boldsymbol{N}$ can couple linearly to $\boldsymbol{M}$ once SOC is included. As the Landau theory now admits magnetocrystalline anisotropy, spin and spatial transformations are coupled and the spin-rotation group IRs reduce to $\Gamma^S_1 \to \Gamma_1$, $\Gamma^S_A \to \Gamma_A$, and $\Gamma^S_Q \to \Gamma_Q$. A linear coupling between $\boldsymbol{N}$ and $\boldsymbol{M}$ thus transforms as

$$(\Gamma_1 \otimes \Gamma_A) \otimes (\Gamma_N \otimes \Gamma_A) = \Gamma_N \otimes (\Gamma_A \otimes \Gamma_A).$$

Using that $\Gamma_A \otimes \Gamma_A = \Gamma_1 \oplus \Gamma_A \oplus \Gamma_Q$, whether this coupling is allowed is determined by whether Γ_N appears in the decomposition of Γ_A or Γ_Q. An identical condition applies for the generation of an anomalous Hall conductivity [2], corresponding to a current transverse to an applied voltage in the absence of an applied magnetic field, $J_\mu = \sum_{\mu\nu} \sigma^{\mu\nu}_H E_\nu$, as it transforms in the same way as $\boldsymbol{M}$. We also note that the Hall conductivity and magnetic circular dichroism transform identically under symmetry, so these conclusions also carry over to this magneto-optical probe.

We can now connect the appearance of a multipolar secondary order parameter to d-wave altermagnetic phenomenology: if $\boldsymbol{N}$ couples linearly to an octupole in the absence of SOC (and thus $\Gamma_N \subset \Gamma_Q$), then it will necessarily have a linear coupling to $\boldsymbol{M}$ and $\sigma^{\mu\nu}_H$ in the presence of SOC. The definition of Šmejkal *et al.* [25] does not *require* inducing an octupole, but instead can involve only higher rank multipoles, corresponding to g- or i-wave altermagnetism. In those cases, the generation of weak ferromagnetism or an anomalous Hall effect can still generically persist. It may still be generated linearly if $\Gamma_N \subset \Gamma_A$, but will necessarily appear nonlinearly otherwise.

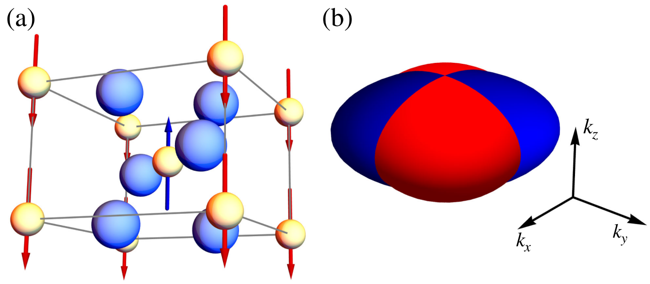

FIG. 1. Illustration of the (a) crystal structure of RuO_2 with magnetic Ru (orange) and oxygens (blue). (b) Fermi surface with d-wave spin splitting (up and down spins in blue and red, respectively) in the model of Eq. (2).

With these core ideas outlined, we apply this framework to understand a few common examples of altermagnetic systems, including rutiles such as RuO_2 and hexagonal MnTe. We will see that by adopting this phenomenological Landau theory, we can clarify the role played by multipolar secondary order parameters and delineate different mechanisms for the generation of characteristic responses when SOC is included.

Rutile altermagnetism—We begin with the canonical example of altermagnetism in rutiles with chemical formula MX_2 where M is the magnetic ion and $X = \mathrm{O}$, F. The most prominent example is currently RuO_2 which is a metallic antiferromagnet with a simple Néel order below the magnetic ordering temperature $T_\mathrm{N} > 300$ K [27–30,36,54–57]. The crystal structure belongs to tetragonal space group $P4_2/mnm$ (No. 136) with the Ru at Wyckoff position $2a$ and the oxygen at Wyckoff position $4f$. The magnetic sublattice is therefore body-centered tetragonal, as shown in Fig. 1(a). The space group has a generator C_{4z} combined with translation through $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ that maps one magnetic sublattice to the other. The inversion center, while present, preserves the magnetic sublattices. Below the magnetic ordering temperature, collinear antiparallel moments appear on the two magnetic sublattices.

