Three-Body Entanglement in Particle Decays

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Quantum entanglement has long served as a foundational pillar in understanding quantum mechanics, with a predominant focus on two-particle systems. We extend the study of entanglement into the realm of three-body decays, offering a more intricate understanding of quantum correlations. We introduce a novel approach for three-particle systems by utilizing the principles of entanglement monotone concurrence and the monogamy property. Our findings highlight the potential of studying deviations from the standard model and emphasize its significance in particle phenomenology. This work paves the way for new insights into particle physics through multiparticle quantum entanglement, particularly in decays of heavy fermions and hadrons.

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Introduction.—The study of quantum entanglement has been a cornerstone of quantum mechanics, providing profound insights into the non-local correlations between quantum systems [1,2]. Historically, much of the focus has been on two-particle entanglement, particularly in the context of the Bell inequalities [3,4]. However, as the field of quantum mechanics evolves, it becomes imperative to explore more complex systems, specifically, the realm of multiparticle entanglement.

Recent research has delved into the intricacies of two-particle entanglement in the context of particle physics. Bipartite systems have been explored for top–antitop quarks [5–10], the Higgs boson [11–13], gauge bosons [14–17] and leptons [18], revealing that the quantum information properties of their spin states at proton colliders are accessible in current data. Furthermore, some studies emphasized the importance of quantum observables in probing the underlying dynamics of quantum systems [19–21].

Given the advancements in studying two-particle entanglement, reflected by the large number of entanglement measures for bipartite systems [22–26], extending this research to three-particle systems is a natural yet surprisingly underexplored advancement. The study of three-particle entanglement presents a richer tapestry of quantum correlations and offers the potential to uncover new insights into the fundamental nature of quantum mechanics. Moreover, extending the Bell inequality tests to three particles can provide a more robust framework for testing the foundational principles of quantum mechanics and exploring potential deviations from the predictions of the standard model.

By building on the entanglement monotone *concurrence* and the *monogamy* property, we propose an approach to extend entanglement to three particles, charting a course for future explorations in the realm of multiparticle quantum entanglement in particle phenomenology. Extending the concept of entanglement to three particles will bolster its applicability to uncharted territories in particle phenomenology, e.g., the decay of heavy fermions and hadrons. (In many high-energy processes, three-body decays are favored phenomenologically over two-body decays, either because of kinematic reasons or improved observability over backgrounds. Among others, those include top decays, $t \to b f \bar{f}'$, and the Higgs boson decay, $H \to Z\mu^+\mu^-$.) This exploration into hadron decays offers a fresh perspective and a deeper understanding of the intricate quantum interactions within these systems.

Additionally, the introduction of the three-particle entanglement measure presents a new observable. This novel measure holds significant promise in aiding the search for unknown heavy resonances and potentially discovering new physics. The entanglement property is based on the fundamental interaction of the process. Thus, for a comprehensive assessment of the expected three-particle entanglement in three-body decays, we calculate its value for the effective Lorentz structures generated by (pseudo)scalars, (pseudo) vectors, and (pseudo)tensors exchanges, respectively.

In the following discussion, we assume these spin states do not decohere by hadronization or interactions with the environment before the measurement. For this assumption to be warranted, the spin-decoherence timescale needs to be significantly longer than the lifetime of the decaying resonance. This is usually the case for electroweak scale and many hadronic resonances. For example, the spin-decoherence timescale in top quark decays is $\mathcal{O}(m_t/\Lambda^2_{\rm QCD})$, while its lifetime is many orders of magnitude shorter, i.e., $1/\Gamma_t$ with $\Gamma_t \simeq 1.4$ GeV.

