Editors' Suggestion

Microscopic Origin of the Entropy of Astrophysical Black Holes

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We construct an infinite family of microstates for black holes in Minkowski spacetime which have effective semiclassical descriptions in terms of collapsing dust shells in the black hole interior. Quantum mechanical wormholes cause these states to have exponentially small, but universal, overlaps. We show that these overlaps imply that the microstates span a Hilbert space of log dimension equal to the event horizon area divided by four times the Newton constant, explaining the statistical origin of the Bekenstein-Hawking entropy.

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Introduction.—Bekenstein and Hawking [1,2] proposed, on the basis of general relativity and quantum mechanics in curved spacetimes, that black holes behave as thermodynamic objects, and carry an entropy $S = A/4G$, where A is the area of the event horizon, G is Newton's constant, and we are working in units where Planck's constant and the speed of light are 1. This remarkable formula is universal. It applies to any black hole regardless of its mass, charge, or angular momentum, and in any spacetime dimension.

What is the origin of this entropy? Statistical mechanics asserts that the thermodynamic entropy of a classical system equals the logarithm of the number of microstates consistent with the macroscopic parameters. Quantum mechanics complicates matters. Quantum states form a Hilbert space; so any suitably normalized linear combination of microstates is also a microstate. Thus, in quantum systems we instead identify entropy as the logarithm of the *dimension* of the Hilbert space. To give a statistical mechanical interpretation of black hole entropy, we have to determine the dimension of the underlying quantum gravity Hilbert space describing a black hole.

This fundamental problem was solved in a special case by Strominger and Vafa [3] who explained the entropy of certain supersymmetric black holes in terms of the Hilbert space of underlying string theoretic microstates. These calculations were possible because the black holes in question (1) have multiple types of electric and magnetic charges (unlike our world where there is one electromagnetic field, and no magnetic charge); (2) are extremal (unlike most astrophysical black holes) so that the mass achieves a certain lower bound in terms of the charges required for avoiding naked spacetime singularities; and (3) are supersymmetric, in that they retain a fraction of the supersymmetry of the theories in which they are defined (unlike real black holes which have no supersymmetry to break), which is central to the computability of the entropy. Furthermore, the analysis relied on technical details of the ultraviolet completion of gravity in string theory, which include many extra dimensions and exotic extended solitonic objects of cosmic scale, thereby obscuring the nature of these microstates in the semiclassical description of the black hole. The fundamental question has thus remained: can we give a universal microscopic explanation for the entropy of astrophysical black holes? Here, we propose an answer to this question.

Briefly, we use the fact that in quantum statistical mechanics, any superposition of microstates is also a microstate, where a microstate is a normalizable vector in the Hilbert space with fixed expectation values for macroscopic observables. Thus, rather than a specific basis of typical black hole microstates, we simply seek any set of states that is large enough to span the entire Hilbert space. We also require this set to be under sufficient control for us to compute the Gram matrix of state overlaps. The rank of the Gram matrix determines the maximum number of linearly independent microstates, giving the dimension of the Hilbert space. Equivalently, the logarithm of this rank quantifies the statistical entropy.

To this end, we construct an infinite family of atypical, but well-controlled, microstates for black holes in Minkowski space with effective semiclassical descriptions,

