Photonic Source of Heralded Greenberger-Horne-Zeilinger States

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Generating large multiphoton entangled states is of main interest due to enabling universal photonic quantum computing and all-optical quantum repeater nodes. These applications exploit measurementbased quantum computation using cluster states. Remarkably, it was shown that photonic cluster states of arbitrary size can be generated by using feasible heralded linear optics fusion gates that act on heralded three-photon Greenberger-Horne-Zeilinger (GHZ) states as the initial resource state. Thus, the capability of generating heralded GHZ states is of great importance for scaling up photonic quantum computing. Here, we experimentally demonstrate this required building block by reporting a polarisation-encoded heralded GHZ state of three photons, for which we build a high-rate six-photon source (547 \pm 2 Hz) from a solid-state quantum emitter and a stable polarization-based interferometer. The detection of three ancillary photons heralds the generation of three-photon GHZ states among the remaining particles with fidelities up to $\mathcal{F} = 0.7278 \pm 0.0106$. Our results initiate a path for scalable entangling operations using heralded linear-optics implementations.

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Introduction.-Quantum entanglement enables the exploration of unique phenomena absent in the classical world, such as nonlocality [1-3] and teleportation [4-6]; and it ultimately provides an advantage to quantum systems over classical ones for various tasks [7,8], ranging from metrology and sensing [9-13], to computation [14–17]. Photonic quantum systems are among the leading physical platforms for large-scale quantum computers [18]. A very promising architecture is based on the concept of measurement-based quantum computing (MBQC) that exploits entangled cluster states as resource for universal quantum computing. A key advantage of this scheme is the superior feasibility and error-correction thresholds [19–21] with respect to gate-based models. For generating photonic cluster states two main encoding schemes exist: analog or continuous variable [22,23]. and digitial or discrete-variables approaches [24,25]. With advantages and disadvantages from both sides, continuous-variable implementations are highly sensitive to losses, degrading the quality of the quantum state. In this regard, discrete-variable encoding constitutes an appealing alternative, as even in the absence of deterministic entangling gates, there exist loss-tolerant schemes for the generation of large entangled quantum states using only probabilistic, but heralded, linear-optics quantum gate operations [26,27].

Additional approaches exist that aim to generate discrete-variable cluster states [28] directly by exploiting quantum emitters [29–34]. However, this requires complex control of atomic structures, solid-state materials, and electro-magnetic fields, to name a few technological challenges. In contrast, linear optics alone also provides a path for the scalable generation of multiphoton cluster states universal for quantum computation [19,27,35]. Thereon, a ballistic (without feed-forward requirements) and loss-tolerant (where losses do not induce logical errors) model for universal quantum computing, named fusionbased quantum computation [36], exploits small resource states made up of a handful of entangled particles [20], and combines them into larger entangled states via boosted (heralded, and at the expense of ancillary photons) entangling gates called fusion operations [19].

The smallest building block in these protocols [27] is the heralded three-photon Greenberger-Horne-Zeilinger (GHZ) state [37,38]. Creating them requires the quantum interference of six separable single photons [26], or a minimum of ten photons from non-linear frequency conversion processes [39,40]. The efficient generation of the necessary input multiphoton states remained to date a major challenge. In this regard, semiconductor quantum dots have recently matured to a point where one handles the interference of single photons at scales of eight particles [41] and even beyond with lost-photons boosted protocols [42],

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FIG. 1. Single-photon quality. (a) Normalized second-order autocorrelation function $g^{(2)}(\Delta t)$ from a Hanbury Brown–Twiss setup, and (b) at the output of a Hong-Ou-Mandel experiment at π -pulse excitation. We measure the single-photon purity $1 - g^{(2)}(0) = 0.981 \pm 0.003$, and two-photon indistinguishability $\mathcal{I} = 0.941 \pm 0.002$.

thus now reaching the necessary scales for these more advanced experiments.

Here, we experimentally demonstrate a heralded threephoton polarization-encoded GHZ state based on the interference of six single photons. We employ a 28.7% fiber-efficient quantum dot single-photon source, actively demultiplexed to produce a source of six indistinguishable photons with 547 ± 2 Hz detected rates. The high quality of the source and interferometric apparatus enable producing heralded three-photon GHZ states at a detection rate of 0.914 ± 0.006 Hz, and fidelities up to $\mathcal{F} = 0.7278 \pm$ 0.0106. Our results mark an important step for enabling the realization of future fusion-based quantum computing protocols.

