Embedding Quantum Many-Body Scars into Decoherence-Free Subspaces

He-Ran Wang,[1,*] Dong Yuan,[2,*] Shun-Yao Zhang,[2,*] Zhong Wang,[1,†] Dong-Ling Deng,[2,3,4,‡] and L.-M. Duan[2,3,4,5]

[1]*Institute for Advanced Study, Tsinghua University, Beijing 100084, People's Republic of China*
[2]*Center for Quantum Information, IIIS, Tsinghua University, Beijing 100084, People's Republic of China*
[3]*Hefei National Laboratory, Hefei 230088, People's Republic of China*
[4]*Shanghai Qi Zhi Institute, 41st Floor, AI Tower, No. 701 Yunjin Road, Xuhui District, Shanghai 200232, China*
[5]*New Cornerstone Science Laboratory, IIIS, Tsinghua University, Beijing 100084, People's Republic of China*

(Received 30 April 2023; accepted 15 March 2024; published 12 April 2024)

Quantum many-body scars are nonthermal excited eigenstates of nonintegrable Hamiltonians, which could support coherent revival dynamics from special initial states when scars form an equally spaced tower in the energy spectrum. For open quantum systems, engineering many-body scarred dynamics by a controlled coupling to the environment remains largely unexplored. Here, we provide a general framework to exactly embed quantum many-body scars into the decoherence-free subspaces of Lindblad master equations. The dissipative scarred dynamics manifest persistent periodic oscillations for *generic* initial states, and can be practically utilized to prepare scar states with potential quantum metrology applications. We construct the Liouvillian dissipators with the local projectors that annihilate the whole scar towers, and utilize the Hamiltonian part to rotate the undesired states out of the null space of dissipators. We demonstrate our protocol through several typical models hosting many-body scar towers and propose an experimental scheme to observe the dissipative scarred dynamics based on digital quantum simulations and resetting ancilla qubits.

DOI: 10.1103/PhysRevLett.132.150401

Isolated quantum many-body systems typically thermalize under Hamiltonian evolution, during which any local information preserved in the initial states scrambles into the entire system. These features of quantum thermalization have been illustrated by the eigenstate thermalization hypothesis (ETH) in the past decades [1,2]. In recent years, studies of weak ergodicity breaking, namely, a small fraction of ETH-violating eigenstates immersed in a sea of thermal eigenstates, dubbed quantum many-body scars, have attracted considerable attention [3–5]. One of the hallmarks of quantum many-body scars, originally discovered in experiments with Rydberg atoms [6,7], is their ability to support long-lived coherent oscillations from initial states that have large overlap with a tower of equally spaced scars in the energy spectrum [8–16]. Despite the fact that such anomalous eigenstates, typically with sub-volume-law entanglement entropy, have been found and carefully analyzed in various Hamiltonians [17–33], the extensions of many-body scars and related coherent revivals into the regime of open quantum systems remain largely unexplored. Here, we add this crucial yet missing block by introducing a general framework to exactly embed quantum many-body scars into the decoherence-free subspaces of Lindblad master equations. See Fig. 1 for a pictorial illustration.

The dynamics of open quantum systems coupled to a Markovian environment are described by the following Lindblad master equation [34],

$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma\sum_j (2L_j\rho L_j^\dagger - \{L_j^\dagger L_j,\rho\}) \equiv \mathcal{L}(\rho), \quad (1)$$

where ρ is the density matrix, H is the Hamiltonian part governing the unitary dynamics, $\{L_j\}$ are jump operators describing the dissipative quantum channels with strength γ, and $\mathcal{L}$ is the Liouvillian superoperator. In particular, if the evolution dynamics governed by $\mathcal{L}$ are purely unitary within a subspace W and do not suffer from dissipation, W is said to be a decoherence-free subspace of this Lindblad master equation [35,36]. One special case is that all the basis elements $\{|S_n\rangle\}$ of the subspace W are annihilated by all the dissipators, and W is closed under the action of the Hamiltonian part, i.e., $L_j|S_n\rangle = 0,\ \forall\ j,n$ ($\{|S_n\rangle\}$ are therefore "dark states" of the jump operators), and $HW \subseteq W$. The decoherence-free subspaces were originally proposed to reduce noises in quantum computation and realize the "passive" quantum error correction codes [35,37,38]. Later works apply similar techniques to realize the dissipative quantum state preparation [39–44].

In this Letter, by designing the dissipators and the Hamiltonian part, we introduce a general protocol to construct local Liouvillians that host *scar-state-only* decoherence-free subspaces. One important consequence reflecting on the Liouvillian spectrum is that all the nondecaying eigenmodes are equally spaced on the imaginary axis, as depicted in Fig. 1(a). Hence, unlike their

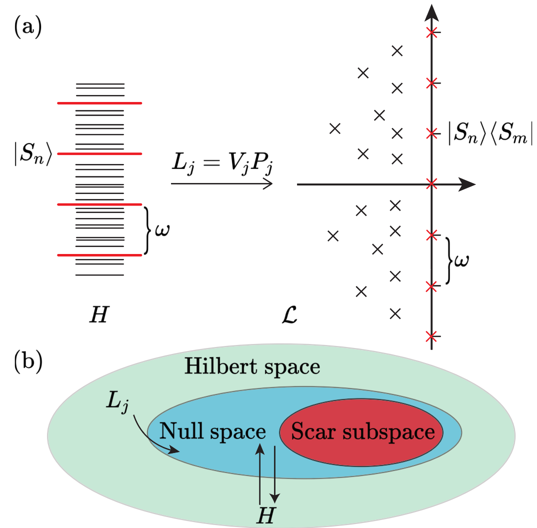

FIG. 1. Schematic illustration of the protocol for embedding quantum many-body scars into decoherence-free subspaces of Lindblad master equations. (a) The equally spaced many-body scar tower $\{|S_n\rangle\}$ (red lines) is embedded onto the imaginary axis of the Liouvillian spectrum as nondecaying eigenmodes in the form of $\{|S_n\rangle\langle S_m|\}$ (red crosses). (b) The dissipators drive the system into their common null space (sometimes equals the scar subspace) and the Hamiltonian part of the Liouvillian rotates the undesired states out of the null space to make them decay away.

closed-system counterparts, which are highly sensitive to the initial states and vulnerable to instantaneous perturbations, the open-system scarred dynamics manifest persistent periodic oscillations for generic initial states (even mixed states) and exhibit intrinsic tolerance to such disturbances. We demonstrate our protocol through four typical models hosting many-body scars, with the constructed Liouvillians summarized in Table I. We show that our dissipative protocol can be further utilized to prepare each scar state that possesses extensive multipartite entanglement with potential applications in quantum enhanced metrology. In addition, we propose an experimental scheme

TABLE I. Summary of the local Hamiltonians and jump operators of the constructed Liouvillians for four typical models. The generic local operators $\{V_j\}$ are specified in the text.