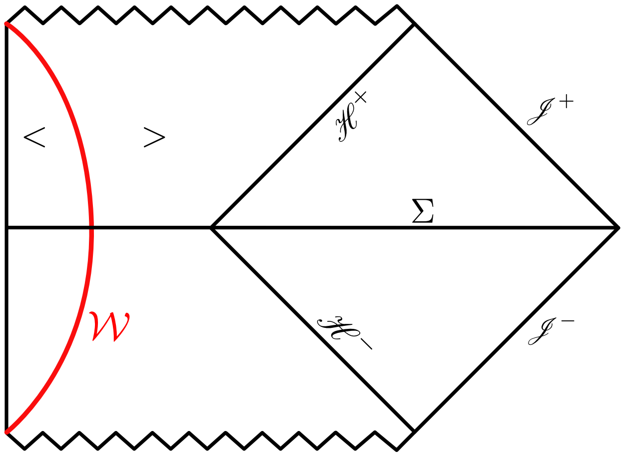

FIG. 1. Penrose diagram of the time evolution of a microstate of an eternal one-sided black hole. The semiclassical state is defined at the time reflection-symmetric Cauchy slice Σ. The exterior geometry extends between the future and past horizons $\mathcal{H}^{\pm}$ and the conformal null boundaries $\mathcal{J}^{\pm}$. The interior contains a thin shell $\mathcal{W}$, which divides the geometry between a region of flat space $<$ inside the shell, and a region of black hole geometry $>$ outside the shell. The zigzag lines at the bottom and top are the white hole and black hole singularities where time starts and ends. The semiclassical state on Σ is nonsingular and perfectly well defined.

which include dust shells in the black hole interior. Our construction follows from general relativity, and does not require any exotic ingredients. Astrophysical black holes generally have some angular momentum, but we will focus on nonrotating black holes for analytical simplicity. Extending methods developed by [4] in the context of two-dimensional gravity and in [5] for general universes with a negative cosmological constant, we compute the overlaps of our microstates in quantum gravity. We find that they span a Hilbert space of dimension precisely equal to the exponential of the Bekenstein-Hawking entropy. This finding explains the microscopic origin of black hole thermodynamics.

Black hole microstates.—We start by constructing an infinite family of microstates for an eternal, asymptotically flat (Minkowski), one-sided black hole (Fig. 1). By microstate we refer to a quantum state in the fundamental Hilbert space of the black hole, with fixed values of the coarse-grained observables, such as the mass of the black hole. The microstates that we will consider will have effective semiclassical descriptions, in terms of black hole solutions of general relativity coupled to matter. In these solutions, the black hole is not formed from collapse—rather, it exists forever, and behind the event horizon there is a "white hole" singularity where time begins, in addition to a black hole singularity where time ends. However, as we will discuss below, their geometry matches the late-time behavior of black holes forming from collapse, and they can account for the entropy associated with the collapsing configuration. All the states we construct have the same geometry between the horizon and the asymptotic spacetime boundary; as such they are microstates of the same black hole as seen by an external observer.

More precisely, outside the horizon all our microstates match the geometry of a Schwarzschild black hole of radius $r_s = 2GM$, where M is the Arnowitt-Deser-Misner (ADM) mass. They are distinguished by their interior geometries: each contains a different configuration of matter which backreacts to generate a distinct interior. The matter emerges out of the past singularity and dives into the future singularity, without leaving the black hole region (Fig. 1). For simplicity, we restrict to matter organized in spherical thin shells of dust particles, with total rest mass m. The states in the family are labeled by the mass m of the shell in the interior.

In detail, the exterior metric is the usual Schwarschild one:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2, \tag{1}$$

where $f(r) = 1 - (r_s/r)$ and $\mathrm{d}\Omega^2 = d\theta^2 + \sin^2\theta\mathrm{d}\varphi^2$ is the round metric of the unit sphere $\mathbf{S}^2$.

This metric can be continued into the interior of the black hole, at $r < r_s$; and further to a second asymptotic region, described by the same metric (1). Our microstates correspond to shells of matter which live in the black hole interior and this second asymptotic region. In the thin-shell limit, the full geometry inside and outside the shell is determined by the Israel junction conditions [6] which fixes the change in the spacetime metric across the shell. Concretely, the world volume $\mathcal{W}$ of the thin shell carries the localized energy momentum of a pressureless perfect fluid

$$T_{\mu\nu}|_{\mathcal{W}} = \sigma u_\mu u_\nu, \tag{2}$$

where σ is the surface density of the fluid, and u^μ is the four-velocity field of the dust, tangent to $\mathcal{W}$. The induced metric on the world volume $\mathcal{W}$ is determined by $R(T)$, the radius of the shell R as a function of its proper time T. From the point of view of the metric (1), the shell will live at $r = R(T)$, with T determined by the proper time along the shell's trajectory. The equation of motion for $R(T)$, determined by the Israel junction conditions, is that of a nonrelativistic particle of zero total energy

$$\dot{R}^2 + V_{\mathrm{eff}}(R) = 0, \tag{3}$$

where we defined the effective potential

$$V_{\mathrm{eff}}(R) = f(R) - \left(\frac{M}{m} - \frac{Gm}{2R}\right)^2. \tag{4}$$

