Editors' Suggestion

# Gravitational Raman Scattering in Effective Field Theory: A Scalar Tidal Matching at $\mathcal{O}(G^3)$

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We present a framework to compute amplitudes for the gravitational analog of the Raman process, a quasielastic scattering of waves off compact objects, in worldline effective field theory. As an example, we calculate third post-Minkowskian order [$\mathcal{O}(G^3)$], or two-loop, phase shifts for the scattering of a massless scalar field including all tidal effects and dissipation. Our calculation unveils two sources of the classical renormalization-group flow of dynamical Love numbers: a universal running independent of the nature of the compact object, and a running self-induced by tides. Restricting to the black hole case, we find that our effective field theory phase shifts agree exactly with those from general relativity, provided that the relevant static Love numbers are set to zero. In addition, we carry out a complete matching of the leading scalar dynamical Love number required to renormalize a universal short scale divergence in the $S$ wave. Our results pave the way for systematic calculations of gravitational Raman scattering at higher post-Minkowskian orders.

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*Introduction.*—Recent advances in gravitational wave astronomy have spurred the development of efficient techniques for precision calculations of binary dynamics. One such technique is worldline effective field theory (EFT) for compact binaries [1–6], wherein a compact object (a neutron star or black hole) is represented at large distances as a point particle, and which provides a systematic program for the perturbative computation of inspiral waveforms. More generally, the EFT paradigm enables an accurate description of a variety of physical effects: tides and dissipation [2,7,8], spin [4,9,10], Hawking radiation [11,12], self-force [13–16], etc.

In this Letter, we use the EFT framework to calculate mildly inelastic gravitational scattering of massless fields off compact objects. This is a direct gravitational analog of Raman scattering of photons that is commonly used to elucidate the internal structure of molecules. Here, we explore its gravitational counterpart to probe the nature of compact relativistic objects.

In the worldline EFT the finite-size structure of compact objects is captured by multipole moments on the particle's worldline nonminimally coupled to the gravitational field [1,2]. The associated Wilson coefficients provide a gauge-invariant definition of the tidal deformability of the objects, also known as Love numbers [1,17–24]. These are free parameters in the EFT that have to be either measured from data or extracted from a matching calculation to a microscopic theory, if the latter is available. Once the values of matching coefficients are determined they can be used to make further predictions. The universality and consistency of the EFT thus guarantee its predictability.

Scattering amplitudes are particularly suitable for matching calculations: they are simple, manifestly gauge-invariant, and field-redefinition independent objects [1,3,25–29]. In addition, in the post-Minkowskian (PM) regime (formal perturbation theory in Newton's constant $G$) they can be directly compared to known amplitudes in full classical general relativity (GR). These matching calculations also provide new insights into the general structure of gravitational scattering amplitudes by confronting them with exact nonperturbative results from black hole solutions. In this vein, partial results on the calibration of Love numbers from scattering amplitudes exploiting the so-called near-far factorization were given in [28,29]. A numerical estimation of tidal effects from scattering of a pointlike particle with scalar charge by black holes at 4PM order was carried out in [30]. Finally, the scattering of photons and gravitons off compact objects is, in principle, an observable phenomenon relevant in astrophysics and cosmology; see, e.g., [31–34].

We present a general framework for systematic computations of EFT amplitudes for gravitational Raman scattering at high PM orders. Our approach makes use of the background field method and advanced multiloop integration techniques. We demonstrate its power by explicitly calculating the amplitudes for spin-0 fields scattering off a nonspinning compact object through 3PM order, $\mathcal{O}(G^3)$, where finite-size effects first appear. We find that the amplitude exhibits ultraviolet (UV) divergences, whose renormalization requires contact worldline operators. They are scalar analogs of the "dynamical Love number," a coefficient that sets the strength of the multipole moment tidally induced by an external time-dependent field. We show that dynamical Love numbers undergo renormalization-group running due to two different effects. The first source of renormalization is the gravitational "dressing" of the point particle action. As such, this running is universal for any compact object. The second source of the running is the gravitational "dressing" of the static Love number. We call such running "self-induced," as its strength is set by the amplitude of lower order tidal Wilson coefficients (see also [29,35–37] for similar discussions).