Model	H_j	L_j
Toy model in [13]	$P_j h_j P_j + \Omega\sigma_j^x/2$	$V_{j,j+1}(1 - \vec{\sigma}_j \cdot \vec{\sigma}_{j+1})$
Spin-1 XY [12]	$(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y)$ $+hS_j^z + D(S_j^z)^2$	$V_{j,j+1}(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y)$
AKLT [10]	$T_{j,j+1}^{S=2}$	$V_{j,j+1}T_{j,j+1}^{S=2,m=-2,-1,0}$, $V'_{j-1,j,j+1}T'_{j-1,j,j+1}$
Domain-wall preserving [14]	$(\sigma_j^x - \sigma_{j-1}^z\sigma_j^x\sigma_{j+1}^z)$ $+\Delta\sigma_j^z + J\sigma_j^z\sigma_{j+1}^z$	$V_{j,j+1}(\lvert\uparrow\uparrow\rangle\langle\uparrow\uparrow\rvert)_{j,j+1}$

to observe such dissipative scarred dynamics on current quantum simulators through digital quantum simulations and resetting ancilla qubits.

Non-Hermitian Shiraishi-Mori embedding.—We motivate our protocol from the non-Hermitian generalization of the Shiraishi-Mori embedding method [17], then extend to the Liouvillian formalism. In Ref. [17] Shiraishi and Mori proposed an approach to embed nonthermal eigenstates into the spectrum of nonintegrable Hamiltonians. The general Shiraishi-Mori Hamiltonians have the form of $H = \sum_j P_j h_j P_j + H'$, where $\{P_j\}$ is a set of local projectors ($P_j^2 = P_j$), $[H', P_j] = 0$, $\forall\ j$, and $\{h_j\}$ are arbitrary local Hamiltonians. We hereafter refer $j = 1, 2, \ldots, L$ to the label of sites in a one-dimensional spin chain with periodic boundary condition. We denote the common null space annihilated by these local projectors $\{P_j\}$ by W'. Since $P_j H|\Psi\rangle = P_j H'|\Psi\rangle = H' P_j|\Psi\rangle = 0$ for $\forall\ |\Psi\rangle \in W'$, W' is closed under the action of H ($HW' \subseteq W'$), and therefore hosts $\dim(W')$ eigenstates of H. For properly chosen $\{P_j\}$ and H', these eigenstates could become many-body scars embedded into the middle of the spectrum of H. Note that in the present case, the scar subspace W [the red circle in Fig. 1(b)] coincides with the common null space W' of the local projectors [the blue circle in Fig. 1(b)].

Now we consider adding some non-Hermitian terms into the Shiraishi-Mori Hamiltonian,

$$H_{\mathrm{NH}} = H - i\sum_j P_j D_j P_j = \sum_j P_j(h_j - iD_j)P_j + H', \qquad (2)$$

where the local Hermitian operators $\{D_j\}$ are positive definite, such that the imaginary parts of the spectrum of H_{NH} are upper bounded by zero. Since the non-Hermitian terms still annihilate the embedded scars, their eigenenergies are kept to be purely real. Other thermal eigenstates now acquire negative imaginary parts for their eigenenergies, therefore they will decay away once we start the dissipative evolution driven by H_{NH}. Through this simple modification, we build up a relationship between thermalization in the closed systems and decoherence in the open systems for quantum many-body scarred models. We note that the addition of non-Hermitian terms into many-body scarred Hamiltonians has been carried out in previous works [45–47] in different frameworks.

However, we emphasize that the description of open quantum dynamics in terms of non-Hermitian Hamiltonians is accurate only for short-time dynamics without quantum jumps, or under postselection. We thus turn to the Lindblad master equation Eq. (1) to describe the full-fledged open quantum scarred dynamics. We take the Hamiltonian part of the Liouvillian as the same H, and choose the jump operators as $L_j = V_j P_j$, where $\{V_j\}$ are generic local operators. Now the Liouvillian could be written as $\mathcal{L}(\rho) = -iH_{\mathrm{eff}}\rho + i\rho H_{\mathrm{eff}}^\dagger + 2\gamma\sum_j L_j \rho L_j^\dagger$ with the effective Hamiltonian

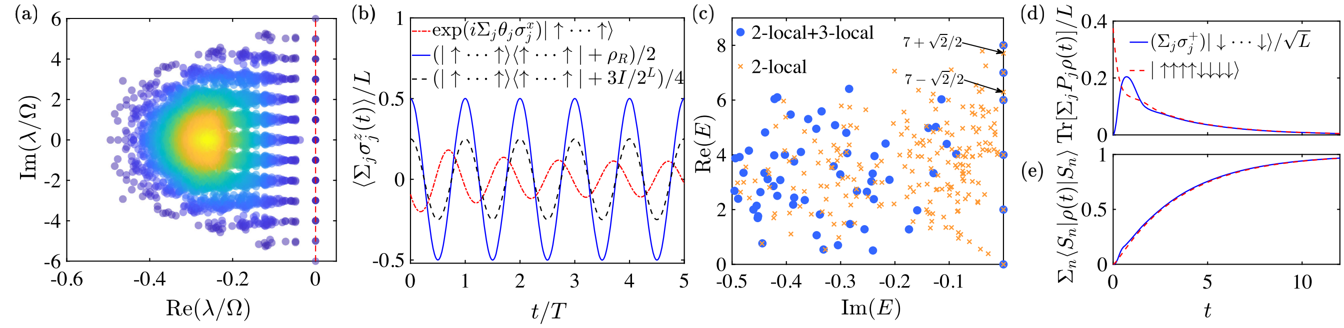

FIG. 2. Numerical results for the Liouvillian spectrum and dissipative scarred dynamics. (a) Liouvillian spectrum of the toy model hosting Dicke states as scars. Scarred eigenmodes are equidistantly embedded on the imaginary axis (the red dotted line) in the form of $\{|S_n\rangle\langle S_m|\}$. $L=6$, $\Omega=2\pi$, $\gamma=1$, $V_{j,j+1}=\sigma_j^x$. (b) Total spin-z dynamics for the toy model, starting from three different initial states. $\theta_j\in[0,\pi]$ are some random rotation angles. ρ_R is a random physical density matrix. (c) Spectrum of the non-Hermitian AKLT Hamiltonian with or without the three-local projectors. $L=8$, $\gamma=2$. Liouvillian dynamics of the quantum jump rate (d) and the scar subspace overlap (e) for the domain-wall preserving model, starting from two initial states. $L=8,\ \Delta=0.5,\ J=1,\ \gamma=1,\ V_{j,j+1}=\sigma_j^x\sigma_{j+1}^x$.