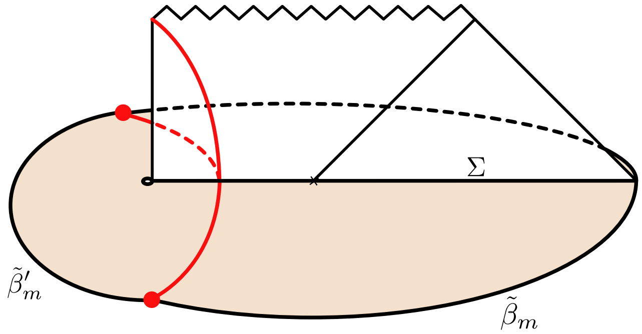

FIG. 2. Euclidean continuation of the spacetime geometry of the microstates along Σ. The Euclidean section consists of a Euclidean black hole (right), and a region of a Euclidean flat space (left), glued together along the trajectory of a thin shell. The shell starts at the asymptotic spatial infinity, bounces back at R_*, and gets back to $R = \infty$. The Euclidean times $\tilde{\beta}_m, \tilde{\beta}'_m \leq \beta$ depend on the mass of the shell.

Provided that $M \leq m$, the shell will expand from the past singularity, located at finite proper time in the past, and enter the second exterior region, where it reaches a maximum radius $R_* \geq r_s$ at which $V_{\mathrm{eff}}(R_*) = 0$. The shell then turns around and dives into the future interior and finally the future singularity. For large proper mass $m \gg M$, the shell recollapses at a radius $R_* \approx Gm/2$, due entirely to its gravitational self-energy.

The geometry inside the shell consists of a portion of flat space

$$\mathrm{d}s^2_< = -\mathrm{d}\tilde{t}^2 + \mathrm{d}\tilde{r}^2 + \tilde{r}^2 \mathrm{d}\Omega^2, \tag{5}$$

which caps of smoothly at $\tilde{r} = 0$. The geometry outside the shell is given by a two-sided black hole geometry, cut off at $r = R_* \geq r_s$ on the left side.

To better understand this geometry, we can focus on the time-reflection symmetric hypersurface Σ (Fig. 1) on which the semiclassical state lives. The induced geometry of this slice resembles a Wheeler's "bag of gold" [7]. When the shell mass is large $m \gg M$, the total interior volume of $\Sigma_{\mathrm{in}} \subset \Sigma$ scales as $\mathrm{Vol}(\Sigma_{\mathrm{in}}) \approx (\pi/3)(Gm)^3$, while the surface of the shell $\sigma = \Sigma \cap \mathcal{W}$ has maximum area scaling as $\mathrm{Area}(\sigma) \approx \pi(Gm)^2$.

The states we have constructed are labeled by the mass of the interior shell. A simple generalization is to consider multiple shells in the interior. We could also consider other configurations of matter, including exotic matter arising from string theory. This choices will generate different interior geometries, while keeping the exterior fixed. As we will argue below, none of these details will matter for counting the microstates.

Quantum gravitational overlaps.—The spacetime geometry X of each microstate can be analytically continued into the Euclidean section along the time reflection-symmetric hypersurface Σ. The Euclidean geometries define a set of asymptotic boundary conditions at Euclidean spatial infinity ∂X. These boundary conditions can be used, within the conventional path integral construction, to prepare the corresponding semiclassical state associated with the microstate of the black hole. Related constructions have been studied in models of 2d gravity [8,9] and in AdS/CFT [5,10].

The semiclassical description of the microstates are constructed on the time reflection symmetric ($t = 0$) slice Σ via the Euclidean path integral. In this way, the construction of the state, and the slice where it lives, are both perfectly regular, analogously to the preparation of the well-known Hartle-Hawking state in the eternal black hole. Our state then defines regular initial data determining Lorentzian time evolution into the future and the past.

In this way, the microstates that we have discussed in the previous section specify an infinite family of quantum states $\{|\Psi_m\rangle\}$ of the Hilbert space of the black hole, where we remind one that m labels the proper mass of the corresponding matter insertion in the black hole interior, and that there is no upper bound on m.

