Parent Hamiltonian Reconstruction via Inverse Quantum Annealing

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Finding a local Hamiltonian $\hat{\mathcal{H}}$ that has a given many-body wave function $|\psi\rangle$ as its ground state, i.e., a parent Hamiltonian, is a challenge of fundamental importance in quantum technologies. Here we introduce a numerical method, inspired by quantum annealing, that efficiently performs this task through an artificial inverse dynamics: a slow deformation of the states $|\psi(\lambda(t))\rangle$, starting from a simple state $|\psi_0\rangle$ with a known $\hat{\mathcal{H}}_0$, generates an adiabatic evolution of the corresponding Hamiltonian. We name this approach inverse quantum annealing. The method, implemented through a projection onto a set of local operators, only requires the knowledge of local expectation values, and, for long annealing times, leads to an approximate parent Hamiltonian whose degree of locality depends on the correlations built up by the states $|\psi(\lambda)\rangle$. We illustrate the method on two paradigmatic models: the Kitaev fermionic chain and a quantum Ising chain in longitudinal and transverse fields.

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Introduction.—The success of quantum technologies ultimately relies on our ability to control increasingly complex artificial quantum systems [1]. This may require, in quantum simulators, the accurate tailoring of a many-body Hamiltonian. Controlling [2,3] and verifying [4,5] the actual functioning of these systems have raised increasing attention to the search for parent Hamiltonians (PHs) [6–15]. This problem consists in finding a local and/or engineerable Hamiltonian having a given wave function as a ground state. The knowledge of a PH is related to Hamiltonian learning [16–18] and verification of quantum devices, and can be exploited to experimentally prepare a target ground state. The search for a PH represents an especially complex instance of the reconstruction of a Hamiltonian from one of its eigenstates [19–23] or time-dependent states [24–29]. In particular, the space of the Hamiltonians having a given state as an eigenstate can be efficiently reconstructed from correlation functions [15,19,20] or expectation values of local commutators [21]. Picking PHs in this space is a hard task since it generally requires the diagonalization of all the candidate PHs to verify that the target state is a ground state [15]. More efficient methods, based on local measurements, have been suggested to obtain approximate PHs [13,14,22,23,30].

Here we introduce a method for obtaining a PH, referred to as inverse quantum annealing (IQA), which is inspired by quantum annealing [31–35], but with the role of states and Hamiltonians swapped. Given a state $|\psi_1\rangle$, whose PH $\hat{\mathcal{H}}_1$ we wish to construct, and starting from a simple state $|\psi_0\rangle$ with a well-known PH $\hat{\mathcal{H}}_0$, we construct a path $|\psi(\lambda(t))\rangle$, connecting $|\psi_0\rangle$ to $|\psi_1\rangle$. We then write down an artificial dynamics for the Hamiltonian, inspired by von Neumann's equation for the density matrix, which is amenable to a well-defined adiabatic limit, and, more importantly, can be approximately solved with a space of local Hamiltonians. In the adiabatic limit we obtain a Hamiltonian having the target state as the ground state. We will illustrate the main ideas of the method with two paradigmatic examples: (1) the "exactly solvable" Kitaev fermionic chain, where $|\psi(\lambda)\rangle$ crosses a second-order transition point, and (2) a quantum Ising chain in the presence of a longitudinal field, where $|\psi(\lambda)\rangle$ crosses a first-order transition point. In case (1) power-law correlations emerge, making the local approximation harder, while in case (2) correlations exponentially decrease and the local approximation is excellent.