Assuming that a compact object is a black hole, and using results from black hole perturbation theory (BHPT) [28,38–46], the EFT scattering amplitudes allow for a complete order-by-order matching of tidal effects, including dissipation. Matching the 3PM scattering amplitudes to BHPT, we prove explicitly that the leading static tidal coefficient is zero and does not run, in agreement with previous off-shell calculations [22,23,47]. This also implies the vanishing of the self-induced tidal coefficients. In addition, we completely match the leading spin-0 dynamical Love number. Finally, we compute the running of the scalar dissipation operators, thus extending the previous calculations from [7,26,29,48]. Our results set the stage for forthcoming spin-2 calculations.

*Worldline EFT and power counting.*—The first ingredient of the worldline EFT is the "bulk" action for the massless scalar and gravitational fields:

$$S_{\text{bulk}} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}(\partial_\mu\phi)^2\right). \tag{1}$$

A compact object of mass $m$ is described by the worldline action

$$S = -m\int d\tau + S_{\text{fs}}, \tag{2}$$

where $\tau$ is proper time. The first term is the relativistic point-particle action and $S_{\text{fs}}$ is an action encoding finite-size effects. As mentioned in the introduction, in the language of effective field theory the latter appear as higher-dimension operators on the worldline and couplings of the fields to dynamical multipole moments describing the internal degrees of freedom of the compact object, $Q_L$. For scalars, this action reads [2,7]

$$S_{\text{fs}} = \sum_\ell \int d\tau\, Q_L \boldsymbol{\partial}_L\phi + S_{\text{fs}}^{\text{ct}}, \tag{3}$$

where in the EFT the multipoles, $Q_L$, are composite operators, $L$ is a multi-index denoting the symmetric traceless combination $\ell$ indices, and $\boldsymbol{\partial} = (g^{\mu\nu} + u^\mu u^\nu)\partial_\mu$, with $u^\rho$ the object's 4-velocity, is the spatial derivative in the rest frame of the compact object. $S_{\text{fs}}^{\text{ct}}$ is the counterterm action discussed shortly. The dynamical dipole coupling $\int d\tau \boldsymbol{Q}\cdot\boldsymbol{\partial}\phi$ is analogous to the familiar dipolar electromagnetic interaction. In the EFT we are ignorant about the microscopic nature of the multipoles. Instead, we are interested in their correlation functions, such as the Fourier transformed time-ordered two-point function

$$\int dt e^{-i\omega t}\langle TQ_{L_1}(t)Q_{L_2}(0)\rangle = -i\delta_{L_1L_2}F_\ell(\omega), \tag{4}$$

which at low frequencies takes the form

$$F_\ell(\omega) = C_{\ell,\omega^0} + iC_{\ell,\omega}|\omega| + C_{\ell,\omega^2}\omega^2 + \cdots. \tag{5}$$

The corresponding Wilson coefficients $C_{\ell,\omega^n}$ are collectively known as Love numbers. The static Love numbers ($n=0$) describe the response of the compact object to time-independent fields (static tides) with different multipolar profiles. These have been extensively studied for neutron star and black holes [18,19,49,50], which yielded a surprising result that they vanish for black holes in $D = 1+3$ [20–24,51] (symmetry explanations were proposed in [47,52–54]). The coefficients $C_{\ell,\omega^{2n}}$ ($n>0$) are called "dynamical Love numbers," as they describe the response to time-dependent fields. We refer to $C_{\ell,\omega^{2n+1}}$ as dissipation numbers.

