Universal Control of Symmetric States Using Spin Squeezing

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The manipulation of quantum many-body systems is a crucial goal in quantum science. Entangled quantum states that are symmetric under qubits permutation are of growing interest. Yet, the creation and control of symmetric states has remained a challenge. Here, we introduce a method to universally control symmetric states, proposing a scheme that relies solely on coherent rotations and spin squeezing. We present protocols for the creation of different symmetric states including Schrödinger's cat and Gottesman-Kitaev-Preskill states. The obtained symmetric states can be transferred to traveling photonic states via spontaneous emission, providing a powerful approach for engineering desired quantum photonic states.

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Control of symmetric systems and their applications.— Universal control of quantum systems is a critical building block in quantum computing and more generally in quantum information processing. In conventional qubit-based quantum systems, single-qubit gates and a two-qubit entangling gate provide all the ingredients to obtain universal control over an arbitrary number of qubits [1]. Despite this, the creation of most states requires the sequential application of a number of gates that grows exponentially with the number of qubits [2]. This becomes impractical for most states, even for a modest number of qubits (e.g., 40 qubits). A large number of gates is incompatible with the current architectures of quantum computers that suffer from short coherent times [3]. Hence, sets of operations that can create desired quantum states with a more scalable, polynomial number of gates are highly sought after.

Recent years have shown increasing interest in quantum states having symmetry to permutation between any two constituent qubits. These states are used in many platforms, such as ones encoding the quantum information in harmonic oscillators (continuous variables), in multi-level systems (qudits), and in ensembles of indistinguishable quantum particles.

Famous systems in which symmetric states naturally arise include nitrogen-vacancy centers [4], nuclear magnetic resonance systems [5], superconducting circuits [6–8], trapped ions [9–11], neutral atoms [12,13], and quantum dots [14,15]. The individual quantum particles comprising these systems are different, yet the combined system is described by the same Hilbert space of symmetric states. For an operation to keep the state inside the symmetric Hilbert space, it should act symmetrically on the entire population of particles. The simplest such operations are coherent rotations acting simultaneously on the state of every particle and spin squeezing of the combined state of all particles. These operations have been thoroughly explored in theory [16,17] and demonstrated in experiments [18–22], for example, enabling the creation of atomic Schrödinger's cat states [8,23]. Nevertheless, it remained unknown whether such operations can create any arbitrary symmetric state and fully control the symmetric system.

Here we show that coherent rotations together with spin squeezing constitute universal control over symmetric states. Any arbitrary symmetric state can be created using a polynomial number of symmetric operations, rather than the exponential number of operations needed in existing conventional approaches. Our findings directly apply to any physical system described by symmetric states that support coherent rotations and squeezing.

Coherent rotations have been implemented in numerous systems [20,21,24,25], but they are not sufficient for universal control. To achieve universality, we find it sufficient to add the spin-squeezing operation. Spin squeezing is a ubiquitous operation that acts not only on spin systems but on any symmetric system, such as atoms, quantum dots, or superconducting circuits [8,25–29]. For example, spin squeezing was implemented on atoms in a cavity using multiple lasers with different frequencies [18,29–31], or by interaction with off-resonant light [32]. Innovative implementations of coherent control and spin squeezing in the last decade [27–29] have made our proposal for universal control relevant to all state-of-the-art platforms in quantum science.

Direct transfer of symmetric states to photonic states.— In cases where the symmetric system describes quantum emitters, our findings have another intriguing implication. We rely on a unique property of symmetric states: when undergoing spontaneous emission, symmetric-state superpositions are precisely the states that get transferred onto a pulse of photons in the form of a *single* quantum-optical mode [33]. This situation is unique because in many other cases, the photonic state emitted in spontaneous emission occupies many spatiotemporal modes, and observing only part of them is expected to create a decohered mixed state, i.e., the initial quantum state of the emitters will not be