Definition of entanglement.—Entanglement can be quantified by a class of non-negative functions called *entanglement monotones* [26,27], whose values do not increase under local operations and classical communication (LOCC). A particularly convenient entanglement monotone is *concurrence* [25,28]. For a mixed state ρ of two qubits the concurrence is defined as

$$\mathcal{C}[\rho] = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \in [0, 1], \tag{1}$$

where η_i ($\eta_i > \eta_j$ for $i < j$) are the eigenvalues of the matrix $R \equiv \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ with $\tilde{\rho} \equiv (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. For separable states $\mathcal{C} = 0$, while $\mathcal{C} = 1$ for maximally entangled states. For a pure state of two qubits, $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, the concurrence can be computed more straightforwardly as

$$\mathcal{C}[|\psi\rangle] = \sqrt{2(1 - \mathrm{Tr}\rho_B^2)}, \tag{2}$$

where ρ_B is the reduced density operator of subsystem B obtained by tracing over subsystem A: $\rho_B \equiv \mathrm{Tr}_A(|\psi\rangle\langle\psi|)$.

For a three-qubit state, $|\Psi\rangle \in \mathcal{H}_i \otimes \mathcal{H}_j \otimes \mathcal{H}_k$, one can consider two types of entanglement. One is an entanglement between two individual particles, say between i and j. This entanglement can be computed by first tracing out subsystem k and use formula (1):

$$\mathcal{C}_{ij} = \mathcal{C}[\rho_{ij}], \qquad \rho_{ij} = \mathrm{Tr}_k(|\Psi\rangle\langle\Psi|). \tag{3}$$

Another type is an entanglement between one particle and the rest of the system, known as *one-to-other* bipartite entanglement. The concurrence between i and the composite subsystem (kj) can be computed using Eq. (2):

$$\mathcal{C}_{i(kj)} = \sqrt{2(1 - \mathrm{Tr}\rho_{kj}^2)}, \qquad \rho_{kj} = \mathrm{Tr}_i(|\Psi\rangle\langle\Psi|). \tag{4}$$

Here we used "qubit power" of the Schmidt theorem [29] (see also, e.g., [30]) and applied the two-qubit formula Eq. (2) to a three-qubit state $|\Psi\rangle$.

The entanglement between i and subsystem (kj) cannot be freely shared between i-j and i-k. Namely, there is a trade-off between i's entanglements with j and k. This property, called *monogamy*, is one of the most fundamental traits of entanglement and formulated by the Coffman-Kundu-Wootters (CKW) monogamy inequality [31,32]:

$$\mathcal{C}^2_{i(kj)} \geq \mathcal{C}^2_{ij} + \mathcal{C}^2_{ik}. \tag{5}$$

For multipartite systems, one can define a so-called *genuine multipartite entanglement* (GME) [30,33,34]. A good GME measure should (i) vanish for all product and biseparable states, (ii) be positive for all non-biseparable states, and (iii) not increase under LOCC. Recently, a GME measure satisfying all these criteria has been found for three-qubit states [35]. It corresponds to the area of the *concurrence triangle*, whose three sides are given by the three one-to-other bipartite entanglements:

$$F_3 = \left[\frac{16}{3} Q(Q - \mathcal{C}_{1(23)})(Q - \mathcal{C}_{2(13)})(Q - \mathcal{C}_{3(12)})\right]^{\frac{1}{2}}, \tag{6}$$

with $Q = \frac{1}{2}[\mathcal{C}_{1(23)} + \mathcal{C}_{2(13)} + \mathcal{C}_{3(12)}]$. With this definition, F_3 takes values between 0 and 1.

Entanglement in three-body decays.—We consider a three-body decay $0 \to 123$ and assume all particles are distinguishable and have spin-1/2. We analyze the entanglement of the spin degrees of freedom (d.o.f.) of the final state particles 1, 2, and 3 at a given phase-space point ($\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$). (As a related topic, the entanglement in a orthopositronium decay into three photons has been studied in Ref. [36].) To parametrize the phase space of the final state we boost into the rest frame of the initial particle 0 and take the z axis in the direction of $\mathbf{p}_1$. The x and y axes are chosen such that the y axis is perpendicular to the decay plane and the $\mathbf{p}_2$ has a positive x component. The opening angles $1 \to 2$ and $1 \to 3$ are denoted by θ_2 and θ_3 ($0 \leq \theta_2$, $\theta_3 \leq \pi$), respectively. We represent the spin polarization $\mathbf{n}$ of the initial particle 0 by the polar and azimuthal angles, θ and ϕ, respectively (see Fig. 1).