This infinite family naively overcounts the Bekenstein-Hawking entropy. But this is only so if the states are orthogonal to each other. To get a correct counting of the dimension of the Hilbert space of the black hole, we must compute the overlaps $\langle \Psi_m | \Psi_{m'} \rangle$ between our microstates. This can be done using the gravitational path integral. The rules are to fix the asymptotic boundary conditions that prepare the respective states, and to fill in the Euclidean geometry with all possible saddle-point manifolds that respect these boundary conditions. We will work in the simple effective description of the microstates, in terms of the Euclidean gravitational action coupled to a thin shell

$$I[X] = -\frac{1}{16\pi G}\int_X R + \frac{1}{8\pi G}\int_{\partial X} K + \int_{\mathcal{W}} \sigma + I_{\mathrm{ct}}. \tag{6}$$

Here, R is the Ricci scalar of the Euclidean manifold X, K is the extrinsic curvature of its boundary ∂X, σ is the density of the shell, $\mathcal{W}$ is the world volume of the shell, and I_{ct} is a background substraction counterterm that removes divergences and renormalizes the value of the on-shell action.

The leading contribution to the overlap comes from the Euclidean manifold in Fig. 2 which has a single asymptotic boundary where the shell trajectory starts and ends. When $m = m'$ these are straightforward to construct and are simple analogs of those constructed in [5,11]. When $m \neq m'$, we need some way to join the shells from he Euclidean boundary. This will come from interactions and we expect the result to be exponentially suppressed in the mass difference. We take this difference to be arbitrarily large, so that at leading order the overlap is

$$\overline{\langle \Psi_m | \Psi_{m'} \rangle} = \delta_{mm'}, \tag{7}$$

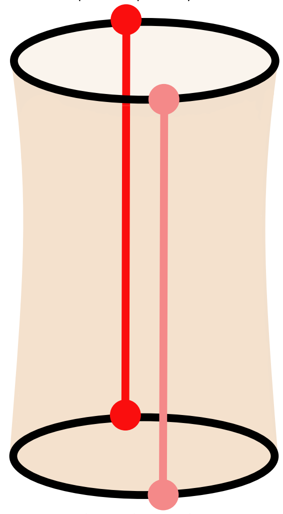

FIG. 3. Euclidean wormhole contribution to the second moment of the overlap. The wormhole has the two inner products as its boundaries. It consists of two Euclidean black holes in flat space, glued along the two shells.

where the overbar denotes that the calculation is performed according to the rules of the gravitational path integral, and where implicitly we have normalized the states using the on-shell action of the corresponding Euclidean manifold $Z_1^{(m)} = e^{-I[X_m]}$. Therefore, microstates representing different classical geometries naively appear to be orthogonal, suggesting that our family of microstates actually spans an infinite dimensional Hilbert space.

However, this conclusion is drastically changed by the appearance of semiclassical wormhole contributions in the Euclidean path integral that compute higher moments of this type of amplitude. These wormholes correspond to nonperturbative effects in quantum gravity. To start with, an explicit two boundary wormhole contributes to the square of the overlap

$$\overline{|\langle\Psi_m|\Psi_{m'}\rangle|^2} = \delta_{mm'} + \frac{Z_2}{Z_1^{(m)} Z_1^{(m')}}. \tag{8}$$

The new contribution is given by the action $Z_2 = e^{-I[X_2]}$, where X_2 is the Euclidean wormhole manifold that extends between the two asymptotic boundaries that prepare each of the overlaps (see Fig. 3). These give order $\mathcal{O}(e^{-S})$ contributions which will dominate any mass-dependent terms in the no-wormhole contribution. Specifically, the wormhole solution is constructed by cutting and gluing two Euclidean black hole solutions along the trajectories of the two thin shells. The detailed construction of such wormholes can be found in [10,11] for the case of AdS space, and we extend them here to the case of Minkowski spacetime.