Inverse quantum annealing protocol.—Given a (many-body) state $|\psi_1\rangle$, the task is to find a (local) Hamiltonian $\hat{\mathcal{H}}_1$ for which $|\psi_1\rangle$ is the ground state. In many cases, this problem has more than one solution [36]. To find a solution, we introduce a method inspired by quantum annealing [31–33], alias adiabatic quantum computation [35]. The first step is to introduce a family of states

$|\psi(\lambda)\rangle$, depending on a parameter λ, with $0 \leq \lambda \leq 1$, such that (i) $|\psi(1)\rangle \equiv |\psi_1\rangle$ is the quantum state whose PH $\hat{\mathcal{H}}_1$ we wish to determine, and (ii) $|\psi(0)\rangle \equiv |\psi_0\rangle$ is a simple initial state whose PH $\hat{\mathcal{H}}_0$ is known. Then we seek a dynamics that, by changing $\lambda(t)$ with time, in the adiabatic limit $\dot{\lambda}(t) \to 0$, leads to the desired $\hat{\mathcal{H}}_1$, starting from $\hat{\mathcal{H}}_0$.

The idea behind this artificial Hamiltonian dynamics is the following. Consider the projector $\hat{\Pi}_{\psi(\lambda)}$ on the selected state path $|\psi(\lambda)\rangle$, suitably redefined as follows:

$$\hat{\Pi}_{\psi(\lambda)} = -J|\psi(\lambda)\rangle\langle\psi(\lambda)|, \tag{1}$$

where J is an arbitrary energy scale which we use as our unit, setting $J = 1$. $\hat{\Pi}_{\psi(\lambda)}$ has $|\psi(\lambda)\rangle$ as its unique ground state, at energy $-J$, while all other states are degenerate, at energy 0.

Regard now $\hat{\Pi}_{\psi(\lambda(t))}$ as a (pseudo-)Hamiltonian—in general, nonlocal—generating a Schrödinger dynamics associated to the evolution operator

$$\hat{U}(t) = T{-}\exp\left(-\frac{i}{\hbar}\int_0^t \mathrm{d}t'\hat{\Pi}_{\psi(\lambda(t'))}\right). \tag{2}$$

By assumption a local PH $\hat{\mathcal{H}}_0$ for $|\psi_0\rangle$ exists. Consider now an "auxiliary Hamiltonian" $\hat{H}_{\mathrm{aux}}(t) = \hat{U}(t)\hat{\mathcal{H}}_0\hat{U}^\dagger(t)$. It will satisfy von Neumann's equation:

$$\partial_t \hat{H}_{\mathrm{aux}}(t) = -\frac{i}{\hbar}\left[\hat{\Pi}_{\psi(\lambda(t))}, \hat{H}_{\mathrm{aux}}(t)\right], \tag{3}$$

with the boundary condition $\hat{H}_{\mathrm{aux}}(0) = \hat{\mathcal{H}}_0$. The presence of the spectral gap J in $\hat{\Pi}_{\psi(\lambda)}$ guarantees that, in the adiabatic limit $\dot{\lambda} \to 0$, the time-evolved state $\hat{U}(t)|\psi_0\rangle$ will be closer and closer to the desired path of states $|\psi(\lambda(t))\rangle$, and, correspondingly, the "Hamiltonian" $\hat{H}_{\mathrm{aux}}(t)$ will approximate a PH $\hat{\mathcal{H}}(\lambda(t))$. The nontrivial issue with such an adiabatically inspired solution for the PH problem is the possible nonlocality of the Hamiltonian determined. We need to devise a further local approximation for the PH problem to guarantee that the solution found is actually a physical local PH.

Before tackling the locality issue, let us rewrite our equation using a fixed basis of Hermitian operators $\mathcal{P} = \{\hat{P}_\mathrm{j}\}$ acting on a system of N particles. As an example, think of a system made of N spin-1/2, where $\hat{P}_\mathrm{j}$ are all possible Pauli string operators made by an arbitrary number of Pauli matrices $\hat{\sigma}_i^{x,y,z}$ at sites i. Without loss of generality, we can assume that the normalization of the operators is such that $\mathrm{Tr}(\hat{P}_\mathrm{j}\hat{P}_{\mathrm{j}'}) = \delta_{\mathrm{j},\mathrm{j}'}$ [37]. For any finite N, the total number of elements in $\mathcal{P}$ is finite, $\mathcal{N} = |\mathcal{P}|$. Any arbitrary Hermitian operator can be expanded in the basis $\mathcal{P}$, for instance, $\hat{H}_{\mathrm{aux}}(t) = \sum_\mathrm{j} h_j(t)\hat{P}_\mathrm{j}$. By substituting in Eq. (3), after simple algebra, see [37], we can rewrite Eq. (3) as