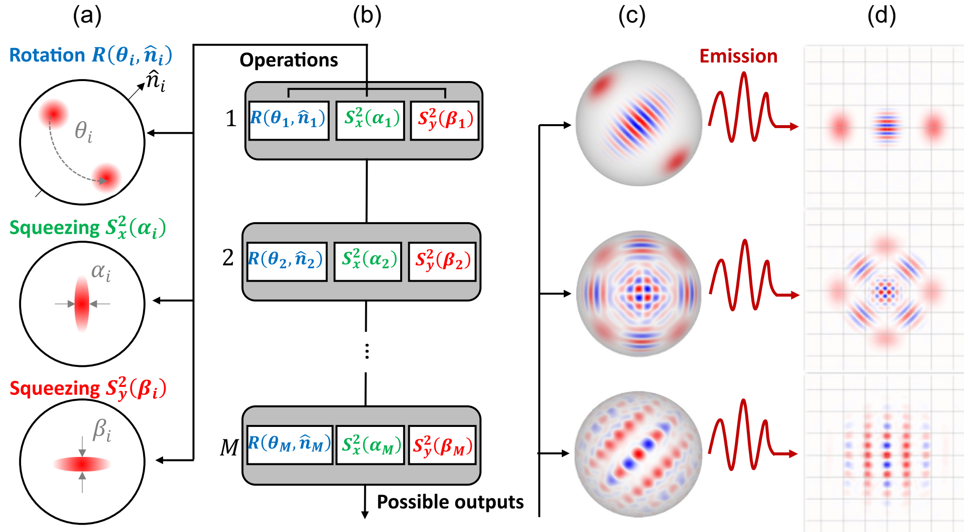

FIG. 1. Universal control of quantum states using spin squeezing and rotations. (a),(b) Each step of the sequence consists of a coherent rotation and squeezing. The coherent rotation is described by the unity vector of the axis $\hat{n}$ and the rotation angle θ around this axis: $R(\theta_i, \hat{n}_i) = \exp[i(\hat{n}_{ix}S_x + \hat{n}_{iy}S_y + \hat{n}_{iz}S_z)\theta_i]$. The squeezing operation is characterized by squeezing in the $\hat{x}$ and $\hat{y}$ directions: $S(\alpha_i, \beta_i) = \exp[i(\alpha_i S_x^2 + \beta_i S_y^2)]$. These two operations are sufficient for efficiently creating any arbitrary state in the Hilbert space of the permutationally symmetric states. (c) Example Wigner functions of the created states can be plotted on Bloch spheres [50], each representing the joint state of indistinguishable emitters. (d) The spontaneous emission by such emitters can be tailored to desired states of light, as illustrated by the corresponding Wigner functions of the emitted light pulses.

preserved due to the inherent "random" nature of spontaneous emission.

In the language of quantum information, spontaneous emission can be considered an error channel that destroys the information embedded in the qubits. However, in our case, spontaneous emission is a promising way to transfer the pure state of emitters (static qubits) to an almost single-mode pure photonic state (flying qubits) that can be used to store and transmit quantum information with minimal loss.

Using this direct transfer of symmetric states to photonic pulses, the universal control of symmetric states allows us to create high-fidelity quantum photonic states that are desired for quantum information processing. Particularly, the fields of photonic quantum computation and communication are in search of an efficient way of creating specific quantum photonic states, such as the Schrödinger cat and Gottesman-Kitaev-Preskill (GKP) states [34]. These photonic states are necessary resources for continuous-variable quantum information processing [35], a rising approach that complements the conventional, discrete-variable, qubit encoding.

To find efficient protocols for creating any arbitrary photonic state and specifically the ones desired for quantum information processing, we develop an optimization protocol for the controllable generation of arbitrary symmetric states. We find sequences of squeezing and rotations that create photonic square and hexagonal GKP states of 10 dB squeezing with 95% and 94% fidelity, as well as two- and four-legged Schrödinger's cat states with 97% and 94% fidelity, respectively. Our results convey the importance of studying new methods for manipulating systems of quantum emitters, especially for the ambitious goal of achieving universal control of light in various spectral regimes.

Creation of quantum photonic states using systems of emitters.—Gaussian states of harmonic oscillators, i.e., states that have Gaussian Wigner functions [36], such as squeezed light, can be beneficial for sensing in spectroscopy and metrology [37]. The creation of these states requires coherent displacements and squeezing. In contrast, non-Gaussian states of light such as Schrödinger's cat and GKP states, cannot be created using only those two operations [36]. Creating non-Gaussian states in conventional photonic systems can only be done by acquiring strong higher-order nonlinearities such as Kerr nonlinearity [36,38] or conditional operators and postselections [39–42]. However, these nonlinearities are relatively weak and inefficient in the optical range. Therefore, the field of quantum optics is in search of new ways to create non-Gaussian quantum photonic states.