We choose the spin quantization axis of each final state particle in the momentum direction of that particle. In this case, the eigenvalues of the spin (multiplied by 2) are called *helicity* and denoted by $\lambda_i = \pm 1$ ($i = 1, 2, 3$).

For a given set of interactions, the quantum field theory framework lets us calculate the transition matrix element (helicity amplitude)

$$\mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} = \langle \lambda_1, \lambda_2, \lambda_3 | \mathbf{n} \rangle, \tag{7}$$

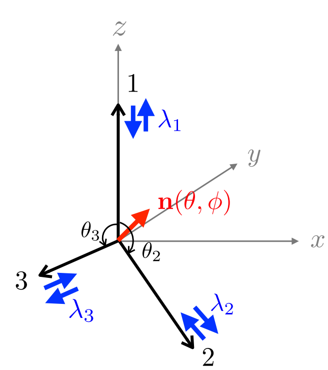

FIG. 1. The momentum and spin configuration in the coordinate system.

where the momentum labels are suppressed. The initial state $|\mathbf{n}\rangle$ is expanded by the final states as

$$|\mathbf{n}\rangle = \sum_{\lambda_1,\lambda_2,\lambda_3} \mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} |\lambda_1,\lambda_2,\lambda_3\rangle + \cdots. \tag{8}$$

The ellipsis represents final states of other phase-space points and other decay modes. Focusing on the spin d.o.f. one can describe the final spin state as

$$|\Psi\rangle = \frac{1}{\mathcal{N}} \sum_{\lambda_1,\lambda_2,\lambda_3} \mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} |\lambda_1,\lambda_2,\lambda_3\rangle, \tag{9}$$

where $\mathcal{N} = (\sum_{\lambda_1,\lambda_2,\lambda_3} |\mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3}|^2)^{1/2}$ is the normalization constant. In general, this is an entangled pure state of three qubits.

The two-particle entanglement and one-to-other entanglement defined in Eqs. (3) and (4) can be readily calculated, respectively.

In the following, we assume, for simplicity, that the final state particles are massless while the extension to the massive case is straightforward. In general, there are 16 nonredundant Lorentz structures formed from bilinear combinations of Dirac spinors $\bar{\psi}\Gamma\psi$ with

$$\Gamma = \{\mathbb{I}, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}, \tag{10}$$

where γ^μ is the Dirac γ matrices, $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\sigma^{\mu\nu} \equiv (i/2)[\gamma^\mu, \gamma^\nu]$.

As a three-body decay $0 \to 123$ of one fermion into three fermions requires two bilinears, 256 Lorentz structures form a complete basis. Instead, we will focus on the matrix elements and Lorentz structures induced by the exchange of (pseudo)scalars, (pseudo)vectors, and (pseudo)tensors.

Scalar and pseudoscalar interaction: We consider the effective interaction operator

$$[\bar{\psi}_1(c_S + ic_A\gamma_5)\psi_0][\bar{\psi}_3(d_S + id_A\gamma_5)\psi_2], \tag{11}$$

where $c_S, c_A, d_S, d_A \in \mathbb{R}$ are coupling constants. We also define $c = c_S + ic_A$ and $d = d_S + id_A$ and take $|c| = |d| = 1$ as we are not interested in the overall scale of the amplitude. For given phase-space point (θ_2, θ_3) and the initial spin (θ, ϕ), the matrix element of $0 \to 123$ can be calculated as

$$\begin{aligned}\mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} \propto{} & 2\sqrt{2mp_1p_2p_3} \cdot s\frac{\theta_2+\theta_3}{2} \\ & \left[-cd \cdot \delta^-_{\lambda_1}\delta^-_{\lambda_2}\delta^-_{\lambda_3} \cdot e^{i\phi} s\frac{\theta}{2} + cd^* \cdot \delta^-_{\lambda_1}\delta^+_{\lambda_2}\delta^+_{\lambda_3} \cdot e^{i\phi} s\frac{\theta}{2} \right. \\ & \left. - c^*d \cdot \delta^+_{\lambda_1}\delta^-_{\lambda_2}\delta^-_{\lambda_3} \cdot c\frac{\theta}{2} + c^*d^* \cdot \delta^+_{\lambda_1}\delta^+_{\lambda_2}\delta^+_{\lambda_3} \cdot c\frac{\theta}{2}\right],\end{aligned} \tag{12}$$