Similarly, the connected contribution to the nth product is nonvanishing due to the appearance of n-boundary wormholes

$$\overline{\langle\Psi_{m_1}||\Psi_{m_2}\rangle\langle\Psi_{m_2}||\Psi_{m_3}\rangle\ldots\langle\Psi_{m_n}||\Psi_{m_1}\rangle}|_c = \frac{Z_n}{Z_1^{(m_1)}\ldots Z_1^{(m_n)}}. \tag{9}$$

The connected contribution $Z_n = e^{-I[X_n]}$ corresponds to a semiclassical wormhole X_n with n boundaries. Again, this wormhole is a classical solution to equations of motion. It consists of two black holes joined through the different shells, see [5,11] for explicit details in AdS.

The "nonfactorization" of the inner products due to nonperturbative effects in quantum gravity might also seem disturbing at first sight. We provide a simple microscopic interpretation of these overlaps and nonfactorization in Supplemental Material [12]. The interpretation is based on the Eigenstate thermalization hypothesis [15,16]. Briefly, applied here it asserts that these amplitudes should be viewed as the statistics of the fine grained microstates, whose precise computation would involve the control of erratic phases. These erratic phases are invisible to the gravity computation which naturally treats them as random and performs an effective average over them. Nonfactorization is naturally associated with an average over random phases.

Below we will need generic n moments of the inner product. These moments can be computed in a straightforward manner for general shell masses, but the expressions are not very illuminating. Luckily we will only need them in the regime of large masses $m_i \gg M$. In this case, the wormhole action simplifies and reduces to

$$\overline{\langle\Psi_{m_1}||\Psi_{m_2}\rangle\langle\Psi_{m_2}||\Psi_{m_3}\rangle\ldots\langle\Psi_{m_n}||\Psi_{m_1}\rangle}|_c \approx \frac{Z_{\mathrm{bh}}(n\beta)}{[Z_{\mathrm{bh}}(\beta)]^n}, \tag{10}$$

where $Z_{\mathrm{bh}}(\beta) = e^{-I_{\mathrm{bh}}(\beta)}$ and $I_{\mathrm{bh}}(\beta)$ is the gravitational action of the Euclidean Schwarzschild black hole of inverse temperature β, where $\beta = 8\pi GM$ is the inverse temperature of the original black hole. Equivalently, the action $I_{\mathrm{bh}}(\beta)$ is the Euclidean gravity action first computed in the seminal article by Gibbons and Hawking [17]. Hence, we conclude that the overlaps become universal in this limit, independent of the actual masses of the shells characterizing the microstates.

Counting microstates.—We now consider an infinite subfamily of black hole microstates $\{|\Psi_{m_j}\rangle\}$ for shells with mass $m_j = jm$ for $j = 1, 2, \ldots$, where m is a sufficiently large value of the mass. The objective is to determine the dimension of the Hilbert space spanned by these microstates. To this end, we consider the Gram matrix of overlaps for a finite subset of these states

$$G_{ij} = \langle \Psi_{m_i} || \Psi_{m_j} \rangle, \tag{11}$$

where $i, j = 1, \ldots, \Omega$. This Gram matrix is Hermitian and positive semidefinite. The number of nonzero eigenvalues of this matrix counts the number of linearly independent microstates in the given subset. Equivalently, the rank of this matrix gives the Hilbert space dimension spanned by the set. We seek to compute the rank of the Gram matrix as a function of Ω.

As mentioned above, the gravitational computations of the moments of this Gram matrix suggest that we interpret it as a random matrix with moments given by the universal overlaps (10). In order to count black hole microstates for a given energy from these overlaps, i.e., the microcanonical degeneracy, we project the previous universal expressions into a given energy via an inverse Laplace transform of the wormhole contributions

$$Z_{\mathrm{bh}}(n\beta) = \int dE\, \rho(E) e^{-n\beta E}, \tag{12}$$

and for a given microcanonical window of energies $[E, E + \Delta E]$, we define the functions

$$e^{\mathbf{S}} \equiv \rho(E)\Delta E, \qquad \mathbf{Z}_n \equiv \rho(E) e^{-n\beta E}\Delta E. \tag{13}$$

The function $\mathbf{S}$ coincides with the Bekenstein-Hawking entropy [1,2]

$$\mathbf{S} = \frac{A}{4G}, \tag{14}$$

but notice that, at this stage, this function lacks the interpretation in terms of the dimension of black hole Hilbert space. Here, it is just recontextualized as a function controlling nonperturbative wormhole contributions to the gravitation path integral.