The approach that we explore here for generating quantum photonic states uses systems of emitters, where the emitters themselves are the nonlinear element. Specifically, quantum emitters can generate non-Gaussian photonic states through their spontaneous emission to waveguides or optical cavities [33]. The simplest example of a non-Gaussian state is the single-photon Fock state, created regularly by spontaneous emission from a single emitter. Similar approaches that rely on spontaneous emission are implemented, for example, in superconducting qubits [41,43], in strongly interacting Rydberg atoms [44–46], and in other atoms coupled to

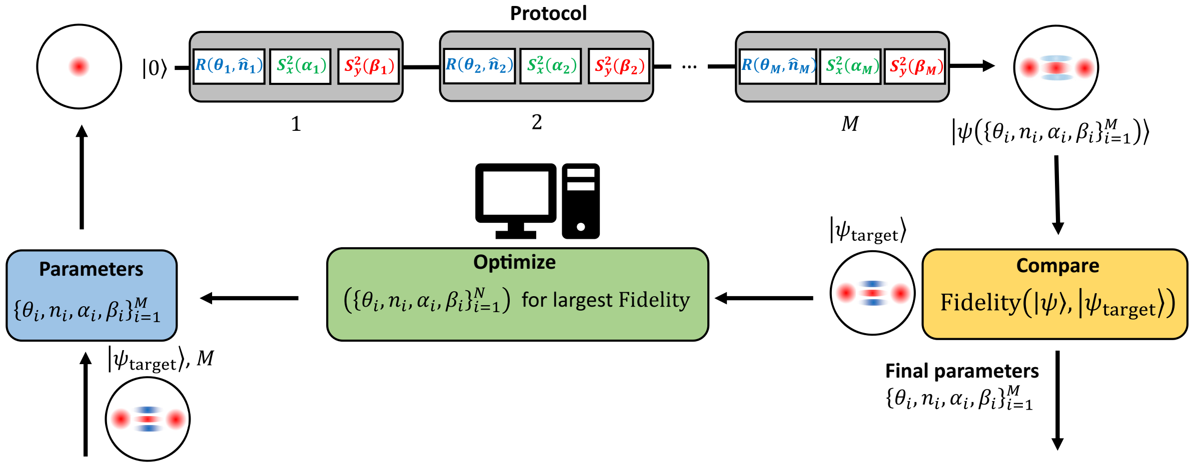

FIG. 2. Protocol for creating target symmetric states. As an input, we set the target state $|\psi_{\text{target}}\rangle$ that we want to get, the desired number of sequences M, and the operations used in each sequence (as shown in Fig. 1). The optimization protocol finds the parameters used in each sequence to maximize the fidelity between the final state and the target state. Although we use pure states in the figure, the code works with density matrices.

optical cavities [47] or to nanophotonic waveguides [38,39]. Despite the success in particular cases of generation of non-Gaussian states, it was unknown to what extent the emitters can be manipulated and what kinds of quantum photonic states can be created this way.

Our work shows that the lowest possible nonlinear operations on the emitters—coherent rotations and spin squeezing—already enable the creation of arbitrary symmetric states (Fig. 1). Then, spontaneous emission transfers the quantum state, enabling controllable generation of arbitrary photonic states [33,48,49].

Theory of symmetric states.—We consider a system of N two-level noninteracting emitters. We use the term emitters to describe general systems such as quantum dots, spins, transmons, trapped ions, etc. Each of these systems can be described in terms of symmetric states [51,52] that are invariant under any permutation of emitters. The basis of the symmetric states, sometimes called the Dicke ladder [52], is defined by

$$
\begin{aligned}
|N\rangle &= |\mathrm{ee}\ldots\mathrm{e}\rangle,\\
|N-1\rangle &= 1/\sqrt{N}(|\mathrm{ge}\ldots\mathrm{e}\rangle + |\mathrm{eg}\ldots\mathrm{e}\rangle + \cdots |\mathrm{ee}\ldots\mathrm{g}\rangle),\\
&\cdots\\
|1\rangle &= 1/\sqrt{N}(|\mathrm{eg}\ldots\mathrm{g}\rangle + |\mathrm{ge}\ldots\mathrm{g}\rangle + \cdots |\mathrm{gg}\ldots\mathrm{e}\rangle),\\
|0\rangle &= |\mathrm{gg}\ldots\mathrm{g}\rangle.
\end{aligned}
$$

Here $|\mathrm{g}\rangle$, $|\mathrm{e}\rangle$ are the ground and excited states of each two-level emitter. Our treatment is valid regardless of their energy gap (and, consequently, the frequency of the emitted photons). We denote the symmetric operators $S_x = \sum_i \sigma_x^{(i)}$, $S_y = \sum_i \sigma_y^{(i)}$, $S_z = \sum_i \sigma_z^{(i)}$, where $\sigma_{x,y,z}^{(i)}$ are the Pauli matrices on the ith emitter. Arbitrary functions of these operators describe all the operators that exist in the Hilbert space of the symmetric states.

