


Gottesman-Kitaev-Preskill State Preparation Using Periodic Driving

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The Gottesman-Kitaev-Preskill (GKP) code may be used to overcome noise in continuous variable quantum systems. However, preparing GKP states remains experimentally challenging. We propose a method for preparing GKP states by engineering a time-periodic Hamiltonian whose Floquet states are GKP states. This Hamiltonian may be realized in a superconducting circuit comprising a SQUID shunted by a superinductor and a capacitor, with a characteristic impedance twice the resistance quantum. The GKP Floquet states can be prepared by adiabatically tuning the frequency of the external magnetic flux drive. We predict that highly squeezed >11.9 dB (10.8 dB) GKP magic states can be prepared on a microsecond timescale, given a quality factor of 10^6 (10^5) and flux noise at typical rates.

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The Gottesman-Kitaev-Preskill (GKP) code encodes discrete variable quantum information into continuous variable quantum systems, and may be used to protect the encoded information from noise [1–4]. GKP states have recently been prepared and stabilized experimentally in trapped ion [5,6] and circuit quantum electrodynamics (QED) architectures [7–9], using an ancillary qubit to perform phase estimation of the GKP code stabilizers [4,10–12]. However, preparing high-quality GKP states remains a central challenge for practical quantum error correction with GKP codes.

An alternative approach has been proposed theoretically, where GKP states arise naturally as eigenstates of superconducting circuits with a carefully engineered phase-charge symmetry [13–15]. This passive approach is appealing, but the proposals rely on exotic circuit elements that may be hard to realize in practice. In Ref. [16], it was recently shown that circuit complexity can be traded for time-dependent control, by demonstrating how GKP states can arise as Floquet states of a periodically displaced harmonic oscillator (HO). Here, the price to pay is instead precise, high-frequency control, as the scheme relies on periodic, near-instantaneous displacements of the HO state (see the Supplemental Material [17] for a detailed comparison to this work).

Building on the idea of trading hardware for control complexity, we propose an ancilla-free method based on modulating a nonlinear element. This scheme may be realized, for example, with a SQUID shunted by a superinductor and a capacitor. We show that with this nonlinear control mechanism, a small number of harmonics of the circuit frequency suffices for the control signal. Numerical simulations indicate that GKP states with large squeezing levels >11.9 dB (10.8 dB) can be realized even in the

presence of flux noise at realistic rates and photon loss assuming a quality factor of 10^6 (10^5).

In this Letter we focus on the single-mode square GKP code, which has commuting stabilizer generators $\hat{D}(\sqrt{2\pi})$ and $\hat{D}(\sqrt{2\pi}i)$, where $\hat{D}(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$ is the displacement operator for a bosonic mode with annihilation operator \hat{a} , satisfying $[\hat{a}, \hat{a}^\dagger] = 1$. Equivalently, the codespace may be defined as the twofold degenerate ground space of the GKP Hamiltonian [1]

$$\hat{H}_{\text{GKP}}/\hbar = -J(\cos(2\sqrt{\pi}\hat{x}) + \cos(2\sqrt{\pi}\hat{p})), \quad (1)$$

where $\hbar J$ defines the energy scale and $\hat{x} = (\hat{a}^\dagger + \hat{a})/\sqrt{2}$ and $\hat{p} = i(\hat{a}^\dagger - \hat{a})/\sqrt{2}$ are the dimensionless position and momentum quadratures. Note that the GKP Hamiltonian is a special case of the Harper model [34,35], which is of importance in studies of quantum chaos and topological insulators [36–39].