$$\partial_t h_\mathrm{j}(t) = \sum_{\mathrm{j}'=1}^{\mathcal{N}} K_{\mathrm{j},\mathrm{j}'}[\psi(\lambda(t))]h_{\mathrm{j}'}(t), \tag{4}$$

where $K_{\mathrm{j},\mathrm{j}'}[\psi] \equiv -i(J/\hbar)\langle\psi|[\hat{P}_\mathrm{j}, \hat{P}_{\mathrm{j}'}]|\psi\rangle$ is a skew-symmetric commutator matrix.

The space of l-local Hamiltonians is formed by linear combinations of a subset $\mathcal{L}^{(l)} \subset \mathcal{P}$ consisting of all Hermitian operators connecting particles within maximum distance l on a lattice, whose number we denote as $\mathcal{N}_l$. (Let us call the maximum coupling distance of an operator as its coupling length.) As an example, a one-local Hamiltonian contains only single-particle terms, while a two-local Hamiltonian will also contain two adjacent particle interactions. In both cases, $\mathcal{N}_l$ scales linearly in N. Any l-local Hamiltonian is written as $\hat{\mathcal{H}}^{(l)} = \sum_\mathrm{j} h_\mathrm{j}\hat{L}_\mathrm{j}^{(l)}$. We call a Hamiltonian *local* if its locality range l does not depend on the system size N.

To find an optimal l-local PH from the adiabatic solution of Eq. (3), we now use a time-dependent variational principle (TDVP) [41], which allows us to determine the Hamiltonian couplings $h_\mathrm{j}(t)$, by projecting the right-hand side of Eq. (3) on the space $\mathcal{L}^{(l)} = \{\hat{L}_\mathrm{j}^{(l)}\}$, through the Hilbert-Schmidt distance $d(\hat{A}, \hat{B}) = \sqrt{\mathrm{Tr}(\hat{A} - \hat{B})^2}$, a natural Euclidean structure in the space of Hermitian operators. As detailed in [37], we can write the resulting projected evolution as

$$\partial_t \hat{\mathcal{H}}^{(l)} = \mathbb{P}_l\left(-\frac{i}{\hbar}\left[\hat{\Pi}_{\psi(\lambda(t))}, \hat{\mathcal{H}}^{(l)}(t)\right]\right), \tag{5}$$

where the projector $\mathbb{P}_l(\hat{A})$ defines the closest l-local operator $\hat{B}$ to a given $\hat{A}$. This leads to the following equation for the coefficients $h_\mathrm{j}(t)$ of the l-local Hamiltonian $\hat{\mathcal{H}}^{(l)}(t)$:

$$\partial_t h_\mathrm{j}(t) = \sum_{\mathrm{j}'=1}^{\mathcal{N}_l} K^{(l)}_{\mathrm{j},\mathrm{j}'}[\psi(\lambda(t))]h_{\mathrm{j}'}(t), \tag{6}$$

where $K^{(l)}_{\mathrm{j},\mathrm{j}'}[\psi] \equiv -i(J/\hbar)\langle\psi|[\hat{L}_\mathrm{j}^{(l)}, \hat{L}_{\mathrm{j}'}^{(l)}]|\psi\rangle$ has a size that scales polynomially with the system size N. Let us note that Eq. (6) is a truncated version of Eq. (4), with the commutator matrix restricted to the space of l-local operators.