Real parts of $F_\ell$ are analytic functions that describe conservative finite-size effects. As such, they can be fully absorbed into the local worldline counterterm action,

$$\begin{aligned} S_{\text{fs}}^{\text{ct}} &= \sum_\ell \frac{1}{2\ell!}\int d\tau\left[C_{\ell,\omega^0}(\boldsymbol{\partial}_L\phi)^2 + C_{\ell,\omega^2}(\boldsymbol{\partial}_L\dot{\phi})^2 + \cdots\right] \\ &= \frac{1}{2}\int d\tau\left[C_{1,\omega^0}(\boldsymbol{\partial}\phi)^2 + C_{0,\omega^2}\dot{\phi}^2 + C_{1,\omega^2}(\boldsymbol{\partial}\dot{\phi})^2 + \cdots\right], \end{aligned} \tag{6}$$

where $\dot{\phi} = \partial_\tau\phi = (u^\mu\partial_\mu)\phi$. Here, in the first line we show operators corresponding to the static and leading dynamical Love numbers, and in the second we show only the leading order operators relevant for our calculation below. Note that the scalar monopole operator $\int d\tau\phi^2$ is forbidden by the shift symmetry $\phi\to\phi+v$ of the massless scalar, i.e., $C_{0,\omega^0} = 0$.

In contrast, $\text{Im}F_\ell$ describes the dissipative part of the response and cannot be written in terms of local worldline operators. This will capture inelastic effects, e.g., absorption or tidal heating. Note that formally $F_\ell$ are correlators of renormalized multipole moments that receive scale dependence through gravitational dressing [55,56].

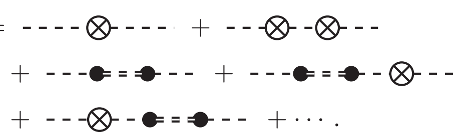
Wilsonian naturalness dictates that $C_{\ell,\omega^n} \sim R^{2\ell+1+n}$, where $R$ is the size of the compact object, $R = AGm$, with $A = 2$ for black holes, and $A \sim 10$ for neutron stars [49], which makes it natural to consider $R$ as a second expansion parameter independent of $Gm$.

*EFT scattering amplitudes.*—The effective action (1)–(2) can be used to calculate the quantum amplitudes for scalars scattering off a compact object. We compute the full EFT amplitudes following the approach of [16,57], i.e., expanding the effective action around a background solution given by the worldline moving in a straight trajectory, $x^\mu(\tau) = u^\mu \tau$, and the metric given by the large-distance expansion of the Schwarzschild metric in isotropic coordinates,

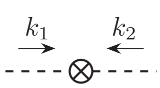
$$\begin{aligned}\bar{g}_{\alpha\beta} &= \eta_{\alpha\beta} + u_\alpha u_\beta\left(-\frac{\mu}{r^{D-3}} + \frac{1}{2}\frac{\mu^2}{r^{2(D-3)}}\right)\\ &\quad + (\eta_{\alpha\beta} + u_\alpha u_\beta)\left(\frac{1}{D-3}\frac{\mu}{r^{D-3}} - \frac{D-7}{8(D-3)^2}\frac{\mu^2}{r^{2(D-3)}}\right)\\ &\quad + \cdots,\end{aligned} \tag{7}$$

where $\mu = [16\pi Gm/(D-2)\Omega_{D-2}]$, and $\Omega_{D-2}$ is the volume of the $(D-2)$-dimensional sphere. Famously, this expansion resums the perturbative solution to Einstein's equation with a point source [58], which corresponds to an infinite number of worldline Feynman diagrams. The recoil of the worldline is subleading in the low-frequency limit, and the metric fluctuations are not relevant for the scalar field amplitude, so we will also ignore them henceforth.