As the eigenstates of the GKP Hamiltonian are not normalizable, they cannot be prepared exactly. Instead one must resort to states that approximate ideal GKP states [1]. To characterize how close a state is to an ideal GKP state, we use the GKP squeezing parameter [40]

$$\Delta = \sqrt{-\frac{1}{2\pi} \log\left(|\langle \hat{D}(\sqrt{2\pi}) \rangle|^2\right)}, \quad (2)$$

which we convert to decibels (dB) via $S = -10\log_{10}(\Delta^2)$, and refer to as squeezing for short. Alternatively, the squeezing may be measured with respect to the orthogonal stabilizer generator $\hat{D}(i\sqrt{2\pi})$, but for symmetrically

squeezed states these two values are equal. Note that Δ approaches zero ($\mathcal{S} \rightarrow \infty$) as $|\langle \hat{D}(\sqrt{2\pi}) \rangle|$ approaches one and the state approaches the GKP codespace. Numerical studies suggest that squeezing in the range 10–12 dB is required for scalable error correction [41–44].

While the squeezing quantifies how close an arbitrary state is to the GKP codespace, it does not give any information about the *logical* information contained in the state. To quantify this, we use the subsystem decomposition introduced in Ref. [45]. Under this decomposition, a partial trace operation amounts to an ideal decoder \mathcal{D} that takes density matrices in the bosonic Hilbert space to logical 2×2 density matrices. This can be used to introduce a *logical* fidelity metric, by computing the fidelity of the decoded state to a target logical state.

In the following, we introduce an approximate form of the GKP Hamiltonian, arising as an effective stroboscopic Hamiltonian in a periodically driven system. At its core, our scheme substitutes the requirement of periodicity in two conjugate variables [cf. Eq. (1)], with periodicity in one variable *and time*. Our approach utilizes similar techniques to Floquet engineering topological phases via driving otherwise trivial and simple systems [46,47], and extends these tools to the domain of quantum error correction.

The kicked HO.—Consider the time-periodic Hamiltonian

$$\hat{H}(t)/\hbar = \omega_0 \hat{a}^\dagger \hat{a} - J f(t) \cos(2\sqrt{\pi} \hat{x}), \quad (3)$$

with the periodic driving function $f(t) = (T/2) \times \sum_{n=0}^{\infty} \delta(t - nT/4)$, where $\delta(t)$ denotes the Dirac delta function, ω_0 is the HO frequency, and T is the driving period [48]. This “kicked HO” model, which represents a freely evolving HO interspersed with four delta-function kicks within one driving period, was previously studied for exploring many-body topological phases [49]. The one-period time-evolution operator (referred to as the Floquet operator) for this model can be written as $\hat{U}_T = (e^{-i\omega_0 \hat{a}^\dagger \hat{a} T/4} e^{-iJ \cos(2\sqrt{\pi} \hat{x}) T/2})^4$.

If the HO frequency is made equal to the driving frequency ($\omega_0 = 2\pi/T$), then via the formula $e^{i\theta \hat{a}^\dagger \hat{a}} \hat{x} e^{-i\theta \hat{a}^\dagger \hat{a}} = \hat{x} \cos \theta + \hat{p} \sin \theta$, we may rotate half of the $\cos(2\sqrt{\pi} \hat{x})$ terms to $\cos(2\sqrt{\pi} \hat{p})$ terms. In this special case, the one-period time evolution becomes equivalent to a kicked Harper model [50,51]. Furthermore, since $\cos(2\sqrt{\pi} \hat{x})$ commutes with $\cos(2\sqrt{\pi} \hat{p})$, the Floquet operator may be written as $\hat{U}_T = e^{-iT \hat{H}_{\text{GKP}}/\hbar}$. That is, time evolution under the time-dependent Hamiltonian for a single period is equivalent to evolving under the static GKP Hamiltonian in Eq. (1) for the same amount of time. This means that the eigenstates of the Floquet operator, referred to as Floquet states, are the ideal GKP states.