Before delving into a phenomenological Landau description, to set the stage we consider a simple model that captures the principal features of rutile altermagnetism. This model, introduced in Ref. [58], consists of noninteracting fermions coupled to classical localized moments on the $2a$ sites through a Hund-like interaction. In real space the Hamiltonian is

$$H = \sum_{n=1,3} \sum_{a} t_n^a \sum_{\langle i,j \rangle_{n,a}} c_{i\sigma}^\dagger c_{j\sigma} - J \sum_i c_{i\alpha}^\dagger (\boldsymbol{S}_i \cdot \boldsymbol{\sigma}_{\alpha\beta}) c_{i\beta}, \quad (2)$$

where $\boldsymbol{S}_i$ are the local moment directions. One important observation is that truncating the model at nearest-neighbor t_1 or second-neighbor t_2 hoppings accidentally realizes the larger symmetry group of the underlying body-centered tetragonal lattice. The (lower) symmetry of the true space group No. 136 manifests first through the presence of two inequivalent third-neighbor hoppings, which generically have different amplitudes absent fine-tuning. The resulting band structure is such that the lowest two bands are split over most of momentum space with degeneracies along $(k, 0, 0)$ that arise from the spin-space symmetry of the system [25,58] and with maximal splitting along $(k, k, 0)$. As spin is a good quantum number, and the two bands correspond to electrons with polarization along $\boldsymbol{S}_i$ in spin space, the resulting splitting is the d-wave pattern shown in Fig. 1(b), characteristic of a d-wave altermagnet [25].

Let us formulate an explicit Landau theory for this class of materials. In this system, $\boldsymbol{N}$ transforms as the nontrivial B_{2g} IR of the point group $4/mmm$ (D_{4h}), satisfying the definition of an altermagnet [25,26]. Direct coupling between $\boldsymbol{M}$ and $\boldsymbol{N}$ is thus forbidden. More precisely, the order parameter transforms under the spin point group $\boldsymbol{b}^\infty \otimes^{\bar{1}} 4/{}^1m{}^1m{}^{\bar{1}}m$ [58,59] where the superscripts refer to spin-space operations coinciding with real-space generators [60].

The Landau theory for the Néel order parameter takes the usual form

$$\Phi = a_2 \boldsymbol{N} \cdot \boldsymbol{N} + a_4 (\boldsymbol{N} \cdot \boldsymbol{N})^2, \quad (3)$$

enforced by spin-rotation and time-reversal symmetry. This conventional Landau theory becomes less standard when couplings to other observables are included. For the D_{4h} point group $\Gamma_Q = A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_g$ and so the only component that transforms like $\boldsymbol{N} \sim B_{2g}$ is the xy spatial quadrupole coupled with the magnetization vector. We thus have a linear coupling $\propto \boldsymbol{N} \cdot \boldsymbol{O}_{xy}$, as defined in Eq. (1). It follows that $\boldsymbol{O}_{xy}$ is a secondary order parameter generated when the primary order parameter $\boldsymbol{N}$ becomes finite. Explicitly, the free energy for $\boldsymbol{O}_{xy}$ would be $\Phi[\boldsymbol{O}_{xy}] = \Phi_0[\boldsymbol{O}_{xy}] - g\boldsymbol{N} \cdot \boldsymbol{O}_{xy} + \cdots$, where $\Phi_0[\boldsymbol{O}_{xy}]$ is the part of the free energy involving $\boldsymbol{O}_{xy}$ alone and g is the linear coupling.