Proof of universality.—In this section, we prove that squeezing and coherent displacements provide universal control of symmetric states. The set of Hamiltonians $\{H_i\}$ is universal if any unitary can be constructed from their respective time evolution operators $U(t) = e^{-iH_it}$. As was shown in [53], to prove universality, it is enough to show that the algebra derived from the set spans the entire Hilbert space, i.e., that we can get any operator in the Hilbert space by using linear combinations and commutators of $\{H_i\}$. Our Hilbert space of matrices of size $(N+1)\times(N+1)$ is spanned by polynomials of the symmetric operators S_x, S_y, S_z, as shown in the Supplemental Material [54]. Hence, it is enough to show that we can construct any polynomial in S_x, S_y, S_z using only the commutations and sums of Hamiltonians from the set $\{H_i\}$. In our case, this set is coherent rotations and squeezing, i.e., $\{H_i\}=\{S_x,S_y,S_x^2,S_y^2\}$.

It is insightful to compare the quantum systems described by symmetric states to those described by harmonic oscillators. In the case of harmonic oscillators, universality cannot be achieved using only coherent rotations and squeezing. The difference can be seen already in the commutation relation $[S_x, S_y] = iS_z$, which differs from that of the analog operators x and p obeying $[x, p] = i\hbar$. For the symmetric operators, the commutation relation of two 1st-order operators yields a 1st-order operator, while for the harmonic oscillator operators, the commutation relation provides a scalar. Introducing the squeezing operators S_x^2 and S_y^2 allows us to reach the commutation relations $[S_x, S_y^2] = i(S_zS_y + S_yS_z) = S_x + iS_yS_z$. So, by subtracting S_x, we can get the operator S_yS_z. The same can be done with $[S_z, S_y^2]$ and $[S_y, S_x^2]$ to get S_xS_y and S_xS_z. Now, we incorporate S_yS_z in the commutation relation $[S_x, S_yS_z] = i(S_z^2 - S_y^2)$. Combining the other operators that we have achieved so far, we see that the algebra contains all polynomials of S_x, S_y, and S_z of the 2nd power.

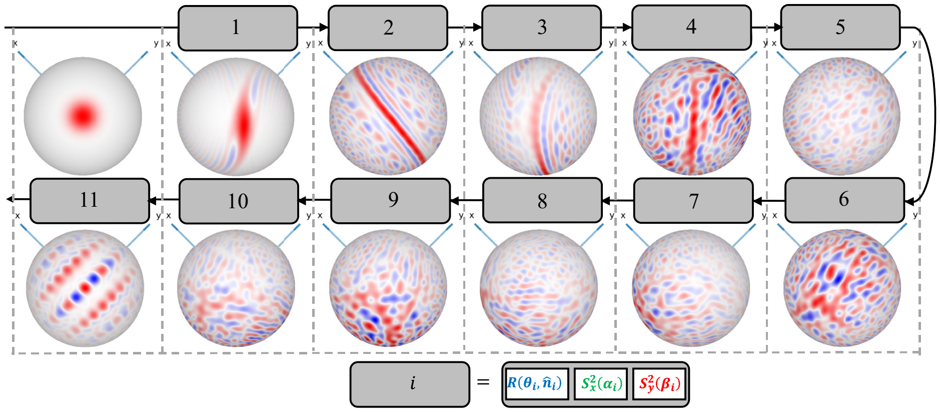

FIG. 3. Example sequence that creates a square GKP state. Sequence for the creation of a GKP symmetric state with 10 dB squeezing using 11 steps displaying the symmetric state of the emitters after each step. Each ith step consists of coherent rotation and squeezing with S_x^2 and S_y^2 operators. The number of emitters in the simulation is 40, and the fidelity of the final state is 98.37% compared to a square GKP symmetric state with 10 dB squeezing. For other parameters, the movie of GKP formation can be found in the supplementary video [54].