Harmonic driving scheme.—Exactly realizing the delta-function drive is impractical. In the following we introduce a new periodically driven model with a harmonic driving

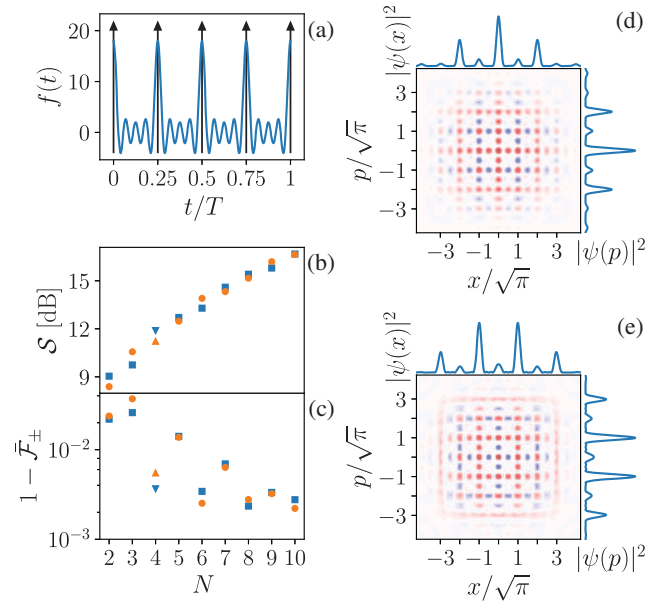


FIG. 1. The harmonic driving scheme. (a) The harmonic driving function with $N = 4$ harmonics (blue), and in the $N \rightarrow \infty$ limit (black). Arrows represent Dirac delta functions. Squeezing (b) and logical fidelity (c) of the $|\psi_{\pm}^{(N)}\rangle$ Floquet states as a function of the number of harmonics N . Blue squares (orange circles) denote values for the $|\psi_{+}^{(N)}\rangle$ ($|\psi_{-}^{(N)}\rangle$) state. (d) [(e)] Wigner function and marginal probability distributions for $|\psi_{+}^{(4)}\rangle$ ($|\psi_{-}^{(4)}\rangle$). Red (blue) represents positive (negative) values of the Wigner function. The blue downward (orange upward) pointing triangles in (b) and (c) correspond to the parameter values for the Floquet state in (d) [(e)]. All results are for $J/\omega_0 = 2.5 \times 10^{-3}$.

scheme, motivated by the kicked HO model. We start by truncating the Fourier series decomposition of the delta-function drive at a finite number of harmonics N [Fig. 1(a)], to obtain

$$f(t) = 2 + 4 \sum_{n=1}^N \cos(4n\omega_0 t). \quad (4)$$

It may then be shown that for fixed N , the Floquet operator takes the form $\hat{U}_T = e^{-iT \hat{H}_F/\hbar}$, where

$$\hat{H}_F/\hbar = \hat{H}_{\text{GKP}}^{(N)}/\hbar + O(J^2/\omega_0), \quad (5)$$

and $\hat{H}_{\text{GKP}}^{(N)}$ is the GKP Hamiltonian [Eq. (1)] truncated at the $\pm 4N$ diagonals in the Fock basis such that $\hat{H}_{\text{GKP}} = \lim_{N \rightarrow \infty} \hat{H}_{\text{GKP}}^{(N)}$. Thus, to zeroth order in J^2/ω_0 , the effective static Hamiltonian converges to the GKP Hamiltonian in the $N \rightarrow \infty$ limit. The details of this calculation are given in the Supplemental Material [17].

The truncation in Fock space leads to two nearly degenerate approximate GKP ground states whose squeezing increases with the number of harmonics. Furthermore,

the truncated GKP Hamiltonian has Fourier transform symmetry for all N , that is $[\hat{H}_{\text{GKP}}^{(N)}, e^{i\pi\hat{a}^\dagger\hat{a}/2}] = 0$, meaning that its eigenstates are invariant under a Fourier transform. Since the Fourier transform operator acts as the Hadamard gate on a square GKP code [1], the two nearly degenerate ground states are approximate GKP magic states [52]. Furthermore, this symmetry ensures that these states are equally squeezed in both quadratures, justifying our choice of metric in Eq. (2).