Equation (6) is the central result of this work. In the adiabatic regime, it allows us to construct an l-local PH $\hat{\mathcal{H}}_1$ for the final state $|\psi_1\rangle$, by integrating the differential equations from $t = 0$, with the initial condition set by the expansion coefficients $h_\mathrm{j}(0)$ of $\hat{\mathcal{H}}_0$, up to a suitably large annealing time T. While the nonprojected adiabatic scheme behind Eqs. (1)–(3) is, by construction, protected by a gap J, there is no guarantee that the projection on the

space of l-local Hamiltonians will not bring in components of higher states.

The error introduced by the l-local approximation is inherently related to the nature of the path of states $|\psi(\lambda)\rangle$. Our physical expectation is that if $|\psi(\lambda)\rangle$ has a finite correlation length, then the adiabatic solution of Eq. (3) is a local Hamiltonian. In this case the matrix $K^{(l)}_{\mathrm{j,j'}}$ connects operators with similar coupling length, and the truncation leading from Eq. (4) to Eq. (6) gives rise to a small error (akin to truncating a short-range Hamiltonian—we discuss this point in [37]). In this case, we can truncate to a coupling length l, independent of the system size, such that the error introduced by the TDVP projection is arbitrarily small. The opposite occurs if there is a λ_c where the correlation length of $|\psi(\lambda)\rangle$ diverges: $K^{(l)}_{\mathrm{j,j'}}$ connects operators with very different coupling lengths, and the truncation gives rise to a large error.

The commutator matrix $K^{(l)}_{\mathrm{j,j'}}$ has been used in previous works to reconstruct local Hamiltonians from their eigenstates [21]. Previous methods, however, could not guarantee finding Hamiltonians having $|\psi(\lambda)\rangle$ as the ground state. The IQA, implemented through Eq. (6), can select a PH, without the need of an explicit diagonalization for checking the solution.

We now illustrate our method on two paradigmatic examples: (1) the integrable quantum Ising chain in a transverse field, where we will exploit the Jordan-Wigner mapping to the 1D Kitaev chain, illustrating the case of a diverging correlation length for $|\psi(\lambda)\rangle$ at a second-order transition; (2) the nonintegrable quantum Ising chain in a transverse and longitudinal field, where the path $|\psi(\lambda)\rangle$ crosses a first-order transition, with a finite correlation length.

(1) IQA with fermionic Gaussian states: To illustrate our IQA method, we apply it here to the BCS-like states

$$|\psi(\lambda)\rangle \equiv \prod_{\substack{k=(2n-1)\pi/N \\ n\in\{1,\ldots,N/2\}}} \left(\sin(\theta_k)+\cos(\theta_k)\hat{c}^\dagger_k\hat{c}^\dagger_{-k}\right)|0\rangle,$$

with

$$\theta_k(\lambda)=\frac{1}{2}\arctan\left(\frac{\sin(\lambda\pi/2)\sin(k)}{\cos(\lambda\pi/2)+\sin(\lambda\pi/2)\cos(k)}\right).$$

Here $|0\rangle$ is the vacuum state, $\hat{c}^\dagger_k=(e^{i\pi/4}/\sqrt{N})\sum_{j=1}^N e^{ikj}\hat{c}^\dagger_j$, and $\hat{c}^\dagger_j$ creates a spinless fermion at site $j=1,\ldots N$. $|\psi(\lambda)\rangle$ is the ground state of the 1D Kitaev model [42]

$$\hat{\mathcal{H}}_{\mathrm{K}}(\lambda)=\sum_{j=1}^{N}\left[\sin\left(\lambda\frac{\pi}{2}\right)(\hat{c}^\dagger_j\hat{c}^\dagger_{j+1}+\hat{c}^\dagger_j\hat{c}_{j+1}+\mathrm{H.c.})+\cos\left(\lambda\frac{\pi}{2}\right)(\hat{c}^\dagger_j\hat{c}_j-\hat{c}_j\hat{c}^\dagger_j)\right], \quad (7)$$