The presence of this magnetic octupole can be directly tied to the structure of the corresponding altermagnetic band spin splitting. When $\boldsymbol{N} \neq 0$, hoppings and on-site terms are allowed that couple linearly to $\boldsymbol{N}$ and thus transform spatially as the same nontrivial one-dimensional IR as $\boldsymbol{N}$. As the physics is independent of spin orientation, without loss of generality, we may consider one orientation of $\boldsymbol{N}$ whereupon the spin components decouple, and the spatial dependence of the new spin-dependent terms follows the *spatial* part of the multipolar secondary order parameter. The spin-splitting of the bands thus has a form factor that mirrors the multipole induced locally. In the case of the rutiles, this gives a spin splitting $\sim k_x k_y$ implying that the spin of the Fermi surface, in itinerant altermagnets, reverses in $\pi/4$ rotations about the c axis, as has been established on the basis of *ab initio* calculations [8].

The nontrivial transformation properties of the Néel order parameter have implications for coupling to other observables even in the zero SOC limit. For example, magnetoelastic couplings and piezomagnetism can be readily understood from this Landau perspective. In the absence of SOC, $|\boldsymbol{N}|^2$ and $|\boldsymbol{M}|^2$ couple trivially to the strains $\epsilon_{xx}+\epsilon_{yy}$ and ϵ_{zz}, as dictated by the underlying tetragonal cell. Remaining in this nonrelativistic setting, the rutile crystal exhibits nontrivial piezomagnetic couplings, even in absence of SOC. To see this, note that $\boldsymbol{N}\cdot\boldsymbol{M}$ transforms like $B_{2g}\otimes\Gamma_1^S$, identical to the strain ϵ_{xy}. In an applied field $\boldsymbol{H}$, the Landau theory thus admits a term of the form $\propto \epsilon_{xy}\boldsymbol{N}\cdot\boldsymbol{H}$ (see also Steward *et al.* [43]). A finite staggered magnetization in the altermagnetic phase then results in a shear distortion under an applied magnetic field. As noted by Dzyaloshinskii [61], the introduction of SOC leads to an additional coupling $\propto (\epsilon_{xz}H_y+\epsilon_{yz}H_x)N_z$.

We can relate the appearance of piezomagnetism to the underlying altermagnetism more generally. Considering the field transforms as $\boldsymbol{H}\sim\Gamma_1\otimes\Gamma_A^S$ and strain as $\epsilon_{\mu\nu}\sim\Gamma_Q\otimes\Gamma_1^S$ (ignoring the uniform strain component) trilinear couplings with $\boldsymbol{N}$ transform as

$$(\Gamma_N\otimes\Gamma_A^S)\otimes(\Gamma_1\otimes\Gamma_A^S)\otimes(\Gamma_Q\otimes\Gamma_1^S).$$

For the spin part we must take the Γ_1^S component of $\Gamma_A^S\otimes\Gamma_A^S$, corresponding to $\boldsymbol{N}\cdot\boldsymbol{H}$, and then we are left with a spatial part $\Gamma_N\otimes\Gamma_Q$. Thus we can conclude: if $\boldsymbol{N}$ couples linearly to an octupole in the absence of SOC, then it will necessarily exhibit piezomagnetism in the absence of SOC, with a trilinear coupling between $\epsilon_{\mu\nu}$, $\boldsymbol{H}$, and $\boldsymbol{N}$, as is the case for a d-wave altermagnet. Note that if an octupole is not generated, for example, for g- or i-wave altermagnetism, the piezomagnetism may still be generated linearly (if $\Gamma_N\subset\Gamma_A$) or nonlinearly (if $\Gamma\not\subset\Gamma_A$) as for the magnetization.