Consistent with the correction term in Eq. (5), for small J/ω_0 , the Floquet states inherit the properties of the eigenstates of $\hat{H}_{\text{GKP}}^{(N)}$ (in the Supplemental Material [17] we quantify this). To quantify the logical information of the Floquet states, we compute their logical fidelity with the Hadamard eigenstates, $\bar{\mathcal{F}}_{\pm} = \max_i \langle H_{\pm} | \mathcal{D} [|\psi_i^{(N)}\rangle \langle \psi_i^{(N)}|] | H_{\pm} \rangle$, where $|H_{\pm}\rangle$ are the ± 1 eigenstates of the 2×2 Hadamard gate, $|\psi_i^{(N)}\rangle$ are the Floquet states for an N -harmonic driving scheme, and we denote by $|\psi_{\pm}^{(N)}\rangle$ the Floquet states that achieve this maximum fidelity (the Supplemental Material [17] contains details on how we numerically implement the decoder \mathcal{D}). Given that the $|H_{\pm}\rangle$ states are magic states, we may also view this metric as quantifying the distillability of the Floquet states [52].

Numerical results for $J/\omega_0 = 2.5 \times 10^{-3}$ are shown in Figs. 1(b)–1(e). Here we find that the squeezing of the $|\psi_{\pm}^{(N)}\rangle$ Floquet states increases monotonically with N [Fig. 1(b)], and that the logical fidelity generally increases with N , albeit not monotonically [Fig. 1(c)]. In particular, the $|\psi_{+}^{(4)}\rangle$ ($|\psi_{-}^{(4)}\rangle$) state, whose Wigner function and marginal probability distributions are shown in Fig. 1(d) [Fig. 1(e)], has logical infidelity 3.7×10^{-3} (5.5×10^{-3}) and squeezing 11.9 dB (11.2 dB). This suggests that $N = 4$ harmonics is sufficient to generate high-quality GKP states. In the next section we discuss how these parameter may be attained in a circuit QED realization.

Circuit QED implementation.—While the driving scheme could be implemented in a variety of quantum computing architectures—in particular cold atoms may be an intriguing platform [49]—here we focus on a proposal using superconducting circuits [53]. We reserve the derivation of the circuit Hamiltonian for the Supplemental Material [17] and outline the main physical requirements here.

Equation (3) can be realized as a symmetric SQUID loop shunted by a linear inductor and a capacitor [Fig. 2(a)] with circuit frequency $1/\sqrt{LC_{\Sigma}} = \omega_0$ and circuit impedance $\sqrt{L/C_{\Sigma}} = 2R_Q$, where $C_{\Sigma} = 2C_J + C$ is the total capacitance in the circuit, and $R_Q = h/(2e)^2$ is the resistance quantum. We note that the same impedance condition was necessary to realize the schemes in Refs. [15,16]. Although an impedance surpassing the resistance quantum is difficult to realize with a conventional LC oscillator, impedances larger than $2R_Q$ have been obtained using chains of Josephson junctions [54–58], thin-film disordered

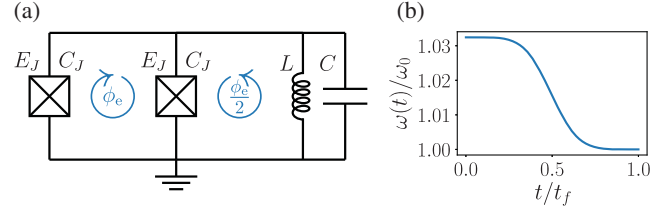


FIG. 2. (a) Circuit diagram for the harmonic driving scheme. The circuit comprises a symmetric SQUID loop with Josephson energies E_J and capacitances C_J shunted by a linear inductor with inductance L and capacitor with capacitance C . The two loops in the circuit are threaded by external fluxes ϕ_e and $\phi_e/2$ in opposite directions (b) Frequency ramp for the external flux drive to prepare GKP states with a preparation time t_f . The driving is switched off at $t = t_f$ once the state is prepared (see discussion and outlook section).

superconductor nanowires [59], and suspended aluminium coils [60]. As for the robustness to deviations from $2R_Q$, we observe a decrease of no more than 0.7 dB (4×10^{-3}) in the squeezing (logical fidelity) of the GKP Floquet states for variations of $\pm 0.1R_Q$ (see the Supplemental Material [17] for more details).