$H_{\mathrm{eff}}=H-i\gamma\sum_j L_j^\dagger L_j$ having the same form as the non-Hermitian Shiraishi-Mori embedding Eq. (2). Given that $L_j|S_n\rangle=0,\ \forall\ j,n,\ H_{\mathrm{eff}}|S_n\rangle=E_n|S_n\rangle$, one can verify that all the $\dim(W)$ scarred eigenstates $\{|S_n\rangle\}$ are embedded into the decoherence-free subspace of the Liouvillian in the form of $\{|S_n\rangle\langle S_m|\}$ [totally $\dim(W)^2$ basis elements]: $\mathcal{L}(|S_n\rangle\langle S_m|)=-i(E_n-E_m)|S_n\rangle\langle S_m|$. We particularly stress that the nonconstant operators V_j in the dissipators $L_j=V_jP_j$ are indispensable. Otherwise, all common eigenstates of $\{P_j\}$ (not necessarily with zero eigenvalues) and H' would enter the decoherence-free subspace, which may include undesired states [40–42].

When the Hamiltonian H (not necessarily following the Shiraishi-Mori formalism) exhibits certain restricted spectrum generating algebra in the scarred subspace W [4,5,13,48], i.e., $([H,Q^\dagger]-\omega Q^\dagger)W=0$ for some ladder operator $Q^\dagger$ generating the tower of scar states $|S_n\rangle=(Q^\dagger)^n|S_0\rangle$, energy levels of scars are evenly spaced by ω in the spectrum of H. In our dissipative protocol, inherited from the Hamiltonian, all the nondecaying eigenmodes located on the imaginary axis are uniformly spaced by the same ω. We emphasize that the aforementioned condition imposes less stringent constraints on the Liouvillians than the dynamical symmetry studied in previous literature [62–66], where the entanglement structure of states in the decoherence-free subspace is not the primary focus either (see [67]).

Within the framework of Shiraishi-Mori embedding, we demonstrate two examples as follows. The first toy model [13] is a one-dimensional spin-1/2 chain with $H_{\mathrm{toy}}=H'+\sum_j P_jh_jP_j$, where $H'=\Omega(\sum_j\sigma_j^x)/2$, $P_j=(1-\vec{\sigma}_j\cdot\vec{\sigma}_{j+1})/4$, and $h_j=\sum_{\mu,\nu}J_{\mu\nu}\sigma_{j-1}^\mu\sigma_{j+2}^\nu$ is a generic two-spin operator. σ_j^μ $(\mu=x,\ y,\ z)$ are standard Pauli matrices. Since $\{P_j\}$ project two adjacent spins onto the singlet states, H_{toy} hosts the x direction Dicke states $|S=L/2,S_x=m\rangle$ as scarred eigenstates with energy spacing $\omega=\Omega$, where S is the total spin and $S_x=\sum_j\sigma_j^x/2$ is the total spin-x polarization, $m=-L/2$, $-L/2+1,\ldots,L/2$. As for the corresponding Liouvillian, we use the same Hamiltonian H_{toy}, together with $L_j=\sigma_j^xP_j$. By exact diagonalization, we obtain the desired Liouvillian spectrum [Fig. 2(a)] and persistent coherent oscillations from generic initial states [Fig. 2(b)]. Moreover, when the Liouvillian superoperator respects the strong symmetry S_x [68], i.e., $[H_{\mathrm{toy}},S_x]=[L_j,S_x]=0,\ \forall\ j$ [the generic forms of local Hamiltonians and dissipators take $h_j=J_1(\sigma_{j-1}^y\sigma_{j+2}^y+\sigma_{j-1}^z\sigma_{j+2}^z)+J_2(\sigma_{j-1}^y\sigma_{j+2}^z-\sigma_{j-1}^z\sigma_{j+2}^y)+J_3\sigma_{j-1}^x\sigma_{j+2}^x$, and $L_j=\sigma_j^xP_j$], the value of S_x is preserved during the open scarred dynamics. In these scenarios, we can effectively prepare any desired x-direction Dicke state by starting the Liouvillian evolution from an x-direction spin product state in the same symmetry sector [48].

The second example is the spin-1 XY model [12] $H_{XY}=\sum_j[S_j^xS_{j+1}^x+S_j^yS_{j+1}^y+hS_j^z+D(S_j^z)^2]$, where there are three degrees of freedom on each site $(|-1\rangle,|0\rangle,|1\rangle)$ and S_j^μ $(\mu=x,y,z)$ are spin-1 operators. The $L+1$ scarred eigenstates are generated from the ferromagnetic state $|S_0\rangle=|-1,\ldots,-1\rangle$ by the ladder operator $Q^\dagger=\sum_j(-1)^j(S_j^+)^2$ with the energy spacing $\omega=2h$. H_{XY} has been shown to be consistent with the Shiraishi-Mori embedding formalism [12,69]. The scar-tower states are annihilated by a set of six orthogonal two-local projectors, which commute with the $\sum_jS_j^z$ and $\sum_j(S_j^z)^2$ terms (see [69] and [48]). Fortunately, the null space of these local projectors coincides with that of the XY interaction term $S_j^xS_{j+1}^x+S_j^yS_{j+1}^y$, so we design the jump operators in a simple form as $L_j=S_j^x(S_j^xS_{j+1}^x+S_j^yS_{j+1}^y)$.

We remark that the success of our dissipative protocol hinges on finding local projectors annihilating the whole scar towers, which could be achieved by compressing the scar tower into a single matrix product state (MPS) $|S(\beta)\rangle = \exp(\beta Q^\dagger)|S_0\rangle = \sum_n \beta^n |S_n\rangle/n!$ and applying standard linear algebra techniques to construct local projectors annihilating the local tensors of $|S(\beta)\rangle$ for any β [15,16,48,69].