With all the 2nd power operators in hand, we can also construct any 3rd power operator by using commutation of all 2nd power operators, for example $[S_x^2, S_xS_y] = iS_x^2S_z + iS_xS_zS_x$. With this polynomial, we can reach S_x^3 since $[[S_x^2, S_xS_y], S_y] = S_x^3 +$ lower order terms. In this manner, we can build 4th power operators from 3rd power operators and so on as shown using an inductive proof in the Supplemental Material [54].

Optimization protocol.—To create arbitrary quantum states, we design and test an optimization protocol for engineering a sequence of coherent rotations $R_{\hat{n}}(\theta) = \exp[i(\hat{n}_{ix}S_x + \hat{n}_{iy}S_y + \hat{n}_{iz}S_z)\theta_i]$ and squeezing operation $S(\alpha,\beta) = \exp[i(\alpha S_x^2 + \beta S_y^2)]$ [Fig. 1(a)]. We demonstrate the protocol to yield several desired symmetric states, which translate to quantum photonic states such as Schrödinger's cat and GKP states. We start from the ground state $|0\rangle$ and at each of M steps, perform coherent rotation $R(\theta,\hat{n})$ followed by squeezing $S(\alpha,\beta)$. We optimize the parameters $\hat{n}, \theta, \alpha, \beta$ of each step to maximize the fidelity between the final state $|\psi(\{\theta_i, n_i, \alpha_i, \beta_i\}_{i=1}^{M})\rangle$ and the target state $|\psi_{\text{target}}\rangle$ (Fig. 2). We base our optimization protocol on a random search algorithm [57] applied over initial random guesses, and we then use the Nelder-Mead method [58] to find the local minimum for each initial guess (see in Supplemental Material [54]).

Illustrating the strength of our approach, the protocol finds an eleven-step sequence that produces a square GKP symmetric state with 98% fidelity. Figure 3 presents the

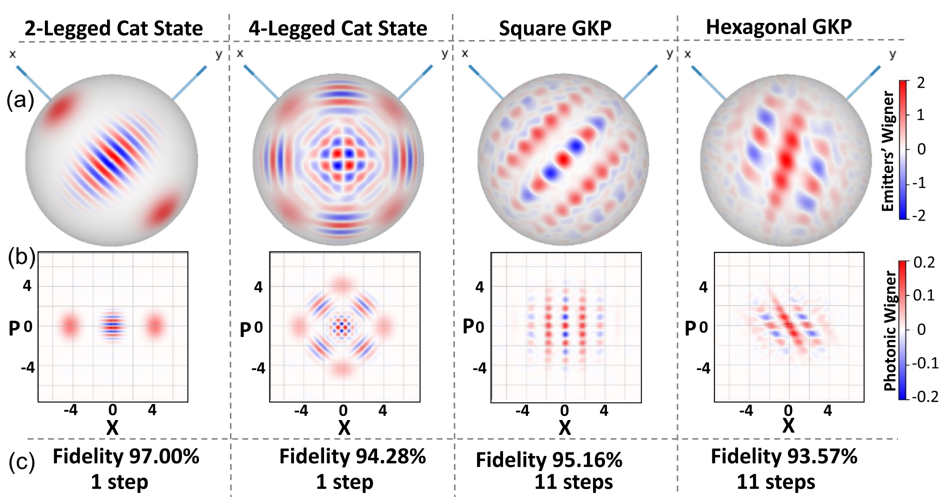

FIG. 4. Example states that can be created using sequences of squeezing and rotation operations. (a) Two-legged and four-legged cat states, as well as square and hexagonal GKP, states with 10 dB squeezing generated with our protocol (the third column corresponds to Fig. 3). (b) Corresponding Wigner functions of the emitted photonic states, assuming unity efficiency of coupling to a waveguide, using the model in [33]. (c) The fidelity between the target light states and the ones achieved by the optimization protocol, denoting the number of steps. (The full list of parameters found by the optimization protocol can be found in Supplemental Material [54]).

Wigner function [59] of the state on the Bloch sphere [50] at each of the steps.

Our protocol also finds useful quantum states with high fidelity, such as two-legged and four-legged Schrödinger's cat states, as well as the hexagonal GKP state. Figure 4 presents the fidelities, the number of steps, the Wigner function of the resulting symmetric states on the sphere, and the corresponding emitted quantum states of the emitted light.