where shorthand notations $c\alpha = \cos\alpha$ and $s\alpha = \sin\alpha$ are used. This corresponds to the spin state

$$\begin{aligned}|\Psi\rangle ={} & M_{LL}|---\rangle + M_{LR}|-++\rangle \\ & + M_{RL}|+--\rangle + M_{RR}|+++\rangle,\end{aligned} \tag{13}$$

with $M_{LL} = -(cd/\sqrt{2}) \cdot e^{i\phi}s(\theta/2)$, $M_{LR} = (cd^*/\sqrt{2}) \cdot e^{i\phi}s(\theta/2)$, $M_{RL} = -(c^*d/\sqrt{2}) \cdot c(\theta/2)$, and $M_{RR} = (c^*d^*/\sqrt{2}) \cdot c(\theta/2)$. We see that this is a biseparable state

$$\begin{aligned}|\Psi\rangle = {} & \left[ce^{i\phi}s\frac{\theta}{2}|-\rangle_1 + c^*c\frac{\theta}{2}|+\rangle_1\right] \\ & \otimes \frac{1}{\sqrt{2}}[d^*|++\rangle_{23} - d|--\rangle_{23}].\end{aligned} \tag{14}$$

Therefore, 1 is entangled neither with 2, 3 nor (23):

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{1(23)} = 0, \tag{15}$$

while 2 and 3 are maximally entangled:

$$\mathcal{C}_{23} = 1. \tag{16}$$

The monogamy inequality (5) implies 2 and 3 must also be maximally entangled with the rest of the system:

$$\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 1, \tag{17}$$

which can be explicitly checked from the formula (4). Because the state is biseparable, the GME measure vanishes

$$F_3 = 0. \tag{18}$$

Vector and axial-vector interaction: We next consider the vector interaction

$$[\bar{\psi}_1\gamma_\mu(c_LP_L + c_RP_R)\psi_0][\bar{\psi}_3\gamma^\mu(d_LP_L + d_RP_R)\psi_2], \tag{19}$$

with $P_{R/L} \equiv (1 \pm \gamma_5)/2$ and $c_L, c_R, d_L, d_R \in \mathbb{R}$. The matrix element is found as

$$\begin{aligned}\mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} \propto{} & 4\sqrt{2mp_1p_2p_3} \\ & \left[\delta^-_{\lambda_1}\delta^+_{\lambda_2}\delta^-_{\lambda_3} \cdot c_Ld_L s\frac{\theta_3}{2}\left[c\frac{\theta}{2}c\frac{\theta_2}{2} + e^{i\phi}s\frac{\theta}{2}s\frac{\theta_2}{2}\right]\right. \\ & - \delta^-_{\lambda_1}\delta^-_{\lambda_2}\delta^+_{\lambda_3} \cdot c_Ld_R s\frac{\theta_2}{2}\left[c\frac{\theta}{2}c\frac{\theta_3}{2} - e^{i\phi}s\frac{\theta}{2}s\frac{\theta_3}{2}\right] \\ & + \delta^+_{\lambda_1}\delta^+_{\lambda_2}\delta^-_{\lambda_3} \cdot c_Rd_L s\frac{\theta_2}{2}\left[c\frac{\theta}{2}s\frac{\theta_3}{2} + e^{i\phi}s\frac{\theta}{2}c\frac{\theta_3}{2}\right] \\ & \left. + \delta^+_{\lambda_1}\delta^-_{\lambda_2}\delta^+_{\lambda_3} \cdot c_Rd_R s\frac{\theta_3}{2}\left[c\frac{\theta}{2}s\frac{\theta_2}{2} - e^{i\phi}s\frac{\theta}{2}c\frac{\theta_2}{2}\right]\right],\end{aligned} \tag{20}$$