Models beyond Shiraishi-Mori embedding.—Our strategy of creating scar-state-only decoherence-free subspace can further apply to many-body scarred models beyond the Shiraishi-Mori embedding formalism. One typical example is the spin-1 Affleck-Kennedy-Lieb-Tasaki (AKLT) model $H_{\mathrm{AKLT}} = \sum_j T^{S=2}_{j,j+1}$, where $T^{S=2}_{j,j+1}$ projects two adjacent spin-1's onto a total spin-2 [70]. A tower of scarred eigenstates with energy spacing $\omega = 2$ is generated from the ground state $|S_0\rangle = |G\rangle$ by the ladder operator $Q^\dagger = \sum_j (-1)^j (S_j^+)^2$ [10,11]. Two-local projectors annihilating the scar tower are known as $P_j = T^{S=2,m=-2}_{j,j+1} + T^{S=2,m=-1}_{j,j+1} + T^{S=2,m=0}_{j,j+1}$ [69], where $T^{S=2,m}_{j,j+1}$ projects two spin-1's onto a total spin-2 with spin-z polarization equal to m. The AKLT Hamiltonian can then be decomposed as $H_{\mathrm{AKLT}} = H' + \sum_j P_j$, where $H' = \sum_j T^{S=2,m=1,2}_{j,j+1}$. However, since $[H', P_j] \neq 0$, H_{AKLT} goes beyond the Shiraishi-Mori framework in the sense that the null space W' is larger than the desired scar subspace W [10,69]. We demonstrate the resulting effect by calculating the spectrum of the non-Hermitian Hamiltonian $H_{\mathrm{NH}} = H_{\mathrm{AKLT}} - i\gamma \sum_j P_j$ [71]. Eigenstates of H_{NH} with purely real eigenvalues are annihilated by $\{P_j\}$ and are eigenstates of the Hermitian part H_{AKLT}, such that they will be embedded in the decoherence-free subspace of the constructed Liouvillian. Apart from the scar states with integer eigenvalues, we observe several undesired irrational eigenvalues on the real axis [orange crosses in Fig. 2(c); see [48] for detailed discussions], which will contaminate the decoherence-free subspace and ruin the periodic oscillations.

To solve the problem, we introduce the three-local projector $T'_{j-1,j,j+1} = (|T'\rangle\langle T'|)_{j-1,j,j+1}$, obtained by the compressed MPS technique,

$$|T'\rangle = \frac{1}{\sqrt{2}}(|0,1,1\rangle + |1,1,0\rangle), \tag{3}$$

to enter the Liouvillian as dissipators. The three-local projectors also annihilate the whole scar tower and they can effectively kill unwanted states in the decoherence-free subspace. As shown by the blue dots in Fig. 2(c), after adding the three-local projectors, irrational eigenvalues disappear from the real axis, and therefore harmonic scarred oscillations are restored. (There still exist a few remaining eigenstates with eigenvalues $L-1$ or $L-2$. See [48] for detailed discussions.) We remark that the common null space W' of two-local and three-local projectors is still larger than the scar subspace W, but the Hamiltonian part H_{AKLT} of the Liouvillian drives unwanted states out of W' to make them decay away [Fig. 1(b)].

To better illustrate the interplay between the dissipators and the Hamiltonian part, we consider the domain-wall preserving model [14] $H_{\mathrm{DW}} = H_0 + H_\Delta + H_J$, where $H_0 = \sum_j (\sigma_j^x - \sigma_{j-1}^z \sigma_j^x \sigma_{j+1}^z)$, $H_\Delta = \Delta \sum_j \sigma_j^z$, and $H_J = J\sum_j \sigma_j^z \sigma_{j+1}^z$. The ladder operator $Q^\dagger = \sum_j (-1)^j P^0_{j-1} \times \sigma_j^+ P^0_{j+1}$ [$P^0_j = (1-\sigma_j^z)/2$] generates the scar tower from the reference state $|S_0\rangle = |{\downarrow\downarrow}\cdots{\downarrow}\rangle$ with energy spacing $\omega = 2\Delta - 4J$. The scar-tower states are subject to the emergent Rydberg-blockade constraints that are absent in H_{DW}: Two neighboring spins cannot both be in the up states. For the constructed Liouvillian, we therefore take $P_j = (|{\uparrow\uparrow}\rangle\langle{\uparrow\uparrow}|)_{j,j+1}$, $V_{j,j+1} = \sigma_j^x \sigma_{j+1}^x$, such that $L_j = V_{j,j+1} P_j = \sigma_j^- \sigma_{j+1}^-$ (in [48] we show that $\{P_j\}$ are the only two-local projectors annihilating the scar tower). We emphasize that $[H_0, P_j] \neq 0$, and the null space W' of $\{P_j\}$ is *exponentially* large with respect to L [8,9], while the dimension of the scar subspace W is only $L/2 + 2$ [48]. The Hamiltonian part of the Liouvillian, H_{DW}, thus plays an indispensable role in creating a scar-state-only decoherence-free subspace, which we demonstrate through the following Liouvillian dynamics. We use the quantum jump rate $\mathrm{Tr}[\sum_j P_j \rho(t)]/L$ to characterize whether a state has reached the null space (zero value implies the state is within W'). As shown in Fig. 2(d), for an initial state in W' but out of W (blue solid line), the quantum jump rate increases up from zero, then decays back to zero, indicating that the state is driven out of W' by the Hamiltonian part and converges to the scar subspace due to dissipation of other eigenmodes. As a comparison, an initial state out of the null space is driven into the scar subspace directly (red dashed line). Meanwhile, we compute the dynamics of the scar subspace overlap for these two initial states, which approaches one monotonically [Fig. 2(e)]. More numerical results are displayed in [48].