In order to implement the driving function, the external flux threading the SQUID loop is varied as

$$\phi_e(t)/\phi_0 = \pi - \epsilon f(t), \quad (6)$$

with $|\epsilon f(t)| \ll 1$ and $\epsilon = \hbar J/E_J$, where E_J is the Josephson energy of each Josephson junction and $\phi_0 = \hbar/2e$ is the reduced flux quantum. It is also necessary to thread the inductive loop of the circuit with an external flux in the opposite direction and with half the amplitude as Eq. (6) to ensure the HO potential remains static [17,61–63]. Here we have assumed the Josephson energies to be equal for simplicity, but we find the squeezing (logical fidelity) of the GKP Floquet states to decrease by less than 0.7 dB (4×10^{-3}) for asymmetries of $\pm 0.05E_J$ (see the Supplemental Material [17] for more details).

As a concrete example, consider a HO frequency $\omega_0/2\pi = 1$ GHz and Josephson energy $E_J/h = 2$ GHz with $\epsilon = 1.25 \times 10^{-3}$, which yields a ratio of $J/\omega_0 = 2.5 \times 10^{-3}$. For modest N , this is a very small modulation of the overall applied field, where $\phi_e(t)$ varies by no more than $0.01N\phi_0$ [Eq. (6)], and satisfies $|\epsilon f(t)| \ll 1$. Since the external flux must be modulated according to Eq. (4), the maximum modulation frequency is $4N\omega_0$. Therefore, the limiting factor for generating highly squeezed GKP states is the maximum modulation frequency that can be achieved in an experiment. For an $\omega_0/2\pi = 1$ GHz HO frequency, $N = 4$ harmonics implies a maximum modulation frequency of 16 GHz.

State preparation.—We now explain how the GKP Floquet states can be prepared. When the driving frequency is detuned from the HO frequency, the Floquet operator

becomes approximately diagonal in the number basis, with nearly HO eigenstates as its Floquet states (see the Supplemental Material [17] for the derivation). By adiabatically tuning the driving frequency into resonance with the HO frequency, the low-energy HO eigenstates are adiabatically evolved into the GKP Floquet states. Therefore, we replace ω_0 in Eq. (4) with a tunable driving frequency $\omega(t)$. To maximize adiabaticity, the initial driving frequency is chosen to be incommensurate with the driving frequency $\omega(0) = \omega_0/(1 - \pi \times 10^{-2})$, and is tuned into resonance using a sigmoid-function ramp with a numerically optimized slope [Fig. 2(b)]. In the Supplemental Material [17] we give the precise equation of the ramp as well as a more detailed explanation for why an incommensurate detuning is chosen.

Using this protocol, we numerically observe that the $|\psi_+^{(N)}\rangle$ ($|\psi_-^{(N)}\rangle$) Floquet state can be prepared from the $|0\rangle$ ($|2\rangle$) eigenstate of the HO. Note that the $|0\rangle$ and $|\psi_+^{(N)}\rangle$ ($|2\rangle$ and $|\psi_-^{(N)}\rangle$) states both have eigenvalue $+1$ (-1) for the Fourier transform operator since the adiabatic process preserves rotation symmetry. Furthermore, an arbitrary superposition $\alpha|0\rangle + \beta|2\rangle$ evolves into $\alpha|\psi_+^{(N)}\rangle + e^{i\phi}\beta|\psi_-^{(N)}\rangle$, where ϕ is a phase factor that accounts for the dynamical phase difference acquired by the two Floquet states throughout the adiabatic evolution, thus allowing an arbitrary logical state to be prepared. In the following we focus on preparation of the $|\psi_+^{(4)}\rangle$ Floquet state, and refer the reader to the Supplemental Material [17] for results on preparation of other states.