corresponding to the state

$$|\Psi\rangle = M_{LL}|-+-\rangle + M_{LR}|--+\rangle + M_{RL}|++-\rangle + M_{RR}|+-+\rangle, \quad (21)$$

with $M_{LL} = \mathcal{M}^{\mathbf{n}}_{-+-}/\mathcal{N}$, $M_{LR} = \mathcal{M}^{\mathbf{n}}_{--+}/\mathcal{N}$, $M_{RL} = \mathcal{M}^{\mathbf{n}}_{++-}/\mathcal{N}$ and $M_{RR} = \mathcal{M}^{\mathbf{n}}_{+-+}/\mathcal{N}$. From explicit calculations, we find

$$\mathcal{C}_{12} = \mathcal{C}_{13} = 0, \qquad \mathcal{C}_{23} = 2|M_{LL}M^*_{LR} + M_{RL}M^*_{RR}|. \quad (22)$$

For one-to-other entanglements, we obtain

$$\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2\sqrt{(|M_{LL}|^2 + |M_{RL}|^2)(|M_{LR}|^2 + |M_{RR}|^2)},$$
$$\mathcal{C}_{1(23)} = 2|M_{RR}M_{LL} - M_{LR}M_{RL}|. \quad (23)$$

Since all three $C_{i(jk)}$ are nonvanishing in general, the GME measure F_3 is also nonvanishing in that case. $M_{XY} \propto c_X d_Y$ ($X, Y = L, R$) and we see that both $\mathcal{C}_{23}$, $\mathcal{C}_{1(23)}$ and F_3 are proportional to $|c_L c_R d_L d_R|$. One the other hand, $\mathcal{C}_{2(13)}$ and $\mathcal{C}_{3(12)}$ vanish only if $|c_L c_R| = |d_L d_R| = 0$.

To discuss the monogamy relation, we define the monogamy measure as

$$M_i = \mathcal{C}^2_{i(jk)} - [\mathcal{C}^2_{ij} + \mathcal{C}^2_{ik}], \quad (24)$$

for $i \neq j \neq k \neq i$. The CKW monogamy inequalities are expressed by $M_i \geq 0$ for $i = 1, 2, 3$. From Eqs. (22) and (23), one can show $\mathcal{C}^2_{23} = \mathcal{C}^2_{2(13)} - \mathcal{C}^2_{1(23)}$. In the vector interaction, we therefore have

$$M_1 = M_2 = M_3 = \mathcal{C}^2_{1(23)} \geq 0. \quad (25)$$

Tensor and pseudotensor interaction: We consider the tensor interaction

$$[\bar{\psi}_1(c_M + ic_E\gamma_5)\sigma^{\mu\nu}\psi_0][\bar{\psi}_3(d_M + id_E\gamma_5)\sigma_{\mu\nu}\psi_2], \quad (26)$$

with $c_M, c_E, d_M, d_E \in \mathbb{R}$. As in the scalar case, we define $c = c_M + ic_E$ and $d = d_M + id_E$ and take $|c| = |d| = 1$. This operator is equivalent to $\alpha[\bar{\psi}_1\sigma^{\mu\nu}\psi_0][\bar{\psi}_3\sigma_{\mu\nu}\psi_2] - (\beta/2)\epsilon^{\mu\nu\rho\sigma}[\bar{\psi}_1\sigma_{\mu\nu}\psi_0][\bar{\psi}_3\sigma_{\rho\sigma}\psi_2]$ with $\alpha = c_M d_M - c_E d_E$ and $\beta = c_M d_E + c_E d_M$. The $0 \to 123$ matrix element is given by