Since $\Gamma_N\subset\Gamma_Q$ here, requiring a octupolar secondary order parameter, we immediately see both weak ferromagnetism and a finite anomalous Hall response linear in $\boldsymbol{N}$ should be expected. More explicitly, when spin and space rotations are coupled $M_x\hat{\boldsymbol{x}}+M_y\hat{\boldsymbol{y}}$ and $N_x\hat{\boldsymbol{x}}+N_y\hat{\boldsymbol{y}}$ both transform like E_g allowing a linear coupling $M_xN_y+M_yN_x$, arising microscopically from Dzyaloshinskii-Moriya exchange. We note that a staggered magnetization along the $\hat{\boldsymbol{z}}$ direction alone does not have a linear coupling to the ferromagnetic moment. For the rutile, σ_H^{xy} transforms as A_{2g} and the other two components σ_H^{yz} and σ_H^{zx} like E_g. Thus, with SOC we see σ_H^{xy} only couples to M_z and σ_H^{yz}, σ_H^{zx} only to the transverse components of both the Néel vector and the magnetization. While the anomalous Hall effect detected in RuO_2 is a conventional symmetry-allowed (not fundamentally altermagnetic) response, we see that it is intimately connected to the presence of a octupolar secondary order parameter and the underlying spin group symmetries.

We have seen that the multipolar secondary order parameter in the rutile case required by $\Gamma_N\subset\Gamma_Q$ fixed many of the phenomenological altermagnetic responses expected both with and without SOC. We will next consider MnTe where the quadrupole Γ_Q does *not* contain Γ_N and the generation of higher multipoles must be considered. We also show that the magnetization, anomalous Hall conductivity, and piezomagnetism all arise nonlinearly in $\boldsymbol{N}$.

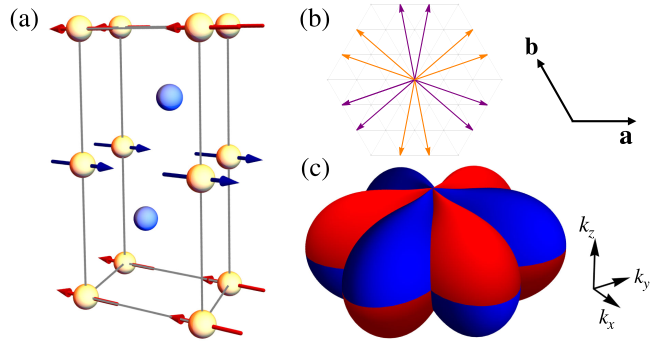

FIG. 2. Illustration of the key features of altermagnetic MnTe including (a) the crystal structure with magnetic Mn ions on an *AA* stacked triangular lattice, (b) the inequivalent bonds connecting neighboring magnetic layers along the $\boldsymbol{c}$ direction that enter into the model of Eq. (2), and (c) the g-wave spin-split Fermi surface expected in weakly doped MnTe.

Hexagonal MnTe.—This material [35,37,62] has magnetic manganese ions on an *AA* stacked triangular lattice. The Mn ions live on the $2a$ Wyckoff positions of space group $P6_3/mmc$ (No. 194) and the Te ions on the $2c$ Wyckoff positions. The magnetic structure is one with in-plane moments that are antialigned between neighboring triangular layers [see Fig. 2(a)] [63]. The primary order parameter is the Néel vector $\boldsymbol{N}$ as in the case of the rutile altermagnet and the Landau theory is therefore identical to Eq. (3). The point group is $6/mmm$ (D_{6h}) and $\boldsymbol{N}$ transforms as B_{1g} and $\boldsymbol{M}$ as A_{1g}. [64] For MnTe, one has that $\Gamma_N\not\subset\Gamma_Q$ and thus a magnetic octupole is not induced. In the language of Šmejkal *et al.* [25], this is g-wave altermagnetism. However, it is straightforward to see there is a higher order rank-5 magnetic multipole

$$\boldsymbol{O}_3^4\equiv\int d^3r[Y_3^4(\hat{\boldsymbol{r}})-Y_{-3}^4(\hat{\boldsymbol{r}})]\boldsymbol{m}(\boldsymbol{r}),\tag{4}$$