Experimental realization.—The dissipative scarred dynamics can be readily implemented [72,73] using currently available quantum simulation technologies, as we demonstrate with the domain-wall preserving model below. Consider a one-dimensional qubit chain coupled to another array of ancilla qubits [Fig. 3(a)]. We digitally simulate the Liouvillian evolution through three steps, similar to the formalism of quantum collision models [66,74–76]. Suppose at time t the entire system has a quantum state in the decoupled form, $|\psi(t)\rangle \otimes |{\downarrow}\cdots{\downarrow}\rangle$, with all the ancilla qubits set to $|{\downarrow}\rangle$. (1) We apply the unitary operator $\exp(-iH_{\mathrm{DW}}\delta t)$ (could be Trotterized to local gates) on $|\psi(t)\rangle$, which plays the role of Hermitian Hamiltonian evolution. (2) We then apply the local unitary gates $\prod_j \exp(-iH^j_{\mathrm{coup}}\sqrt{\delta t})$ with

$$H^j_{\mathrm{coup}} = \sqrt{2\gamma}(L_j \tau_j^+ + L_j^\dagger \tau_j^-), \tag{4}$$

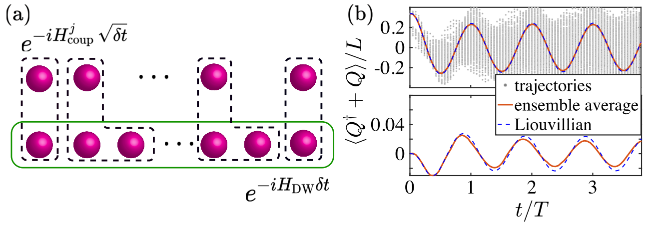

FIG. 3. (a) An illustration of the experimental scheme to implement the dissipative scarred dynamics. (b) Observable dynamics simulated by the quantum trajectory method with the initial state $\exp(i\Sigma_j\theta_j\sigma_j^x)\{\prod_{j=2}^{L-1}[1+(-1)^j P^0_{j-1}\sigma_j^+ P^0_{j+1}]|\downarrow\cdots\downarrow\rangle\}$ [14], $\theta_j\in[0,0.2\pi]$ (top panel), and $|\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\rangle$ (bottom panel; trajectories omitted due to the plot range). For both panels we take 1000 trajectories. $\delta t=0.1$, $L_{j=1,L}=\sigma_j^-$, other $L_{j\neq 1,L}=\sigma_j^-\sigma_{j+1}^-$, $L=8$, $\Delta=0.5$, $J=1$, $\gamma=1$.

which couple the system and ancilla ($\tau_j^\pm$) qubits to create probabilistic quantum jumps induced by $\{L_j\}$. (3) Finally, we reset all the ancilla qubits back to $|\downarrow\rangle$ via measurements or optical pumping [77–84]. We rigorously prove that the above protocol faithfully reproduces the many-body Liouvillian dynamics up to error of order $O(\delta t^2)$ [48]. Moreover, we numerically simulate the three-step dynamical process by the quantum trajectory method [85]. As shown in Fig. 3(b), with a moderate δt, the observable dynamics of $Q^\dagger+Q=\sum_j(-1)^j P^0_{j-1}\sigma_j^x P^0_{j+1}$ (requiring only two measurement settings) computed by the ensemble average of trajectories agree well with the exact Liouvillian evolution. We particularly choose two initial states that are easy to prepare on experimental platforms—the first one mimics an imperfectly prepared bond-dimension-two MPS, and the second one is a product state.

Conclusions.—In summary, our protocol utilizes the synergy between the dissipators and the Hamiltonian part of the Liouvillian to create a scar-state-only decoherence-free subspace. We systematically obtain local projectors annihilating the whole scar towers by the compressed MPS technique. Meanwhile, maximizing the power of the Hamiltonian part is crucial to keep the designed dissipators as local as possible. On the one hand, our framework introduces many-body scarred dynamics into the open quantum system regime. An intriguing advantage compared to the closed-system counterpart is that the dissipative scarred dynamics is independent of the initial states and naturally tolerate instantaneous perturbations. The constructed decoherence-free subspaces can be utilized to prepare scar states with extensive multipartite entanglement [86–88] by engineered short-range dissipation, which makes them promising candidates for quantum enhanced metrology [89]. On the other hand, our work introduces as well new principles and techniques to construct local Liouvillians hosting decoherence-free subspaces with special entanglement structures and equally spaced nondecaying eigenmodes. These scar-state-only decoherence-free subspaces support nonstationary coherent many-body dynamics under dissipation, which have profound connections with certain dissipative kinetically constrained models [90,91] and open up an avenue toward the realization of dissipative time crystals [62,63,92–94]. In the current work the nondecaying scarred eigenmodes of Liouvillians are inherited from the original Hamiltonians. It is also interesting to consider the intrinsic scarred eigenmodes in open quantum systems, which could possibly be distinguished by relatively small operator entanglement [95–99].

We acknowledge helpful discussions with Berislav Buča, Alexey Gorshkov, Fernando Iemini, Francisco Machado, Lei Ying, and Yukai Wu, communications with Juan P. Garrahan, and previous collaborations with Thomas Iadecola and Shenglong Xu. This work was supported by the National Natural Science Foundation of China (Grants No. 12125405, No. 12075128, and No. T2225008), Shanghai Qi Zhi Institute, the Innovation Program for Quantum Science and Technology (No. 2021ZD0302203, No. 2021ZD0301601, and No. 2021ZD0302502), National Key R&D Program of China (No. 2023YFA1406702), the Tsinghua University Initiative Scientific Research Program, Tsinghua University Dushi Program, and the Ministry of Education of China.

[*]These authors contributed equally to this letter.
[†]wangzhongemail@tsinghua.edu.cn
[‡]dldeng@tsinghua.edu.cn

[1] J. M. Deutsch, Quantum statistical mechanics in a closed system, Phys. Rev. A **43**, 2046 (1991).
[2] M. Srednicki, Chaos and quantum thermalization, Phys. Rev. E **50**, 888 (1994).
[3] M. Serbyn, D. A. Abanin, and Z. Papić, Quantum many-body scars and weak breaking of ergodicity, Nat. Phys. **17**, 675 (2021).
[4] S. Moudgalya, B. A. Bernevig, and N. Regnault, Quantum many-body scars and hilbert space fragmentation: A review of exact results, Rep. Prog. Phys. **85**, 086501 (2022).
[5] A. Chandran, T. Iadecola, V. Khemani, and R. Moessner, Quantum many-body scars: A quasiparticle perspective, Annu. Rev. Condens. Matter Phys. **14**, 443 (2022).
[6] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner *et al.*, Probing many-body dynamics on a 51-atom quantum simulator, Nature (London) **551**, 579 (2017).
[7] D. Bluvstein, A. Omran, H. Levine, A. Keesling, G. Semeghini, S. Ebadi, T. T. Wang, A. A. Michailidis, N. Maskara, W. W. Ho *et al.*, Controlling quantum many-body dynamics in driven Rydberg atom arrays, Science **371**, 1355 (2021).
[8] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Weak ergodicity breaking from quantum many-body scars, Nat. Phys. **14**, 745 (2018).
[9] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization,

and stability to perturbations, Phys. Rev. B **98**, 155134 (2018).

