Erratum: Massive Gravitons as Feebly Interacting Dark Matter Candidates [Phys. Rev. Lett. 128, 081806 (2022)]

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There is a sign error in the fermion propagator in our calculation of $\bar{q}(p_1) + q(p_2) \to g^{\rho}(k_1) + G^{\mu\nu}(k_2)$. We listed our explicit amplitudes in the Supplemental Material, where the correct coupling vertices under the $\xi = 1$ gauge are applied for all channels. However the numerator of fermion propagator should be corrected to be $(\not{p}_1 - \not{k}_1 + m_q)$ in Eq. (3) and $(\not{p}_1 - \not{k}_2 + m_q)$ in Eq. (4) of the Supplemental Material. As per the energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$, the amplitude of single production of graviton will always satisfy $k_2^{\mu}\mathcal{M}_{\mu\nu} = 0$ assuming that the massive graviton universally couples to all standard model (SM) particles. In the high energy limit $s \to \infty$, the amplitude squared for $\bar{q}(p_1) + q(p_2) \to g^{\rho}(k_1) + G^{\mu\nu}(k_2)$ is given by

$$|\mathcal{M}|^2 = 8g_s^2\left[(1+\cos^2\theta)s - \left(\cos(2\theta)(M_G^2 + 2m_q^2) + \frac{2}{3}\csc^2\theta(10m_q^2 - 3M_G^2) - 10m_q^2\right)\right] + \mathcal{O}(s^{-1}). \tag{1}$$

As expected, the perturbative unitarity is respected by Eq. (1). In particular since the result for $m_q = 0$ is not influenced by the sign flip of m_q, Eq. (7) in the Supplemental Material exactly reproduces the differential cross section in Refs. [9–10]. It should be noted that the cancellation in the enhanced terms of the cross section is crucially due to the universal couplings of the massive gravitons to matter, i.e., quarks and gluons in the case under study.

In models where nonuniversal couplings of the massive graviton are allowed, enhanced terms do survive. For instance, in the three-brane Randall–Sundrum (RS) model considered in the Letter, we can envision a situation where the gauge bosons propagate within the first slice, while all other SM particles reside on the middle brane. The brane coupling in the multibrane RS models is

$$C_H = \frac{1}{M_{pl}}\frac{e^{2kr_1\pi}}{N_n}\left[J_2\left(M_G\frac{e^{kr_1\pi}}{k}\right) + \alpha_n Y_2\left(M_G\frac{e^{kr_1\pi}}{k}\right)\right] \simeq \frac{1}{\Lambda_H}\frac{x_n^2}{4\sqrt{2}J_2(x_n)}, \tag{2}$$

where J_2 and Y_2 are Bessel functions with $N_n \simeq (e^{kr_2\pi}/\sqrt{2})J_2(x_n)$ and $J_1(x_n) = 0$. Instead, the graviton coupling to two bulk gluons in the first slice is

$$C_V = \frac{1}{\pi M_{pl}}\int_0^{\pi} d\phi \frac{e^{2kr_1\phi}}{N_n}\left[J_2\left(M_G\frac{e^{kr_1\phi}}{k}\right) + \alpha_n Y_2\left(M_G\frac{e^{kr_1\phi}}{k}\right)\right] \simeq \frac{1}{4kr_1\pi}C_H. \tag{3}$$

As the bulk coupling is highly suppressed, i.e., $C_V \ll C_H$, the coupling difference simply is $\Delta_C = C_H - C_V \simeq C_H$.

Henceforth, in the nonuniversal coupling scenario, the amplitude squared for $q\bar{q} \to gG$ at the leading order yields

$$|\mathcal{M}|^2_{q\bar{q}} = \frac{8g_s^2\Delta_C^2 s\left(2(m_q^2 - t)^2 + s^2 + 2st\right)}{3M_G^4}. \tag{4}$$

Then integrating over the angular phase space gives

$$\mathcal{A}_{q\bar{q}} = \int d\Omega|\mathcal{M}|^2_{q\bar{q}} = 2\pi\Delta_C^2\frac{32g_s^2s^2(s+2m_q^2)}{9M_G^4}, \tag{5}$$

where we can clearly see the emergence of a leading term enhanced by $1/M_G^4$, as discussed in the Letter. For the process of $qg \to qG$, one can use the cross symmetry of $|\mathcal{M}|^2_{qg} = -|\mathcal{M}|^2_{q\bar{q}}(s \leftrightarrow t)$. It is easy to derive that

$$\mathcal{A}_{qg} = \int d\Omega |\mathcal{M}|^2_{qg} = 2\pi \Delta_C^2 \frac{4g_s^2(s - m_q^2)^4 \left(3(s - m_q^2)^2 + 4s^2\right)}{9M_G^4 s^3}. \tag{6}$$

With the Y_{IR} formula, Eqs. (5) and (6) lead to an estimate of the relic density that qualitatively matches the result presented in the Letter. Hence, the main conclusion remains unchanged as long as nonuniversal cases are considered.

We identified another minor mistake: we listed a nonzero coupling of C_r due to a wrong reference. The correct interaction between two gravitons and one radion r, after radion stabilization, is [1]

$$\mathcal{L}_{G^2 r} = \frac{3}{4\kappa^2} \int dy\, e^{-4A} r \left(G'^{\mu\nu} G'_{\mu\nu} - G'^2\right) \tag{7}$$

where the prime stands for the ∂/∂_y derivative in the fifth dimension and $A(y)$ is the warped metric. In fact, the Ref. [1] proves that other interaction forms proportional to $(r' - 2A'r)$ and A'' will exactly cancel the expansions from the Goldberger-Wise scalar terms, hence the coupling is the same as the one prior to radion stabilization [2,3]. Since the 5d profile of the zero mode graviton is a constant, the effective coupling $G^{(n)}$-$G^{(0)}$-r is zero. This means that a massive graviton cannot decay into a massless graviton plus a radion in the RS model. This property generally permits a light radion in the regime of $m_r > M_{G^{(1)}}/2$, while keeping our dark matter candidate stable over the Hubble timescale. This change does not affect the decay width of the DM graviton $G^{(1)}$ at all.

[1] H. Cai, Diffeomorphism on-shell breaking from radion stabilization, arXiv:2309.07904.
[2] A. de Giorgi and S. Vogl, Unitarity in KK-graviton production: A case study in warped extra dimensions, J. High Energy Phys. 04 (2021) 143.
[3] Note that the r^3 interaction without GW stabilization in Ref. [2] omitted a term of $\mathcal{L}_{r^3} \supset -(64/3\kappa^2) \int dy e^{-4A} A'^2 r^3$ that is necessary to preserve the 5d diffeomorphism.