High-rate multiphoton source.—We first describe our source of multiphoton states. An InAs/GaAs quantum dot coupled to a micropillar cavity is kept in a cryostat at ~4 K, and is resonantly driven using a standard crossed-polarized excitation scheme. An efficient collection setup allows us to measure 19.5 MHz of single photons with simultaneous purity $1 - g^2(0) = 0.981 \pm 0.003$ and indistinguishability [43] $\mathcal{I} = 0.941 \pm 0.002$, see Fig. 1, when pumped with π pulses at 80 MHz repetition rate and using a detection system of 85% efficiency, thus corresponding to a 28.7% fiber-efficient single-photon source.

Subsequently, we utilize a time-to-space demultiplexer composed of resonant-enhanced electro-optic modulators (r-EOMs) and polarizing beam splitters (PBSs) arranged in



FIG. 2. Multiphoton source. (a) Resonant demultiplexer. Seven synchronized r-EOMs—one driven at 40 MHz, two at 20 MHz, and four at 10 MHz—and polarizing beam splitters, deterministically demultiplex eight consecutive time bins. Fiber-based delays and translation stages are used to correct the initial temporal mismatch of the time bins. (b) Measured coincidence rates. Multiphoton rates at the output of the demultiplexer, up to a number of n = 8 photons.

a tree structure [44,45], see Fig. 2(a). As a result, a number of input time bins separated from each other by 12.5 ns are deterministically routed towards, in this implementation, eight different outputs, which thereon follow suitable fiberbased temporal delays to result in a source of eight indistinguishable single photons traveling simultaneously. Figure 2(b) shows the measured multiphoton coincidence rates using eight superconducting nanowire single-photon detectors (SNSPDs) directly at the output of the demultiplexed source. In particular, the resulting six-photon source is detected at a rate of 547 ± 2 Hz, and the eight-photon source at 15.7 ± 0.4 Hz. We note that these are the highest rates of multiphoton sources reported to date.

Heralded entanglement.—We use six single photons from this source as input to a polarization-based interferometer, as depicted in Fig. 3(a), such that the detection of three photons heralds an entangled GHZ state among the other three [26], see Supplemental Material [46]. The six input photons are labeled $i_1, \ldots i_6$, and are first initialized in horizontal polarization. When one, and only one, photon propagates towards each of the heralding outputs o_4 , o_5 , and o_6 , then the signal output photons o_1 , o_2 , and o_3 are left



FIG. 3. Polarization six-photon interferometer. (a) Depiction of experimental setup. Three pairs of single photons $(i_1, i_6), (i_2, i_5), (i_3, i_4)$ probabilistically generate three bell pairs, together with other unwanted terms, after interfering on PBSs 1,2,3. Subsequently, a nonheralded six-photon entangled state, and further unwanted states, is probabilistically generated after PBSs 4,5. The polarization and number-resolved detection at the output of the type I and type II fusion operation among outputs o_4 , o_5 , and o_6 corrects for the unwanted terms and leaves the remaining photons at outputs o_1 , o_2 , and o_3 in a probabilistic but heralded three-photon GHZ state. Quarter-wave (QWP) and half-wave plates (HWP) together with extra PBSs are used to perform three-qubit quantum state tomography. (b) Photons' indistinguishability. We use the same six-photon interferometer to measure all 15 pairwise two-photon indistinguishabilities, resulting in an average value of 0.923 ± 0.009 across all combinations.

in a three-particle entangled state. Note that the successful implementation of this protocol requires that all six photons are highly indistinguishable from each other. To confirm that this is the case, we measure all 15 cases of pairwise indistinguishabilities among the six input photons, and find an average indistinguishability of 0.923 ± 0.009 across all combinations, see Fig. 3(b).