For the remaining numerical results, we set $N = 4$ and $J/\omega_0 = 2.5 \times 10^{-3}$. To assess the quality of the prepared state, $\hat{\rho}(t_f)$, we compute its squeezing and logical infidelity $\{\bar{\mathcal{F}}_+^{\text{prep}} = \langle H_+ | \mathcal{D}[\hat{\rho}(t_f)] | H_+ \rangle\}$ for different preparation times t_f (Fig. 3). We observe both the squeezing and logical infidelity to increase over the range of preparation times of $1000 \lesssim t_f/T \lesssim 2000$ (blue solid lines in Fig. 3), after which they plateau and converge to the values for $|\psi_+^{(4)}\rangle$ (dotted black lines in Fig. 3), consistent with the adiabatic theorem (in the Supplemental Material [17] we also plot the direct fidelity between the prepared state and the Floquet state). Interestingly, both the squeezing and logical infidelity of the prepared state reach higher values than for the true Floquet state $|\psi_+^{(4)}\rangle$ before converging to their values, and suggest an optimal preparation time of $t_f/T \approx 2000$.

Noisy state preparation.—In a closed environment, the adiabatic theorem dictates that the Floquet states may be prepared from the HO eigenstates with perfect fidelity by increasing the preparation time indefinitely. However, in the presence of decoherence processes, a longer preparation time may degrade the quality of the prepared state. Here we consider two decoherence processes: photon loss and dephasing due to flux noise.

To model photon loss, we consider the system Hamiltonian in Eq. (3) coupled to a zero-temperature bath,

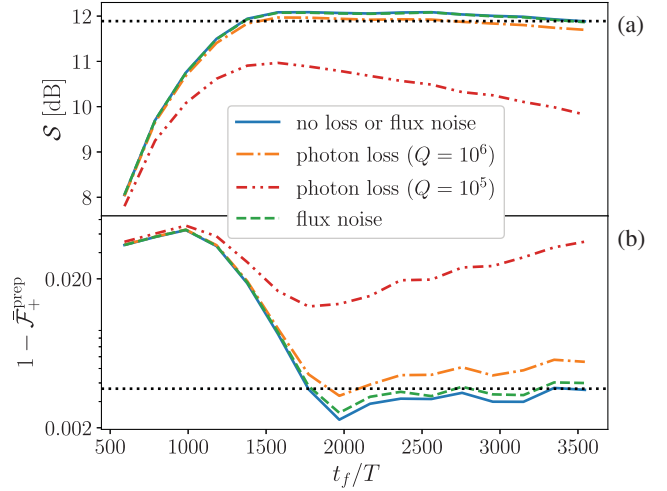


FIG. 3. Quality of prepared GKP states. Squeezing (a) and logical infidelity (b) of the prepared GKP state as a function of preparation time. Blue solid lines denote preparation with no photon loss and no flux noise, orange (red) (dot)-dot-dashed lines represent the preparation with photon loss at a quality factor of $Q = \omega_0/\kappa = 10^6$ (10^5), and green dashed lines represent preparation with flux noise. The black dotted lines represent the values for the $|\psi_+^{(4)}\rangle$ Floquet state [blue downward triangles in Figs. 1(b) and 1(c)]. All results are for $J/\omega_0 = 2.5 \times 10^{-3}$ and $N = 4$.

described by the Lindblad master equation [64]

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar} [\hat{H}(t), \hat{\rho}(t)] + \kappa \left(\hat{a} \hat{\rho}(t) \hat{a}^\dagger - \frac{1}{2} \{ \hat{a}^\dagger \hat{a}, \hat{\rho}(t) \} \right), \quad (7)$$

where $\hat{\rho}(t)$ is the reduced density matrix for the system at time t and κ is the photon-loss rate. We simulate photon loss using Monte Carlo sampling [65] to generate the density matrix as an ensemble average of many quantum trajectories (see the Supplemental Material [17] for more details).