[10] S. Moudgalya, S. Rachel, B. A. Bernevig, and N. Regnault, Exact excited states of nonintegrable models, Phys. Rev. B **98**, 235155 (2018).

[11] S. Moudgalya, N. Regnault, and B. A. Bernevig, Entanglement of exact excited states of Affleck-Kennedy-Lieb-Tasaki models: Exact results, many-body scars, and violation of the strong eigenstate thermalization hypothesis, Phys. Rev. B **98**, 235156 (2018).

[12] M. Schecter and T. Iadecola, Weak ergodicity breaking and quantum many-body scars in spin-1 xy magnets, Phys. Rev. Lett. **123**, 147201 (2019).

[13] S. Choi, C. J. Turner, H. Pichler, W. W. Ho, A. A. Michailidis, Z. Papić, M. Serbyn, M. D. Lukin, and D. A. Abanin, Emergent SU(2) dynamics and perfect quantum many-body scars, Phys. Rev. Lett. **122**, 220603 (2019).

[14] T. Iadecola and M. Schecter, Quantum many-body scar states with emergent kinetic constraints and finite-entanglement revivals, Phys. Rev. B **101**, 024306 (2020).

[15] N. Shibata, N. Yoshioka, and H. Katsura, Onsager's scars in disordered spin chains, Phys. Rev. Lett. **124**, 180604 (2020).

[16] S. Chattopadhyay, H. Pichler, M. D. Lukin, and W. W. Ho, Quantum many-body scars from virtual entangled pairs, Phys. Rev. B **101**, 174308 (2020).

[17] N. Shiraishi and T. Mori, Systematic construction of counterexamples to the eigenstate thermalization hypothesis, Phys. Rev. Lett. **119**, 030601 (2017).

[18] C.-J. Lin and O. I. Motrunich, Exact quantum many-body scar states in the Rydberg-blockaded atom chain, Phys. Rev. Lett. **122**, 173401 (2019).

[19] S. Ok, K. Choo, C. Mudry, C. Castelnovo, C. Chamon, and T. Neupert, Topological many-body scar states in dimensions one, two, and three, Phys. Rev. Res. **1**, 033144 (2019).

[20] K. Bull, I. Martin, and Z. Papić, Systematic construction of scarred many-body dynamics in 1D lattice models, Phys. Rev. Lett. **123**, 030601 (2019).

[21] F. M. Surace, P. P. Mazza, G. Giudici, A. Lerose, A. Gambassi, and M. Dalmonte, Lattice gauge theories and string dynamics in Rydberg atom quantum simulators, Phys. Rev. X **10**, 021041 (2020).

[22] A. Hudomal, I. Vasić, N. Regnault, and Z. Papić, Quantum scars of bosons with correlated hopping, Commun. Phys. **3**, 1 (2020).

[23] S. Moudgalya, N. Regnault, and B. A. Bernevig, η-pairing in Hubbard models: From spectrum generating algebras to quantum many-body scars, Phys. Rev. B **102**, 085140 (2020).

[24] D. K. Mark and O. I. Motrunich, η-pairing states as true scars in an extended Hubbard model, Phys. Rev. B **102**, 075132 (2020).

[25] S. Scherg, T. Kohlert, P. Sala, F. Pollmann, B. H. Madhusudhana, I. Bloch, and M. Aidelsburger, Observing non-ergodicity due to kinetic constraints in tilted Fermi-Hubbard chains, Nat. Commun. **12**, 1 (2021).

[26] J.-Y. Desaules, A. Hudomal, C. J. Turner, and Z. Papić, Proposal for realizing quantum scars in the tilted 1D Fermi-Hubbard model, Phys. Rev. Lett. **126**, 210601 (2021).

[27] D. Banerjee and A. Sen, Quantum scars from zero modes in an Abelian lattice gauge theory on ladders, Phys. Rev. Lett. **126**, 220601 (2021).

[28] C. M. Langlett, Z.-C. Yang, J. Wildeboer, A. V. Gorshkov, T. Iadecola, and S. Xu, Rainbow scars: From area to volume law, Phys. Rev. B **105**, L060301 (2022).

[29] C. M. Langlett and S. Xu, Hilbert space fragmentation and exact scars of generalized Fredkin spin chains, Phys. Rev. B **103**, L220304 (2021).

[30] F. Schindler, N. Regnault, and B. A. Bernevig, Exact quantum scars in the chiral nonlinear Luttinger liquid, Phys. Rev. B **105**, 035146 (2022).

[31] J.-Y. Desaules, D. Banerjee, A. Hudomal, Z. Papić, A. Sen, and J. C. Halimeh, Weak ergodicity breaking in the Schwinger model, Phys. Rev. B **107**, L201105 (2023).

[32] P. Zhang, H. Dong, Y. Gao, L. Zhao, J. Hao, J.-Y. Desaules, Q. Guo, J. Chen, J. Deng, B. Liu *et al.*, Many-body Hilbert space scarring on a superconducting processor, Nat. Phys. **19**, 120 (2022).

[33] G.-X. Su, H. Sun, A. Hudomal, J.-Y. Desaules, Z.-Y. Zhou, B. Yang, J. C. Halimeh, Z.-S. Yuan, Z. Papić, and J.-W. Pan, Observation of many-body scarring in a Bose-Hubbard quantum simulator, Phys. Rev. Res. **5**, 023010 (2023).

[34] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, England, on Demand, 2002).

[35] D. A. Lidar, I. L. Chuang, and K. B. Whaley, Decoherence-free subspaces for quantum computation, Phys. Rev. Lett. **81**, 2594 (1998).

[36] D. Bacon, D. A. Lidar, and K. B. Whaley, Robustness of decoherence-free subspaces for quantum computation, Phys. Rev. A **60**, 1944 (1999).

[37] L.-M. Duan and G.-C. Guo, Preserving coherence in quantum computation by pairing quantum bits, Phys. Rev. Lett. **79**, 1953 (1997).

[38] P. Zanardi and M. Rasetti, Noiseless quantum codes, Phys. Rev. Lett. **79**, 3306 (1997).