with antiperiodic boundary conditions $\hat{c}_{N+1}=-\hat{c}_1$, hence a two-local PH exists. The goal is to use the dynamics defined in Eq. (6) to find a PH, using the exactly known results to quantify the accuracy of the IQA. The annealing schedule is $\lambda(t)=t/T$, where T is the final time so that the state interpolates between $|\psi(0)\rangle$, the ground state of $\hat{\mathcal{H}}_{\mathrm{K}}(0)$, and $|\psi(1)\rangle$, the ground state of $\hat{\mathcal{H}}_{\mathrm{K}}(1)$. Note that $|\psi(\lambda)\rangle$ passes through an emerging second-order phase transition critical point at $\lambda_c=1/2$ where the correlation length diverges. Therefore, as a consequence of the TDVP approximation, we expect IQA to work very well for paths such that $\lambda<\lambda_c$ while it may be less accurate if $\lambda\geq\lambda_c$.

We perform IQA with different annealing times T to study the convergence to the adiabatic limit, analyzing different ranges l of the interactions in $\mathcal{L}^{(l)}$, and different system sizes N. The basis $\mathcal{L}^{(l)}$ of translation and reflection invariant l-interacting quadratic fermions is $\{\Sigma^z_0/\sqrt{2},\Sigma^\alpha_1,\ldots,\Sigma^{\alpha'}_l,\ldots\}$ for $l<N/2$, and $\mathcal{L}^{(N/2-1)}\cup\{\Sigma^x_{N/2}/\sqrt{2},\Sigma^y_{N/2}/\sqrt{2}\}$ for $l=N/2$, where $\alpha,\alpha'\in\{x,y,z\}$ and $\Sigma^x_m=(1/2\sqrt{N})\sum_j(\hat{c}^\dagger_j\hat{c}^\dagger_{j+m}+\mathrm{H.c.})$, $\Sigma^y_m=(i/2\sqrt{N})\sum_j(\hat{c}^\dagger_j\hat{c}^\dagger_{j+m}-\mathrm{H.c.})$, and $\Sigma^z_m=(1/2\sqrt{N})\sum_j(\hat{c}^\dagger_j\hat{c}_{j+m}+\mathrm{H.c.})$. The antiperiodic boundary conditions imply $\hat{c}_{N+m}\equiv-\hat{c}_m$. The commutator matrix that generates the IQA is explicitly calculated in [37], where we also investigate the locality of the adiabatic solution of Eq. (3).

To verify when the adiabatic evolution is achieved, we compute the relative Hilbert-Schmidt operator distance

$$R_{T,\Delta T}(\lambda)=\sqrt{\mathrm{Tr}\left(\hat{\mathcal{H}}_{T+\Delta T}(\lambda)-\hat{\mathcal{H}}_T(\lambda)\right)^2/\mathrm{Tr}\hat{\mathcal{H}}^2_T(\lambda)} \quad (8)$$

between the Hamiltonians $\hat{\mathcal{H}}_T(\lambda)$ found by IQA, at fixed l and different final times T and $T+\Delta T$. Since the operators in $\mathcal{L}$ are orthonormal, this is the distance between the couplings vectors $(h_1,h_2,\ldots)$ of the two Hamiltonians. When the annealing time T is sufficiently large, the couplings $\{h_{\mathrm{j}}\}$ converge to those of the adiabatic Hamiltonian and $R_{T,\Delta T}(\lambda)$ goes to zero. In Fig. 1(a), we show the maximum value of $R_{T,\Delta T}(\lambda)$, for different annealing times and system sizes. The functions $\max_\lambda[R_{T,\Delta T}(\lambda)]$ for different values of N overlap and fit to $\max_\lambda[R_{T,\Delta T}(\lambda)]\propto 1/T$. The error is inversely proportional to the annealing time and is independent of the system size because the expectation values corresponding to the entries of the matrix $K^{(l)}_{\mathrm{j,j'}}$ converge for large N. Having determined the adiabatic regime, we fix $T=16\,000$ and investigate the properties of the PH obtained from the IQA.