where Y_m^l is a spherical harmonic that transforms as B_{1g}—identically to $\boldsymbol{N}$. The Landau free energy for this multipolar order parameter would then include a linear coupling. Explicitly, one would write $\Phi[\boldsymbol{O}_3^4]=\Phi_0[\boldsymbol{O}_3^4]-g\boldsymbol{N}\cdot\boldsymbol{O}_3^4+\cdots$, where $\Phi_0[\boldsymbol{O}_3^4]$ is the part of the free energy involving $\boldsymbol{O}_3^4$ alone and g is the coupling. This magnetic multipole is therefore a secondary order parameter with a g-wave symmetry. The higher rank of this multipole is reflected in the nature of the band spin splitting [see Fig. 2(c)] which contains lines where the spin splitting vanishes. For this case, a toy model can be formulated along the same lines as the rutile example, but with the essential inequivalent bonds lying at relatively long range [see illustration in Fig. 2(b)]. This case highlights the potential for sufficiently long-range symmetry inequivalent hoppings to be important for altermagnetism in materials.

In contrast to the rutile case, symmetry does not permit a direct coupling between the magnetization and the staggered magnetization even in the presence of SOC as $\Gamma_N \not\subset \Gamma_Q$ or Γ_A. Therefore, altermagnetism does not coincide, in general, with weak ferromagnetism or with an anomalous Hall conductivity appearing linearly in $\boldsymbol{N}$. Explicit symmetry analysis reveals that coupling between $\boldsymbol{N}$ and $\boldsymbol{M}$ or $\sigma_H^{\mu\nu}$ appears first at *third* order in $\boldsymbol{N}$. Restricting to an in-plane $\boldsymbol{N} = N_x\hat{\boldsymbol{x}} + N_y\hat{\boldsymbol{y}}$, as is relevant experimentally for MnTe [63], one finds a single allowed coupling,

$$\sigma_H^{xy} = a_3 N_y (3N_x^2 - N_y^2) + \cdots, \qquad (5)$$

between $\boldsymbol{N}$ and $\sigma_H^{\mu\nu}$ with an identical relation holding for the weak ferromagnetic moment M_z. From the perspective of this Landau theory the generation of higher multipolar secondary order parameters thus leads to cubic (or higher) couplings between the Néel vector and the magnetization or Hall conductivity. We note that, experimentally, the observed temperature dependence of the Hall signal σ_H^{xy} in MnTe appears convex near T_{N}, perhaps consistent with a nonlinear dependence [Eq. (5)] on the order parameter [46]. Similarly, unlike for the rutile case, MnTe piezomagnetism, reported in Aoyama and Ohgushi [62], appears only in the presence of SOC or involves nonlinear couplings to $\boldsymbol{N}$ or $\boldsymbol{H}$.

Discussion.—The ideas of the previous sections can be used straightforwardly to formulate Landau theories for other candidate altermagnetic materials, d, g, and i wave, with or without SOC, as well as predict how they will couple to new physical observables.

For example, CuF_2 has the Néel vector transforming as the B_g IR of $C_{2\mathrm{h}}$ $(2/m)$. Since Γ_Q contains two copies of B_g [65], its Néel vector $\boldsymbol{N}$ can couple separately to the $\boldsymbol{O}_{21}$ and $\boldsymbol{O}_{21}^s$ time odd multipoles ($l = 2$, $m = 1$ Stevens operators for the spatial quadrupole), with two sets of inequivalent bonds in the xz and yz planes. We can thus infer that CuF_2 should exhibit weak ferromagnetism and an anomalous Hall effect linear in the Néel order parameter, as well as piezomagnetism in the absence of SOC.