The heralded generation of the entangled quantum state requires that no more than one photon is detected at each heralding output. For example, without number resolution, a pattern with four photons among the three heralding spatial trajectories cannot be discerned from another pattern with an exact number of three photons. In such cases, the state produced at signal outputs is not solely the target GHZ state, but it also contains other components with a different number of photons. Therefore, only nonheralded (postselected) states are generated in the absence of numberresolving detection. Our implementation makes use of pseudo photon-number resolution (PPNR) at every heralding output, $o_{4,h}$, $o_{4,v}$, $o_{5,h}$, $o_{5,v}$, $o_{6,h}$, $o_{6,v}$, by further splitting each of them into two new outputs and SNSPDs; where h and v denote horizontal and vertical polarization, respectively. Therefore, we use 18 SNSPDs in total: six detectors to cover both polarizations of the three signal outputs, and twelve detectors for implementing the polarization and pseudo number-resolved measurement of the three heralding outputs. At each PPNR stage (six in total), we condition a measurement such that one of the detectors clicks and the other one does not, which performs the pseudo number resolution of one, and no more, photon. The heralded generation of the target state occurs then by imposing that each of the heralding stages measures at most one photon.

The specific 3-GHZ state generated depends on the polarization pattern that clicks at the heralding outputs. For instance, the patterns {*hhh*, *hvv*, *vhv*, *vvh*}, where *ijk* denotes the condition $o_{4,i}$, $o_{5,j}$, $o_{6,k}$, herald the signal state $|\text{GHZ}^+\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ with an accumulated success probability of 1/64, and $|0\rangle$ ($|1\rangle$) denoting horisontal (vertical) polarization. Conversely, the complementary patterns {*hhv*, *hvh*, *vhh*, *vvv*} herald the state $|\text{GHZ}^-\rangle = (|000\rangle - |111\rangle)/\sqrt{2}$ with an equal success probability. Accordingly, the total success probability of generating a three-GHZ state with this protocol is 1/32.

In our experiment, we start by measuring the witness $W_{GHZ} = 3\mathbb{I}/2 - X_1X_2X_3 - (Z_1Z_2 + Z_2Z_3 + Z_1Z_3)/2$, whose negative value verifies the presence of genuine three-particle entanglement for GHZ states [47]. Figure 4 displays the measured mean values of the involved observables, from where we obtain $\langle W_{GHZ} \rangle = -0.2613 \pm 0.0335$, confirming three-partite entanglement by more than seven standard deviations.

Moreover, given the high count rates of the available sixphoton source, we are also able to perform overcomplete three-qubit quantum state tomography at the signal outputs, with all heralding patterns simultaneously. That is, by using 18 SNSPDs, we reconstruct both $|\text{GHZ}^+\rangle$ and $|\text{GHZ}^-\rangle$ from the measurement of $3^3 = 27$ three-qubit observables that result from all combinations of *Z*, *X*, *Y* Pauli matrices



FIG. 4. Entanglement witness. Measured mean values of the observables forming W_{GHZ} . We obtain $\langle X_1 X_2 X_3 \rangle = 0.5629 \pm 0.0252$, $\langle Z_1 Z_2 \rangle = 0.7971 \pm 0.0166$, $\langle Z_2 Z_3 \rangle = 0.8151 \pm 0.0191$, and $\langle Z_1 Z_3 \rangle = 0.7846 \pm 0.0199$.

among the 3 signal qubits. Figure 5 shows the reconstructed density matrices of the heralded entangled states, from where we extract a fidelity of $\mathcal{F}^+ = 0.7189 \pm 0.0109$ to the $|\text{GHZ}^+\rangle$ state when using the corresponding four heralding conditions, as well as $\mathcal{F}^- = 0.6995 \pm 0.0116$ to $|\text{GHZ}^-\rangle$ by using the respective other four heralding patterns. Note that small terms are present in the imaginary part of the density matrices, showing that the prepared states contain a small relative phase between the state basis, which can be compensated for via local unitaries. Taking this into account, we obtain fidelities of $\bar{\mathcal{F}}^+ = 0.7278 \pm 0.0106$ and $\bar{\mathcal{F}}^- = 0.7083 \pm 0.0120$ to the GHZ states ($|000\rangle \pm e^{i(0.04 \times 2\pi)}|111\rangle$)/ $\sqrt{2}$, respectively.