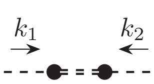
The full scalar amplitude is simply given by iterative scattering against the background plus the scattering off the dynamical multipole moments. The corresponding background-field Feynman diagrams are

$i\mathcal{M} =$  (8)

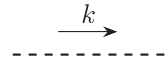
The first background-field vertex Feynman rule is given in momentum space by the Fourier transform of the scalar action

 $= i(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}(q) - \eta^{\mu\nu})k_{1\mu}k_{2\nu}$, (9)

with momentum transfer $q = k_1 + k_2$. The other vertex is just the correlator $\langle Q_{L_1} Q_{L_2}\rangle$ that captures the dynamical multipolar tidal response

 $= i(-1)^\ell k_1^{L_1} k_2^{L_2} \delta_{L_1 L_2} F_\ell(u\cdot k_1)$ (10)

They are connected by ordinary flat-space propagators

 $= \dfrac{-i}{k^2 - i0}$. (11)

The background-field diagrams can be recast in terms of ordinary flat-space Feynman integrals [16,57]. In our case, at 3PM, all such integrals belong to the family

$$G_{a_1a_2a_3a_4a_5a_6a_7} = \int_{\ell_1\ell_2} \frac{\delta(u\cdot\ell_1)\delta(u\cdot\ell_2)D_7^{-a_7}}{D_1^{a_1}D_2^{a_2}D_3^{a_3}D_4^{a_4}D_5^{a_5}D_6^{a_6}}, \tag{12}$$

with a basis of propagators and invariant products

$$\begin{aligned}&D_1 = \ell_1^2, \qquad D_2 = \ell_2^2, \qquad D_3 = (\ell_1 + k_1)^2,\\ &D_4 = (\ell_2 + k_2)^2, \qquad D_5 = (\ell_1 + \ell_2 + k_1 + k_2)^2,\\ &D_6 = (\ell_1 + k_2)^2, \qquad D_7 = (\ell_2 + k_1)^2.\end{aligned} \tag{13}$$

We will compute all integrals in dimensional regularization with $D = 4 - 2\epsilon$. Using integration-by-parts identities [59] we can reduce any integral in such family to a basis of *master integrals* given by

$$\begin{aligned}&\{G_{0011000}, G_{0110100}, G_{1001100}, G_{1100100},\\ &G_{1101100}, G_{1110100}, G_{1111100}, G_{2111100}\},\end{aligned} \tag{14}$$

where we use the notation in Eq. (12). Their dependence on the frequency $\omega$ is fixed by the dimensional analysis, so they are only nontrivial functions of the scattering angle, which we parametrize by $x = \sin(\theta/2)$. We compute the dependence on $x$ by using the method of differential equations for Feynman integrals [60–62]. Indeed, it is not difficult to find a basis $\vec{f} = \{f_{i=1,\ldots,8}\}$ that satisfies canonical differential equations [63,64]

$$\frac{d\vec{f}}{dx} = \epsilon\mathbb{A}(x)\vec{f} = \epsilon\left(\frac{A_0}{x} + \frac{A_1}{x^2-1}\right)\vec{f} \tag{15}$$

with matrices $A_i$ independent of $x$ and $\epsilon$. The solution easily obtained order by order in $\epsilon$ (see, e.g., [64])

$$\vec{f}(x,\epsilon) = \vec{f}(0,\epsilon) + \epsilon\int_0^x dx'\mathbb{A}(x')\vec{f}(0,\epsilon) + \cdots. \tag{16}$$

The boundary conditions are fixed by requiring the absence of singularities in the backward limit $x = \pm 1$ and expanding around the forward limit $x = 0$ [65].

*Results.*—Since the worldline operators in (7) furnish irreducible representations of the rotation group, it is natural to consider scattering amplitudes in the partial wave basis

$$i\mathcal{M}(\omega,\theta) = \frac{2\pi}{\omega}\sum_{\ell=0}^{\infty}(2\ell+1)(\eta_\ell e^{2i\delta_\ell} - 1)P_\ell(\cos\theta), \tag{17}$$

where $\theta$ is the scattering angle and $P_\ell$ are Legendre polynomials; see Supplemental Material [66] for a derivation of our partial wave expansion. The partial wave coefficients are parametrized in terms of real scattering phase shifts $\delta_\ell$ and inelasticity parameters $\eta_\ell$ (equivalently, $\Delta\eta_\ell \equiv 1 - \eta_\ell$). In our basis, an operator with a multipole number $\ell$ contributes only to the $\ell$th wave.