[39] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, Cavity-loss-induced generation of entangled atoms, Phys. Rev. A **59**, 2468 (1999).

[40] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. Büchler, and P. Zoller, Quantum states and phases in driven open quantum systems with cold atoms, Nat. Phys. **4**, 878 (2008).

[41] B. Kraus, H. P. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, Preparation of entangled states by quantum Markov processes, Phys. Rev. A **78**, 042307 (2008).

[42] F. Verstraete, M. M. Wolf, and J. I. Cirac, Quantum computation and quantum-state engineering driven by dissipation, Nat. Phys. **5**, 633 (2009).

[43] S. Diehl, W. Yi, A. J. Daley, and P. Zoller, Dissipation-induced d-wave pairing of fermionic atoms in an optical lattice, Phys. Rev. Lett. **105**, 227001 (2010).

[44] S. Diehl, E. Rico, M. A. Baranov, and P. Zoller, Topology by dissipation in atomic quantum wires, Nat. Phys. **7**, 971 (2011).

[45] K. Pakrouski, P. N. Pallegar, F. K. Popov, and I. R. Klebanov, Group theoretic approach to many-body scar states in fermionic lattice models, Phys. Rev. Res. **3**, 043156 (2021).

[46] Q. Chen, S. A. Chen, and Z. Zhu, Weak ergodicity breaking in non-Hermitian many-body systems, SciPost Phys. **15**, 052 (2023).
[47] K. Omiya and M. Müller, Quantum many-body scars in bipartite Rydberg arrays originating from hidden projector embedding, Phys. Rev. A **107**, 023318 (2023).
[48] See Supplemental Materials at http://link.aps.org/supplemental/10.1103/PhysRevLett.132.150401 for details about the compressed matrix product state technique to obtain the local projectors annihilating the whole scar towers, the special states in the AKLT model, more numerical results, more discussions about the experimental realizations, and the dissipative preparation of scar states with quantum metrology applications, which includes Refs. [49–61].
[49] D. Perez-Garcia, F. Verstraete, M. Wolf, and J. Cirac, Matrix product state representations, Quantum Inf. Comput. **7**, 401 (2007).
[50] R. Shen, Y. Guo, and S. Yang, Construction of non-Hermitian parent Hamiltonian from matrix product states, Phys. Rev. Lett. **130**, 220401 (2023).
[51] Google Quantum AI and Collaborators, Non-Abelian braiding of graph vertices in a superconducting processor, Nature (London) **618**, 264 (2023).
[52] M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, and P. Zoller, Mesoscopic Rydberg gate based on electromagnetically induced transparency, Phys. Rev. Lett. **102**, 170502 (2009).
[53] S. J. Evered, D. Bluvstein, M. Kalinowski, S. Ebadi, T. Manovitz, H. Zhou, S. H. Li, A. A. Geim, T. T. Wang, N. Maskara *et al.*, High-fidelity parallel entangling gates on a neutral atom quantum computer, Nature (London) **622**, 268 (2023).
[54] M. McEwen, D. Kafri, Z. Chen, J. Atalaya, K. Satzinger, C. Quintana, P. V. Klimov, D. Sank, C. Gidney, A. Fowler *et al.*, Removing leakage-induced correlated errors in superconducting quantum error correction, Nat. Commun. **12**, 1761 (2021).
[55] K. C. Miao, M. McEwen, J. Atalaya, D. Kafri, L. P. Pryadko, A. Bengtsson, A. Opremcak, K. J. Satzinger, Z. Chen, P. V. Klimov *et al.*, Overcoming leakage in scalable quantum error correction, arXiv:2211.04728.
[56] S. L. Braunstein and C. M. Caves, Statistical distance and the geometry of quantum states, Phys. Rev. Lett. **72**, 3439 (1994).
[57] L. Pezzé and A. Smerzi, Entanglement, nonlinear dynamics, and the Heisenberg limit, Phys. Rev. Lett. **102**, 100401 (2009).
[58] P. Hyllus, W. Laskowski, R. Krischek, C. Schwemmer, W. Wieczorek, H. Weinfurter, L. Pezzé, and A. Smerzi, Fisher information and multiparticle entanglement, Phys. Rev. A **85**, 022321 (2012).
[59] G. Tóth, Multipartite entanglement and high-precision metrology, Phys. Rev. A **85**, 022322 (2012).
[60] T. Iadecola, M. Schecter, and S. Xu, Quantum many-body scars from magnon condensation, Phys. Rev. B **100**, 184312 (2019).
[61] M. Kitagawa and M. Ueda, Squeezed spin states, Phys. Rev. A **47**, 5138 (1993).
[62] B. Buča, J. Tindall, and D. Jaksch, Non-stationary coherent quantum many-body dynamics through dissipation, Nat. Commun. **10**, 1730 (2019).
[63] C. Booker, B. Buča, and D. Jaksch, Non-stationarity and dissipative time crystals: Spectral properties and finite-size effects, New J. Phys. **22**, 085007 (2020).
[64] B. Buča, C. Booker, M. Medenjak, and D. Jaksch, Bethe ansatz approach for dissipation: Exact solutions of quantum many-body dynamics under loss, New J. Phys. **22**, 123040 (2020).
[65] K. Chinzei and T. N. Ikeda, Time crystals protected by Floquet dynamical symmetry in Hubbard models, Phys. Rev. Lett. **125**, 060601 (2020).
[66] G. Guarnieri, M. T. Mitchison, A. Purkayastha, D. Jaksch, B. Buča, and J. Goold, Time periodicity from randomness in quantum systems, Phys. Rev. A **106**, 022209 (2022).
[67] Despite the fact that both conditions lead to equally spaced nondecaying eigenmodes and nonstationary periodic dynamics, the dynamical symmetry necessitates $[H, Q^\dagger] = \omega Q^\dagger$ to be valid in the entire Hilbert space, and $[L_j, Q^\dagger] = [L_j^\dagger, Q^\dagger] = 0$ for all the dissipators. In contrast, our protocol requires only the first condition to hold in the scar subspace W and does not need the second one. Furthermore, $[H, Q^\dagger] = \omega Q^\dagger$ poses too strong symmetry in the Hamiltonian to qualify the tower of states $\{|S_n\rangle\}$ as genuine scar states [4,5,23,24].
[68] B. Buča and T. Prosen, A note on symmetry reductions of the Lindblad equation: Transport in constrained open spin chains, New J. Phys. **14**, 073007 (2012).
[69] D. K. Mark, C.-J. Lin, and O. I. Motrunich, Unified structure for exact towers of scar states in the Affleck-Kennedy-Lieb-Tasaki and other models, Phys. Rev. B **101**, 195131 (2020).
[70] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Rigorous results on valence-bond ground states in antiferromagnets, Phys. Rev. Lett. **59**, 799 (1987).
[71] We compute the spectrum of the non-Hermitian AKLT Hamiltonian instead of the whole Liouvillian, in order to access to larger system sizes ($L \geq 8$) to observe the irrational real eigenvalues (see [48] for details).
[72] S. Lloyd and L. Viola, Engineering quantum dynamics, Phys. Rev. A **65**, 010101(R) (2001).
[73] D. Bacon, A. M. Childs, I. L. Chuang, J. Kempe, D. W. Leung, and X. Zhou, Universal simulation of Markovian quantum dynamics, Phys. Rev. A **64**, 062302 (2001).
[74] F. Ciccarello, S. Lorenzo, V. Giovannetti, and G. M. Palma, Quantum collision models: Open system dynamics from repeated interactions, Phys. Rep. **954**, 1 (2022).
[75] M. Cattaneo, G. L. Giorgi, R. Zambrini, and S. Maniscalco, A brief journey through collision models for multipartite open quantum dynamics, Open Syst. Inf. Dyn. **29**, 2250015 (2022).
[76] E. Gillman, F. Carollo, and I. Lesanovsky, Using $(1+1)d$ quantum cellular automata for exploring collective effects in large-scale quantum neural networks, Phys. Rev. E **107**, L022102 (2023).
[77] S. Shankar, M. Hatridge, Z. Leghtas, K. Sliwa, A. Narla, U. Vool, S. M. Girvin, L. Frunzio, M. Mirrahimi, and M. H. Devoret, Autonomously stabilized entanglement between