In Fig. 3 we consider quality factors of $Q = \omega_0/\kappa = 10^6$ (10^5), which correspond to T_1 times of $1/\kappa \approx 160 \mu\text{s}$ ($16 \mu\text{s}$) for an $\omega_0/2\pi = 1$ GHz HO. At a quality factor of $Q = 10^5$, our numerical results (red dot-dot-dashed lines) reveal an optimal preparation time region at $1600 \lesssim t_f/T \lesssim 1800$, with a maximum squeezing (minimum logical infidelity) of 11.0 dB (1.3×10^{-2}) achieved at the lower (upper) bound of this range. Increasing the quality factor to $Q = 10^6$ improves these values (orange dot-dot-dashed lines), with a maximum squeezing (minimum logical infidelity) of 12.0 dB (3.2×10^{-3}) attained at a preparation time of $t_f/T \approx 1600$ ($t_f/T \approx 2000$).

In addition to photon loss, state preparation may be affected by noise from the external flux, which is used to modulate $f(t)$ in Eq. (3). However, when considering combined $1/f$ and white noise at experimentally measured noise amplitudes [66–69] (green dashed lines in Fig. 3), we find flux noise to have negligible impact on the quality of the prepared state, with a decrease of less than 0.02 dB

(6×10^{-4}) in the squeezing (logical fidelity) relative to the lossless case over all preparation times $t_f/T \lesssim 3500$. Thus, at quality factors of $Q \leq 10^6$, photon loss will be the dominant source of decoherence for state preparation. In the Supplemental Material [17] we provide details on how this flux noise was modeled, as well as results at different flux noise strengths.

Overall, our results show that with flux noise and a quality factor of $Q = 10^6$ ($Q = 10^5$), a GKP state with squeezing >11.9 dB (10.8 dB) and logical infidelity $< 4 \times 10^{-3}$ (2×10^{-2}) may be prepared with a preparation time of $t_f/T \approx 1800$ (2000), which corresponds to 1.8 μ s (2.0 μ s) for an $\omega_0/2\pi = 1$ GHz HO.

Discussion and outlook.—In this Letter we introduced a time-periodic Hamiltonian that has GKP states as its Floquet states, and showed how these states may be prepared by adiabatic evolution of low-energy HO eigenstates. We showed that this scheme may be realized in a superconducting circuit with a SQUID shunted by a linear superinductor and a capacitor. Numerical analysis of the impact of photon loss and flux noise on the squeezing and logical fidelity of the prepared state revealed that high-quality GKP states may be prepared on a microsecond timescale with experimentally achievable quality factors and typical flux noise amplitudes [54–56,60,66–69]. This represents an efficient way to prepare GKP states without the need for an ancilla qubit. As our protocol naturally gives rise to GKP magic states, universality can be achieved using only Clifford gates, which are realized with Gaussian bosonic operations on GKP states [1,70].

A natural question is whether the periodic driving may also be used to stabilize the GKP state once it has been prepared, providing a form of autonomous error correction. Because of the Floquet heating phenomenon [71,72], we find that the periodic drive decreases the lifetime of the GKP Floquet states (see the Supplemental Material [17] for further details). Thus, once the states are prepared, the drive should be turned off, and active error correction applied to stabilize the GKP states. This can be achieved via coupling to an ancillary qubit, as has been done experimentally [7,9], or by interacting GKP modes, as has been studied in many theoretical works [4,42–44].

Our work also opens opportunities for realizing more general Harper models beyond the GKP Hamiltonian, using simple superconducting circuits. Besides the rich and fascinating physics of the Harper model and its topological features, an intriguing avenue for future work is to investigate this broader class of models in the light of quantum error correction, and the potential to realize new quantum error correction schemes via periodic driving.

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