In Fig. 1(b), we plot the fidelity $F(\lambda)=|\langle\psi^{(l)}_{\mathrm{GS}}(\lambda)|\psi(\lambda)\rangle|^2$ between the target state $|\psi(\lambda)\rangle$ and the unique ground state $|\psi^{(l)}_{\mathrm{GS}}(\lambda)\rangle$ of the adiabatic l-local Hamiltonian $\hat{\mathcal{H}}^{(l)}(\lambda)$ obtained from the IQA. If $\lambda<\lambda_c=1/2$ the fidelity $F(\lambda)$ is close to 1 even for relatively small values of l. This means

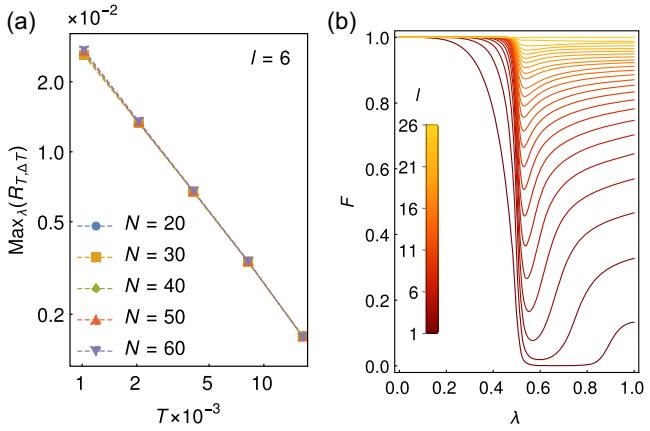

FIG. 1. IQA with fermionic Gaussian states. Panel (a): maximum value $\max_\lambda[R_{T,\Delta T}(\lambda)]$ of the relative distance $R_{T,\Delta T}(\lambda)$ between solutions of Eq. (6) with different annealing times T and $T+\Delta T=2T$, for $l=6$, as a function of the annealing time, for systems of different sizes. Panel (b): fidelity between the target state $|\psi(\lambda)\rangle$ and the ground state $|\psi^{(l)}_{\mathrm{GS}}(\lambda)\rangle$ of the adiabatic l-local Hamiltonian $\hat{\mathcal{H}}^{(l)}(\lambda)$ obtained via IQA. We consider a system of 50 sites and interaction ranges from $l=1$ to $l=26$.

that our algorithm finds an optimal l-local PH for the target state. The scenario changes when $\lambda>\lambda_c$. Indeed, as we show in [37], the capability of the evolution in Eq. (4) of connecting local and nonlocal operators, as well as the effective interaction range of its adiabatic solution, scales with the correlation length. As a consequence, the effects of long-range correlations can be accounted for by the TDVP only by increasing the value of l. This happens even though the Kitaev Hamiltonian is two-local, and is reminiscent of what happens in quantum annealing at phase transitions [43].

A more quantitative analysis is obtained by looking at the fidelity as a function of l for different system sizes. If we restrict our target state to $\lambda<\lambda_c$, the fidelity is close to 1 even at small values of l, and almost independent of N, see Fig. 2(a). In Fig. 2(b), the fidelity is shown for a target state $|\psi(\lambda)\rangle$ with λ close but beyond λ_c, $\lambda=1.1\lambda_c$. In this case, the larger l the better the fidelity, as expected.