Other observables can also be treated within this framework. For example, one can consider the generation of spin currents [47], characterized by a spin conductivity tensor defined through $\boldsymbol{J}_\mu^S = \sum_{\mu\nu} \boldsymbol{\sigma}_S^{\mu\nu} E_\nu$ where $\boldsymbol{E}$ is the electric field and the vector index encodes the spin direction. This transforms as $\boldsymbol{\sigma}_S^{\mu\nu} \sim (\Gamma_V \otimes \Gamma_V) \otimes \Gamma_A^S = (\Gamma_1 \oplus \Gamma_A \oplus \Gamma_Q) \otimes \Gamma_A^S$. Thus, if $\Gamma_N \subset \Gamma_Q$ or Γ_A then $\boldsymbol{N}$ can appear linearly in $\boldsymbol{\sigma}_S^{\mu\nu}$ in the absence of SOC. For the rutile case, we would thus expect a spin conductivity $\boldsymbol{\sigma}_S^{xy} \propto \boldsymbol{N}$. For cases where $\Gamma_N \not\subset \Gamma_Q$ or Γ_A, like in MnTe, this would necessarily involve a higher polynomial in the Néel vector $\boldsymbol{N}$.

While we have considered multipolar secondary order parameters that are even in their spatial components, when the magnetic structure lacks inversion we may find odd spatial multipoles as well. For example, point group $C_{6\mathrm{v}}$ $(6mm)$, admits collinear antiferromagnetic spin groups and has IRs B_1 and B_2 that allow linear couplings between certain time odd, space odd multipolar order parameters and the appropriate Néel order parameter. We leave the exploration of these multipoles for future work.

These ideas can also be generalized to noncollinear magnets. There is recent literature cataloging the possible multipolar orders in noncollinear magnets and connecting their symmetries to their response (see, for example, Refs. [66–70]). These noncollinear magnets can also be described using the framework discussed here. For example, the kagome lattice with $\boldsymbol{Q} = 0$, 120° order [67,71,72] has a two component order parameter that can be encoded in a complex vector

$$\boldsymbol{\Psi} = e^{+2\pi i/3}\boldsymbol{S}_1 + e^{-2\pi i/3}\boldsymbol{S}_2 + \boldsymbol{S}_3,$$

leading to quadratic invariant $\propto \boldsymbol{\Psi}^* \cdot \boldsymbol{\Psi}$. In the Landau theory this can couple linearly to a d-wave multipole in IR E_{2g} of $D_{6\mathrm{h}}$ $(6/mmm)$ with components $k_x^2 - k_y^2$ and $k_x k_y$ that itself is reflected in the spin expectation value within each band.

Conclusion.—In this Letter, we have explored the application of Landau theory to altermagnets. This framework ties together several key ideas that have arisen in this burgeoning field including spin-split bands, spin symmetries, multipolar order parameters, and the phenomenology of these materials both with and without SOC. We have given examples of spin symmetric time odd multipolar order parameters that characterize these magnets, as well as outlining their generalization to noncollinear altermagnetic behavior. These techniques are straightforwardly generalizable to the many candidate altermagnetic materials [25,26] and we hope they will prove useful in sharpening predictions of altermagnetic phenomenology.

More broadly, the considerations underpinning our Landau theory, and altermagnets viewed widely, flow from the need to generalize magnetic symmetries from the magnetic space groups to spin symmetry groups when SOC is weak [58,59,72–83]. The induction of multipolar secondary order parameters would likely also need to be revisited in this broader context, especially as SOC is reintroduced [11,12,72,84,85] as has been recently discussed in Ref. [86]. Altermagnets provide a striking demonstration that there is much to be gained by thinking about novel phases, band structures, and response functions in the context of these higher symmetries. Landau theories built from order parameters with given spin symmetries are the natural language to explore the resulting new physics and reveal how these symmetries control the altermagnetic phenomenology when SOC is introduced.

P. M. acknowledges useful discussions with Libor Šmejkal and many interesting and informative talks at the SPICE workshop "Altermagnetism: Emerging Opportunities in a New Magnetic Phase" 2023. Work at the University of Windsor (J. G. R.) was funded by the Natural Sciences and Engineering Research Council of Canada (NSERC) (Funding Reference No. RGPIN-2020-04970).

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