two superconducting quantum bits, Nature (London) **504**, 419 (2013).

[78] J. Han, W. Cai, L. Hu, X. Mu, Y. Ma, Y. Xu, W. Wang, H. Wang, Y. P. Song, C.-L. Zou, and L. Sun, Experimental simulation of open quantum system dynamics via Trotterization, Phys. Rev. Lett. **127**, 020504 (2021).

[79] W. Cai, J. Han, L. Hu, Y. Ma, X. Mu, W. Wang, Y. Xu, Z. Hua, H. Wang, Y. P. Song, J.-N. Zhang, C.-L. Zou, and L. Sun, High-efficiency arbitrary quantum operation on a high-dimensional quantum system, Phys. Rev. Lett. **127**, 090504 (2021).

[80] X. Mi, A. Michailidis, S. Shabani, K. Miao, P. Klimov, J. Lloyd, E. Rosenberg, R. Acharya, I. Aleiner, T. Andersen *et al.*, Stable quantum-correlated many body states via engineered dissipation, Science **383**, 1332 (2024).

[81] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, An open-system quantum simulator with trapped ions, Nature (London) **470**, 486 (2011).

[82] Y. Lin, J. Gaebler, F. Reiter, T. R. Tan, R. Bowler, A. Sørensen, D. Leibfried, and D. J. Wineland, Dissipative production of a maximally entangled steady state of two quantum bits, Nature (London) **504**, 415 (2013).

[83] P. Schindler, M. Müller, D. Nigg, J. T. Barreiro, E. A. Martinez, M. Hennrich, T. Monz, S. Diehl, P. Zoller, and R. Blatt, Quantum simulation of dynamical maps with trapped ions, Nat. Phys. **9**, 361 (2013).

[84] H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, A Rydberg quantum simulator, Nat. Phys. **6**, 382 (2010).

[85] A. J. Daley, Quantum trajectories and open many-body quantum systems, Adv. Phys. **63**, 77 (2014).

[86] S. Dooley, Robust quantum sensing in strongly interacting systems with many-body scars, PRX Quantum **2**, 020330 (2021).

[87] J.-Y. Desaules, F. Pietracaprina, Z. Papić, J. Goold, and S. Pappalardi, Extensive multipartite entanglement from SU(2) quantum many-body scars, Phys. Rev. Lett. **129**, 020601 (2022).

[88] S. Dooley, S. Pappalardi, and J. Goold, Entanglement enhanced metrology with quantum many-body scars, Phys. Rev. B **107**, 035123 (2023).

[89] L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Quantum metrology with nonclassical states of atomic ensembles, Rev. Mod. Phys. **90**, 035005 (2018).

[90] B. Olmos, I. Lesanovsky, and J. P. Garrahan, Facilitated spin models of dissipative quantum glasses, Phys. Rev. Lett. **109**, 020403 (2012).

[91] K. Macieszczak, M. Guţă, I. Lesanovsky, and J. P. Garrahan, Towards a theory of metastability in open quantum dynamics, Phys. Rev. Lett. **116**, 240404 (2016).

[92] N. Maskara, A. A. Michailidis, W. W. Ho, D. Bluvstein, S. Choi, M. D. Lukin, and M. Serbyn, Discrete time-crystalline order enabled by quantum many-body scars: Entanglement steering via periodic driving, Phys. Rev. Lett. **127**, 090602 (2021).

[93] Z. Gong, R. Hamazaki, and M. Ueda, Discrete time-crystalline order in cavity and circuit QED systems, Phys. Rev. Lett. **120**, 040404 (2018).

[94] H. Keßler, P. Kongkhambut, C. Georges, L. Mathey, J. G. Cosme, and A. Hemmerich, Observation of a dissipative time crystal, Phys. Rev. Lett. **127**, 043602 (2021).

[95] P. Zanardi, C. Zalka, and L. Faoro, Entangling power of quantum evolutions, Phys. Rev. A **62**, 030301(R) (2000).

[96] P. Zanardi, Entanglement of quantum evolutions, Phys. Rev. A **63**, 040304(R) (2001).

[97] T. Prosen and I. Pižorn, Operator space entanglement entropy in a transverse Ising chain, Phys. Rev. A **76**, 032316 (2007).

[98] I. Pižorn and T. Prosen, Operator space entanglement entropy in *xy* spin chains, Phys. Rev. B **79**, 184416 (2009).

[99] T. Zhou and D. J. Luitz, Operator entanglement entropy of the time evolution operator in chaotic systems, Phys. Rev. B **95**, 094206 (2017).