In order to better quantify when $\hat{\mathcal{H}}^{(l)}(\lambda)$ is a suitable PH, let us fix a target accuracy ϵ. We can accept $\hat{\mathcal{H}}^{(l)}(\lambda)$ as a PH if the fidelity between $|\psi(\lambda)\rangle$ and the ground state $|\psi^{(l)}_{\mathrm{GS}}(\lambda)\rangle$ of $\hat{\mathcal{H}}^{(l)}(\lambda)$, $F_l(\lambda)=|\langle\psi(\lambda)|\psi^{(l)}_{\mathrm{GS}}(\lambda)\rangle|^2$ is larger than $1-\epsilon$. This condition defines a minimal interaction range l_ϵ required to adiabatically find the PH within the given accuracy ϵ, i.e., $F_l(\lambda)\geq 1-\epsilon\ \forall\ l\geq l_\epsilon$. This length l_ϵ is plotted versus N in Fig. 2(c) for $\epsilon=0.005$ and different values of λ. For $\lambda<\lambda_c$, l_ϵ weakly depends on the system size, while, for $\lambda>\lambda_c$, l_ϵ scales almost linearly with the system size.

(2) IQA with a nonintegrable Ising chain: We apply the IQA on a path of ground states $|\psi(\lambda)\rangle$ of a one-dimensional Ising chain in transverse and longitudinal field

$$\hat{\mathcal{H}}_{\mathrm{I}}(\lambda)=\sum_{j=1}^{N}\left(\hat{\sigma}^x_j\hat{\sigma}^x_{j+1}-B_z\hat{\sigma}^z_j+B_x(\lambda)\hat{\sigma}^x_j\right),\tag{9}$$

where $\hat{\sigma}^{x,z}_j$ are Pauli matrices, and $\hat{\sigma}^x_{N+1}\equiv\hat{\sigma}^x_1$. We choose $B_x(\lambda)=0.9\cdot(\lambda-\lambda_c)$ with $\lambda_c=0.5$. Here, a first-order quantum phase emerges at $\lambda=\lambda_c$ and allows us to further challenge our method. We perform the IQA for $\lambda\in[0,1]$, $N\in\{4,6,8,10\}$ and $\mathcal{L}^{(2)}$ containing all the local translationally invariant operators up to range $l=2$. The time schedule is $\lambda(t)=4(t/T)^3-6(t/T)^2+3(t/T)$ and has a null derivative when $\lambda=\lambda_c$ to slowly cross the emerging criticality.

Results of the IQA are summarized in Fig. 3 for $B_z=0.8$ (see [37] for $B_z=0.9$) corroborating our considerations regarding the effectiveness of our method and the effect of different order phase transitions on the adiabatic time and