Using $\lambda \equiv 2Gm\omega$, our final 3PM EFT phase shift for the $\ell$ wave can be written as

$$\delta_\ell\Big|_{\mathrm{EFT}} = -\frac{\lambda}{2\epsilon_{\mathrm{IR}}} + \frac{\lambda}{2}\ln\left(\frac{4\omega^2}{\bar{\mu}_{\mathrm{IR}}^2}\right) + \sum_{n=1}^{3}\nu_n^\ell \lambda^n + \delta_\ell^{G^3},$$
$$\Delta\eta_\ell\Big|_{\mathrm{EFT}} = \frac{\ell!\omega^{2\ell+1}\mathrm{Im}F_\ell(\omega)}{2\pi(2\ell+1)!!}\left(1 + \pi\lambda + \lambda^2\eta_\ell^{G^2}\right), \tag{18}$$

where $\nu_n^\ell$ are $\mathcal{O}(1)$ numerical constants, e.g.,

$$\nu_2^\ell = \frac{-11 + 15\ell(1+\ell)}{4(-1+2\ell)(1+2\ell)(3+2\ell)}\pi, \tag{19}$$

and the rest are given in Supplemental Material. $\bar{\mu}_{\mathrm{IR}}$ is the IR matching scale. $\delta_\ell^{G^3}$ and $\eta_\ell^{G^2}$ contain UV divergences. Elastic terms $\delta_\ell^{G^3}$ are nonzero only for $\ell = 0$, 1:

$$\delta_0^{G^3}\Big|_{\mathrm{EFT}} = \lambda^3\left[\frac{1}{4\epsilon_{\mathrm{UV}}} + \frac{49}{24} - \frac{1}{2}\ln\left(\frac{4\omega^2}{\bar{\mu}^2}\right)\right] + \frac{C_{0,\omega^2}\omega^3}{4\pi},$$
$$\delta_1^{G^3}\Big|_{\mathrm{EFT}} = \frac{C_{1,\omega^0}\omega^3}{12\pi}\left(1 + \pi\lambda + \lambda^2\eta_1^{G^2}\right) + \frac{C_{1,\omega^2}\omega^5}{12\pi}, \tag{20}$$

where $\bar{\mu}$ is the matching scale in the minimal subtraction ($\overline{\mathrm{MS}}$) scheme. The expression for $\eta_\ell^{G^3}$ is given in the Supplemental Material. For $\ell = 0$, 1 we have

$$\eta_0^{G^2}\Big|_{\mathrm{EFT}} = \frac{67}{12} - \frac{11}{6}\left[-\frac{1}{2\epsilon_{\mathrm{UV}}} + \ln\left(\frac{4\omega^2}{\bar{\mu}^2}\right)\right] + \frac{\pi^2}{3}$$
$$\eta_1^{G^2}\Big|_{\mathrm{EFT}} = \frac{413}{100} - \frac{19}{30}\left[-\frac{1}{2\epsilon_{\mathrm{UV}}} + \ln\left(\frac{4\omega^2}{\bar{\mu}^2}\right)\right] + \frac{\pi^2}{3}. \tag{21}$$