To take decoherence mechanisms into account, we investigate in Supplemental Material [54] how dephasing influences the emitters' state during the preparation stage, showing that for realistic systems such as superconducting qubits and trapped ions, our protocol is feasible even with current levels of decoherence. Specifically, we find fidelities of 80%, 65%, and 30% for GKP states in superconducting circuits [8], in trapped ions [11], and in Rydberg atoms [46], respectively. The rapid developments of these platforms should increase the fidelities for creating GKP and cat states in the near future. In [54], we also simulate how unwanted decay channels affect the emitted photonic state. We show that if the rate of unwanted channels is smaller than the spontaneous emission rate, then the emitted photonic state is very close to the single mode calculation presented in Fig. 4(b).

Discussion.—In this work, we showed that the simplest possible 1st and 2nd order operations (i.e., coherent rotations and spin squeezing) provide universal control over symmetric states of emitters. In contrast to standard two-qubit gate approaches, our method requires only a polynomial number of steps to create any symmetric state (see Supplemental Material [54]). We designed an optimization protocol that finds efficient sequences of such operations for the creation of arbitrary superpositions of symmetric states. Consequently, the creation of quantum states of emitters that are useful for quantum computation is possible without addressing the emitters individually (as in Refs. [60,61]).

States of emitters that are symmetric to permutations are especially interesting as they are the states that can be naturally mapped to arbitrary single-mode pulses of photons [33]. Using a full multimode theory [33], we analyze the spontaneous emission from the symmetric states created using our protocol. We find that the emission takes the form of single-mode traveling Schrödinger's cat and GKP states, which are desirable for quantum technologies. This result is in striking difference to most current methods that can create GKP states only in cavities in microwave frequencies (e.g., [41]).

Compared to previous works (e.g., [44]), which were so far limited to a linear regime where the number of emitters is greatly larger than the number of excitations, our approach explicitly utilizes the nonlinearity that is intrinsic to the emitters at the high excitation regime. In addition, as we manipulate the emitters using rotations and squeezing, the transition into the coherent basis of the emitted light is natural, and thus the creation of Schrödinger's cat and GKP states takes a relatively small number of steps.

Importantly, our protocol is also more efficient (i.e., requires fewer steps) for a small number of emitters N, when compared to the conventional single- and two-qubit gate approaches. For example, to create a square GKP for $N = 10$, we need three steps (see the Supplemental Material [54]), far fewer than the number of single and two-qubit gates needed in conventional methods: at least N (and in some cases 2^N) gates for all-to-all entanglement. The relative efficiency of our protocol is higher for large N due to the polynomial scaling with N, compared to exponential in conventional protocols, as we show in Supplemental Material [54].

Outlook.—Our protocols considered manipulation of the emitters only before their emission. We suspect that by manipulating the emitters during emission, using nonlinear operations like squeezing, as well as detuning the emitter frequencies, exotic multimode photonic states can be created efficiently, such as NOON, multimode squeezed states, and cluster states.

The protocol presented in this work can be implemented in many modern quantum information processing platforms. For example, platforms of trapped ions [9,11], superconducting circuits [6,8], and Rydberg atoms [46] could be ideal candidates as the symmetric operations considered in this work naturally appear in them. Furthermore, the rising field of waveguide quantum electrodynamics (QED) [62], along with the more mature field of cavity QED [13,63] are promising platforms for implementing our proposal, providing high collection and detection efficiency of the emitted photonic state. We note that the decoherence mechanisms in each of these platforms are analyzed in in Supplemental Material [54] and show that even with current levels of decoherence our proposal is feasible at least for superconducting circuits. The rapid development of all the platforms makes our proposal even more feasible in the near future.

Looking forward, we see the utilization of low-order nonlinearities such as spin squeezing in different platforms as an efficient method for universal control of emitters and for creating sources of both single and multimode photonic states. Even though our protocol was numerically tested for a moderate number of emitters, it can be applied to large systems with thousands of emitters. For a large number of emitters (e.g., $N = 10^3$) the realization of our protocol can be difficult in the presence of decoherence. The investigation of decoherence mechanisms and scaling for a large number of emitters is an important aspect for future research. Our results emphasize that multipartite entangled states of emitters can be achieved without the need to control the emitters individually.

Supplementary calculations and derivations can be found at [64].

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