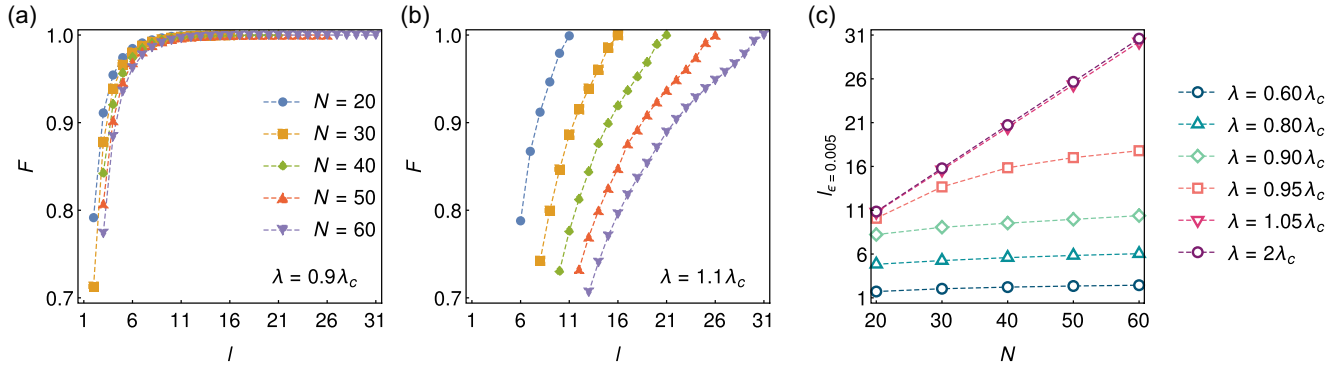

FIG. 2. IQA with fermionic Gaussian states. Panels (a) and (b): fidelity between the target state $|\psi(\lambda)\rangle$ and the ground state $|\psi^{(l)}_{\mathrm{GS}}(\lambda)\rangle$ of the adiabatic l-local Hamiltonian $\hat{\mathcal{H}}^{(l)}(\lambda)$ as a function of the interaction range l and for different system sizes N [legend in panel (a)]. Panel (a) λ is just before the phase transition, i.e., at $\lambda=0.9\lambda_c$, in panel (b) just after the phase transition, i.e., at $\lambda=1.1\lambda_c$. Panel (c): minimal interaction range l_ϵ required to ensure a fidelity $F\geq 1-\epsilon$ ($\epsilon=0.005$) between the target state and the ground state of the Hamiltonian obtained via the IQA, as a function of the system size and for different values of λ.

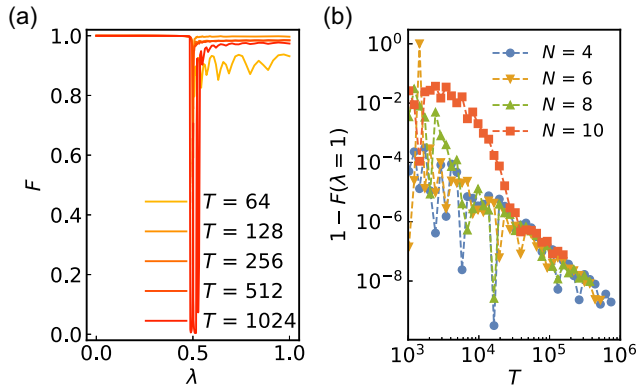

FIG. 3. IQA with a nonintegrable Ising chain ($B_z = 0.8$). Panel (a): fidelity between the target state $|\psi(\lambda)\rangle$ and the ground state $|\psi^{(l)}_{\mathrm{GS}}(\lambda)\rangle$ of the adiabatic two-local Hamiltonian, for a system of 10 spins and different annealing times. Panel (b): final infidelity at $\lambda = 1$ as a function of the annealing time, for different system sizes.

on the error introduced by the TDVP. In panel (a) we plot the fidelity $F(\lambda)$ between the target state and the ground state of the IQA Hamiltonian, as a function of the control parameter λ for different annealing times. We see that before the phase transition at λ_c, a large fidelity is reached also for short annealing times. Near λ_c the fidelity drops, but, differently from the second order transition case, we observe a large recovery after λ_c.

In Fig. 3(b), where we plot the final infidelity $1 - F(\lambda)$ at $\lambda = 1$ for different annealing times, we observe that after the critical point, the infidelity decreases polynomially with the annealing time and becomes independent on the system size. Remarkably, a small interaction range $l = 2$ is sufficient to construct an excellent PH in the adiabatic limit.

Discussion and conclusions.—Quantum annealing represents one of the major examples of the computational potential of quantum many-body systems. In this work, exploiting a combination of adiabatic approximation and TDVP, we introduced an annealing technique for approximating the l-local parent Hamiltonian of a target many-body state. We exemplified our method by reconstructing PHs for the Kitaev fermionic chain and a quantum Ising chain in longitudinal and transverse fields. The IQA allows for efficient reconstruction of PHs for paths of many-body states with finite correlation lengths, independently of the integrability of the model or the emergence of a first-order phase transition. The next step is to use IQA to design Hamiltonians that would allow for the experimental preparation of quantum states relevant in many-body physics and quantum information, directly from their wave functions. Remarkable examples include quantum spin liquid variational wave functions [44]. Finally, IQA only relies on the knowledge of local expectation values. This can be relevant for applications to the quantum marginal problem [30,45–50].

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