$$\mathcal{M}^{\mathbf{n}}_{\lambda_1,\lambda_2,\lambda_3} \propto -8\sqrt{2mp_1p_2p_3}\left[c^*d^*\cdot\delta^+_{\lambda_1}\delta^+_{\lambda_2}\delta^+_{\lambda_3}\cdot\left[2e^{i\phi}s\frac{\theta}{2}s\frac{\theta_2}{2}s\frac{\theta_3}{2} - c\frac{\theta}{2}s\frac{\theta_2-\theta_3}{2}\right] + cd\cdot\delta^-_{\lambda_1}\delta^-_{\lambda_2}\delta^-_{\lambda_3}\cdot\left[e^{i\phi}s\frac{\theta}{2}s\frac{\theta_2-\theta_3}{2} + 2c\frac{\theta}{2}s\frac{\theta_2}{2}s\frac{\theta_3}{2}\right]\right], \quad (27)$$

implying the spin quantum state

$$|\Psi\rangle = M_R|+++\rangle + M_L|---\rangle, \quad (28)$$

with $M_R = \mathcal{M}^{\mathbf{n}}_{+++}/\mathcal{N}$ and $M_L = \mathcal{M}^{\mathbf{n}}_{---}/\mathcal{N}$. This state interpolates between the separable states, $|+++\rangle$ and $|---\rangle$, and the maximally entangled Greenberger Horne Zeilinger state, $|\mathrm{GHZ}\rangle = (|+++\rangle + |---\rangle)/\sqrt{2}$ [37]. For the tensor interaction, there are no entanglements between two individual particles

$$\mathcal{C}_{12} = \mathcal{C}_{13} = \mathcal{C}_{23} = 0, \quad (29)$$

while one-to-other entanglements are universal

$$\mathcal{C}_{1(23)} = \mathcal{C}_{2(13)} = \mathcal{C}_{3(12)} = 2|M_R M_L|. \quad (30)$$

The GME measure in this case is

$$F_3 = 4|M_R M_L|^2. \quad (31)$$

The monogamy inequalities are trivially satisfied because no entanglement of one-to-other is shared by the individual pairs [cf. Eq. (29)].

As in the scalar case, all entanglement measures are independent of the phases of c and d.

Numerical results: In Fig. 2, we show the GME measure F_3 as a function of θ_2 and θ_3 for vector (upper panel) and tensor (lower panel) interactions. The lower left plane is empty because this region is unphysical, $\theta_2 + \theta_3 < \pi$. All couplings, c_X, d_X ($X = L, R, M, E$), are fixed to $1/\sqrt{2}$. In the left panel, the spin direction $\mathbf{n}$ of the initial particle is set to the y direction (perpendicular to the decay plane), while in the right panel, it is tilted with $\theta = \phi = (\pi/4)$ (see Fig. 1). We see that when $\mathbf{n}$ is perpendicular to the decay plane, F_3 for the vector interaction depends only on the combination $\theta_2 + \theta_3$ and symmetric under $2 \leftrightarrow 3$ exchange. For the tensor interaction, in this case, the system is maximally entangled, $F_3 = 1$, regardless of the decay angles. This results in $|M_L| = |M_R| = 1/\sqrt{2}$, as inferred from Eq. (27). When the initial spin is tilted, F_3 behaves asymmetrically under $2 \leftrightarrow 3$ both for vector and tensor interactions, as shown in the two right plots of Fig. 2.

Figure 3 shows various entanglement measures as a function of the initial spin direction $\mathbf{n}$. For vector

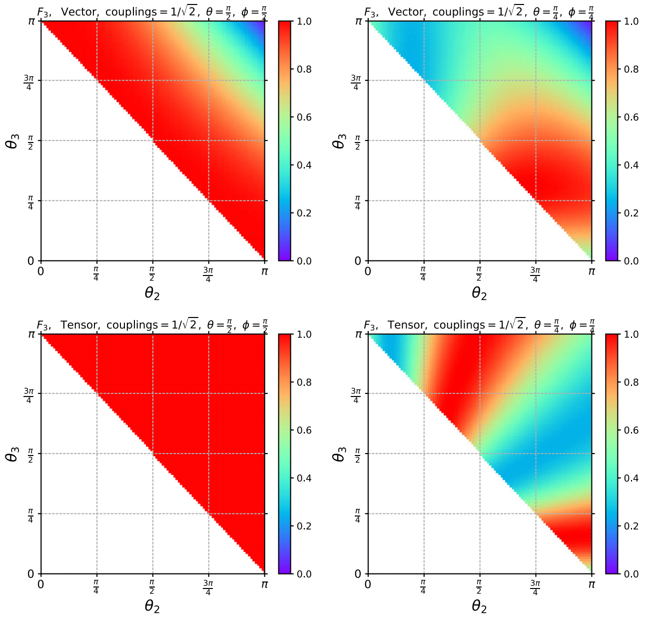

FIG. 2. F_3 on the (θ_2, θ_3) planes.