The IR divergence and the $\bar{\mu}_{\mathrm{IR}}$ dependence in the first two terms of Eq. (18) are unobservable [70,71] because they appear multiplicatively in the $S$ matrix and hence do no affect the physical cross section. The third term is a sum of the finite Feynman diagrams. The last term in the first line of Eq. (18) contains the UV singularity in the single insertion of the background metric at $O(G^3)$ and the relevant tree-level worldline counterterms (Love numbers), displayed in Eq. (20). Since there are no divergences in the $P$ wave at $\mathcal{O}(G^3)$, $C_{1,\omega^0}$ is just a constant. In contrast, $C_{0,\omega^2}(\bar{\mu})$ is a running coupling, which we use to renormalize the $S$-wave divergence in the $\overline{\mathrm{MS}}$ scheme. Its $\beta$ function is given by

$$\frac{dC_{0,\omega^2}(\bar{\mu})}{d\ln\bar{\mu}} = -4\pi(2Gm)^3. \tag{22}$$

It is convenient now to absorb all local counterterms into the real part of internal multipole moments defined in Eq. (3). Then we can write down a unified expression for the two-loop beta function of all scalar tidal operators:

$$\frac{dF_\ell(\omega;\bar{\mu})}{d\ln\bar{\mu}} = -(2Gm\omega)^2\left[\frac{4\nu_2^\ell}{\pi}F_\ell(\omega;\bar{\mu}) + 8\pi Gm\delta_{[0\ell]}\right], \tag{23}$$

where $\delta_{[\ell\ell']}$ is the Kronecker delta. The first term in the rhs above describes the running of self-induced tidal effects, both conservative and dissipative. The EFT elegantly explains this homogeneity: both effects stem from the correlators $\langle QQ\rangle$ that pick up the same running from gravitational two-loop diagrams attached to them. Interestingly, the two-loop beta function is proportional to the one-loop (2PM) phase shift $\nu_2^\ell$. This can be explained by the fact that unitarity fixes the coefficient of the $\ln\omega^2$ in the dressed correlator in terms of the lower-PM amplitude. In contrast, the rightmost term above is a *universal* conservative contribution that arises from the PM expansion. The EFT dictates that this part does not depend on the nature of the compact object.

*Matching to black holes.*—Let us compare our EFT phase shift (18) with the analytic expression known from BHPT in GR [38–44,46]. Truncating this expression at 3PM and introducing the Schwarzschild radius $r_s = 2Gm$ as the only scale of static black holes, we find

$$\delta_\ell\Big|_{\mathrm{GR}} = (r_s\omega)\ln(2\omega r_s) + \sum_{n=1}^{3}\nu_n^\ell(r_s\omega)^n + \delta_\ell^{G^3} \tag{24}$$

where $\delta_\ell^{G^3}\Big|_{\mathrm{GR}} = 0$ for $\ell > 0$ and

$$\delta_0^{G^3}\Big|_{\mathrm{GR}} = (r_s\omega)^3\left[\frac{7}{12} - \gamma_E - \ln(2r_s\omega)\right], \tag{25}$$

while the 3PM inelasticity parameters are given by

$$\Delta\eta_\ell\Big|_{\mathrm{GR}} = \frac{2^{2\ell+1}(\ell!)^4(r_s\omega)^{2\ell+2}}{[(2\ell)!(2\ell+1)!]^2(2\ell+1)}\left(1 + \pi\lambda + \lambda^2\eta_\ell^{G^2}\right), \tag{26}$$

with

$$\eta_0^{G^2}\Big|_{\rm GR} = -\frac{11}{6}\ln\left(4r_s^2\omega^2 e^{2\gamma_E}\right) + \frac{2\pi^2}{3} + \frac{191}{36},$$
$$\eta_1^{G^2}\Big|_{\rm GR} = -\frac{19}{30}\ln\left(4r_s^2\omega^2 e^{2\gamma_E}\right) + \frac{2\pi^2}{3} + \frac{6853}{900}, \quad (27)$$

and the rest are given in Supplemental Material.