Using the microcanonical expressions (13), in Supplemental Material [12] we show that the density of states of the Gram matrix is given by

$$D(\lambda) = \frac{e^{\mathbf{S}}}{2\pi\lambda}\sqrt{\left[\lambda - \left(1 - \sqrt{\frac{\Omega}{e^{\mathbf{S}}}}\right)^2\right]\left[\left(1 + \sqrt{\frac{\Omega}{e^{\mathbf{S}}}}\right)^2 - \lambda\right]} + \delta(\lambda)(\Omega - e^{\mathbf{S}})\theta(\Omega - e^{\mathbf{S}}). \tag{15}$$

This density of states has a continuous part and a singular part. The eigenvalues accounted by the continuous part are all positive definite. The singular part counts the number of zero eigenvalues. Therefore, the rank of the Gram matrix is the number of eigenvalues contained in the continuous part of the distribution. We thus conclude (i) For $\Omega < e^{\mathbf{S}}$, the rank of G is given by Ω. (ii) For $\Omega > e^{\mathbf{S}}$, the rank of G is given by $e^{\mathbf{S}}$.

For $\Omega < e^{\mathbf{S}}$, we can thus use the Gram-Schmidt procedure to construct an orthonormal set of vectors. For $\Omega > e^{\mathbf{S}}$ this will fail, as the microstates will no longer be linearly independent. This gives the main result of this Letter, namely that the black hole microstate degeneracy, equal to the number of possible orthogonal states in a given energy band is equal to the exponential of the Bekenstein-Hawking entropy (14). Equivalently, the present microstate construction provides a microscopic statistical understanding of the entropy of black holes in Minkowski spacetime.

Let us summarize the intuition behind our result. Even if we keep adding potential microstates, there is a point at which these states cannot be orthogonal anymore. This point is controlled by the universal statistics of the inner product, themselves controlled by the Bekenstein-Hawking entropy. As proposed in the introduction, the solution to the problem of the microscopic origin of the entropy of general black holes is not to construct a *specific* set of $e^{\mathbf{S}}$ microstates. Indeed, there are infinite numbers of such sets, even when they are constrained to be semiclassical and geometrical. The problem is to count how many orthogonal states we can build out of those, and prove this counting gives rise to the right Bekenstein-Hawking dimension.

Discussion.—Gibbons and Hawking [17] proposed that the Euclidean gravitational path integral should be understood as a thermal free energy, and extracted an entropy from this. In our calculation, the same path integral appears in a different way that is manifestly tied to a microcanonical state counting interpretation. Specifically, we used this path integral to prepare semiclassically well-controlled microstates and compute their overlaps.

The microcanonical Hilbert space dimension then follows from a universal norm of the overlaps of our microstates, an *output* of the gravitational path integral that we discover. We propose an interpretation: the microstates become "random" relative to each other, if the masses of their respective shells are very different. The average norm of the overlap between two random states is universal and only depends on the Hilbert space dimension. It would be interesting to understand how some UV information about the statistics of state overlaps makes its way into the semiclassical path integral.

We do not use details of string theory, AdS/CFT, or any other UV formulation of quantum gravity. Our assumptions are (a) there is some UV completion, and (b) the semiclassical Euclidean path integral provides sensible information about this completion. Our results then explain black hole entropy in any theory that has general relativity coupled to massive matter as a low-energy limit. In particular, the construction works in top-down models such as those appearing in string theory and AdS/CFT, since these theories contain the ingredients to construct our microstates. Perhaps this explains the universality of the Bekenstein-Hawking entropy formula. Of course, the black hole entropy will have subleading, nonuniversal corrections. Some of these can be computed explicitly from the semiclassical matter, by including one-loop determinants correcting the universal overlaps. Still other corrections, related for example to the discreteness of the exact quantum density of states, will depend on the precise UV completion, and will require a microscopic understanding of these states. It would be very interesting to achieve such an understanding.

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[12] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.132.141501 for an argument describing why the gravitational path integral provides information about the typical magnitude of the overlaps between different microstates and not about their fine-grained phases, which includes Refs. [13,14].

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