interaction, we show F_3 (red-solid), $\mathcal{C}_{1(23)}$ (blue-dashed), $\mathcal{C}_{2(13)} = \mathcal{C}_{3(12)}$ (green-dashed), $\mathcal{C}_{23}$ (purple-dotted), and $M_i = \mathcal{C}^2_{1(23)}$ (orange-dotted) lines, while only F_3 is shown for the tensor interaction. All couplings are set to $1/\sqrt{2}$. In the right and left panels, the decay angles are fixed to $(\theta_2, \theta_3) = [(4\pi/6), (5\pi/6)]$ and $[(2\pi/6), (5\pi/6)]$, respectively. The horizontal axes of the plots represent the angle between the z axis and $\mathbf{n}$. In the upper and lower panels, $\mathbf{n}$ rotates about the y and x axes in the right-handed way, respectively. We observe that F_3 responds differently to the

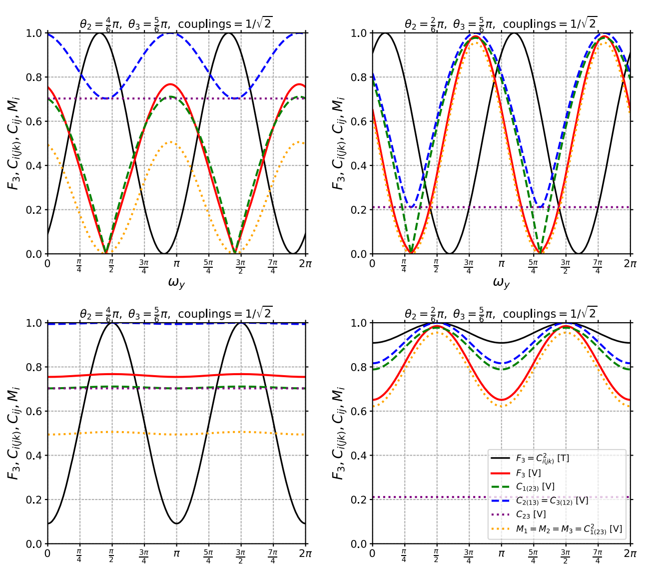

FIG. 3. Various entanglement measures as functions of the initial spin direction $\mathbf{n}$.

rotations of $\mathbf{n}$ between the vector and tensor cases. The only nonvanishing two-particle entanglement $\mathcal{C}_{23}$ in the vector case is constant with respect to $\mathbf{n}$.

Conclusion.—The exploration of quantum entanglement has been a cornerstone in understanding the nonlocal correlations inherent in quantum systems. While much of the historical focus has been on two-particle entanglement, we expanded its realm to three-particle systems, revealing a richer tapestry of quantum correlations. This advancement offers profound insights into the fundamental nature of quantum mechanics, extending beyond the traditionally studied bipartite systems.

Building upon the foundational concepts of entanglement monotone concurrence and the monogamy property, we propose a novel approach to understanding entanglement in three-body decays. This exploration paves the way for future studies in multiparticle quantum entanglement and emphasizes its significance in particle phenomenology, particularly in the decay dynamics of heavy fermions and hadrons. Having explicitly calculated the expected entanglement for the three-body decay via (pseudo)scalar, (pseudo)vector, and (pseudo)tensor mediators, this approach can potentially uncover deviations from the predictions of the standard model, shedding light on uncharted territories within the quantum realm.

Thus, we emphasize the pivotal role of three-particle entanglement in particle physics, suggesting new avenues for exploring new observables and novel search strategies in high-energy physics.

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