As a first check of our calculation, we verify that infrared divergences in the EFT match those in the full theory by choosing $\bar{\mu}_{\rm IR} = 1/r_s$. A second important observation is that the coefficients in front of the UV logs in the EFT expression (20) match those in GR (25), (27), as expected by consistency of the EFT. Matching the $P$-wave ($\ell = 1$) phase shift we obtain the vanishing of the scalar dipole static Love number, $C_{1,\omega^0} = 0$, consistent with previous results [22–24,28]. This is the first rigorous on-shell proof of the vanishing of Love numbers. The contribution of the dipolar dynamical Love number $C_{1,\omega^2}$ in Eq. (20) shifts to 5PM for black holes, which is beyond the scope of our work.

Matching the $S$ wave, we extract the monopole dynamical Love number

$$C_{0,\omega^2}(\bar{\mu})^{\overline{\rm MS}} = -4\pi r_s^3\left[\frac{1}{4\epsilon_{\rm UV}} + \ln(\bar{\mu}r_s) + \frac{35}{24} + \gamma_E\right] \quad (28)$$

obtained in the conventional dimensional regularization $+\overline{\rm MS}$ scheme. This is one of our main results. Note it is broadly consistent with the numerical estimate from [30].

Finally, matching $\eta_\ell$ we get the renormalized ${\rm Im}F_\ell$, e.g., for the $S$ wave we have

$$\begin{aligned}{\rm Im}F_0(\omega;\bar{\mu})^{\overline{\rm MS}} = 4\pi r_s^2|\omega|\Bigg\{1 + (r_s\omega)^2\Bigg[\frac{\pi^2}{3} - \frac{5}{18} \\ - \frac{11}{3}\left(\ln(\bar{\mu}r_s) + \gamma_E\right)\Bigg]\Bigg\}. \quad (29)\end{aligned}$$

Concluding, we note that comparison with GR demonstrates the utility of the EFT. Although the full GR phase shift is known, the physical interpretation of individual terms in it, especially logarithms, is difficult. In contrast, the EFT clearly classifies all logs into IR and UV ones, and also distinguishes universal and self-induced tidal effects. Finally, the EFT nicely explains the apparent conspiracy between coefficients in front of dissipative logs and conservative phase shifts at lower loop orders.

*Generalizations.*—Our method can be used to study tides in higher spacetime dimension $D$. For general $D$, it is trivial to match finite-size couplings because tidal effects do not scale as integer powers of $G$ [22,23,28,29]. UV divergences and nontrivial matching conditions arise if $2\ell/(D-3)$ is integer. In particular, in $D = 5$ we find divergences for both $S$ and $P$ waves,

$$\delta_\ell\Big|_{\rm EFT}^{D=5} \supset -\frac{(Gm\omega^2)^2}{72\pi}(64\delta_{[\ell 0]} + \delta_{[\ell 1]})\ln\left(\frac{\omega}{\bar{\mu}}\right). \quad (30)$$

Their renormalization requires the following universal running of the worldline couplings:

$$\frac{dC_{1,\omega^0}}{d\ln\bar{\mu}} = -\frac{8}{9}(Gm)^2, \qquad \frac{dC_{0,\omega^2}}{d\ln\bar{\mu}} = -\frac{128}{9}(Gm)^2. \quad (31)$$

The beta function for $C_{1,\omega^0}$ matches the known results from GR [22,23,28]. The running part of the dynamical love number $C_{0,\omega^2}$ is obtained for the first time. Since full BHPT results are not readily available in the literature for $D = 5$, we leave a complete matching for future work.

*Conclusions.*—We have presented a new systematic framework to match tidal responses of compact objects from probability amplitudes of massless waves to scatter off these objects. Our method is free of gauge dependence and field-redefinition ambiguities that plague the standard off-shell matching techniques commonly used to extract tidal effects (Love numbers). We illustrated the power of our approach by calculating a full 3PM amplitude for a scalar field to quasielastically scatter off a generic compact object. Our technique leads to rich implications for black holes, for which analytic GR results are available for comparison. In particular, we clarified the IR and UV origin of different terms in the GR expressions. Overall, our findings presented here give new insights into the form of gravitational scattering amplitudes, and serve as a prototype for the upcoming spin-2 Raman scattering calculations.

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