We measure both heralded states at a combined rate of 0.914 ± 0.006 Hz. This value is expected considering: ~80% throughput efficiency of the polarization interferometer (affecting six photons), ~85% throughput efficiency of

the pseudo number-resolving detection setup (three photons), and ~85% throughput given by multiple fiber mating connections (six photons). Together with a 1/32 success probability of producing both GHZ states, results in an expected rate of $(547 \text{ Hz}) \times 0.8^6 \times 0.85^3 \times 0.85^6/32 \sim 1 \text{ Hz}.$

Discussion.-We have experimentally demonstrated a building block for ballistic and all linear-optical photonic quantum computing: the heralded three-photon Greenberger-Horne-Zeilinger state. First, we developed a high-rate (547 \pm 2 Hz) source of six photons from a solidstate quantum emitter, with an average pairwise indistinguishability of 0.923 ± 0.009 . Subsequently, these photons propagated through a polarization-based multimode interferometer, where the pseudo number-resolved detection of three of them heralded the generation of three-GHZ states among the remaining particles. Thanks to the high rate of the generated multiphoton source, we were able to perform three-qubit overcomplete quantum state tomography, reaching fidelities up to $\mathcal{F} = 0.7278 \pm 0.0106$. Moreover, the efficient multiphoton source presented here reached an eight-photon rate of 15.7 ± 0.4 Hz, readily enabling the implementation of more complex experiments at scales soon beyond ten photons.

Our results represent a significant advancement in the active field of photonic quantum computing. We anticipate that near-term future improvements will overcome limitations of our present experiment. An important issue will be the realization of photon detectors with true photonnumber resolution. Otherwise, the heralding events cannot guarantee the presence of GHZ states with unity probability, even if all other elements are perfect in principle. However, despite these constraints, our efforts to utilize photon number resolving detectors (PNRDs) remain highly



FIG. 5. Quantum state tomography of the heralded GHZ states. (a),(b) Real (left) and imaginary part (right) of the reconstructed density matrices at the signal outputs o_1 , o_2 , and o_3 , see Fig. 3(a), when the heralding patterns produce either the $|\text{GHZ}^+\rangle$ or $|\text{GHZ}^-\rangle$ state. We collect 27 observables for an overcomplete quantum state tomography. To obtain the mean value of a particular observable, we measure simultaneously $2^3 = 8$ polarization projections among the three signal outputs, times four heralding polarization combinations that produce the same target state, times $2^3 = 8$ PPNR configurations for number-resolving the three heralding outputs. That is, a total of $2^3 \times 4 \times 2^3 = 256$ cases of six-photon coincidence patterns collected simultaneously among 18 output SNSPDs reconstruct a single three-qubit observable of one heralded state. Accordingly, 512 six-photon coincidence patterns are measured simultaneously to obtain the same observable for both heralded states. Each observable is measured for 900 s, which produced density matrices with a total of 11 038 sixfold coincidence counts for the $|\text{GHZ}^+\rangle$ state, and 11 178 sixfolds for $|\text{GHZ}^-\rangle$. Accordingly, each heralded state is measured at a rate of 0.454 ± 0.004 and 0.460 ± 0.004 Hz, respectively, resulting in a total measured rate of 0.914 ± 0.006 Hz.

valuable. The utilization of pseudo photon-number resolution enhances the probability of generating heralded GHZ states, conditioned on the heralding event, by excluding undesired states. This represents a significant advancement beyond non-PNRDs, which are theoretically unsuitable for heralded GHZ protocols. Our endeavors undeniably push the boundaries of multiphoton entanglement generation in a heralded manner, setting a benchmark for future experiments in this research direction. We foresee that further experiments employing true PNRDs [48] may open up new horizons in this field.

Photonic cluster states have gained considerable attention owing to their remarkable properties and versatility in quantum science. Since the inception of one-way quantum computing [49], progress in cluster states has expanded significantly. These achievements range from basic cluster state preparation to blind delegated computation [50], as well as diverse applications beyond [51,52]. Particularly promising is the use of integrated chips to implement complex interferometers and programmable circuits for cluster state preparation, which has led to more practical demonstrations [53-56]. These advancements have primarily been accomplished via non-deterministic, postselected cluster states. The heralded generation of multiphoton entangled states not only offers a building block for scalable, ballistic universal quantum computation but also provides new insights into relevant applications and fundamental research. For example, the use of genuine multipartite sources, achieved through the combination of bipartite resources and postselection, for certifying multipartite nonlocality has been a notorious strategy in these experimental endeavors. Their validity has been questioned due to selection bias introduced by postselection [57]. However, the generation of multiphoton entangled states in an event-ready manner may pave the way to close the postselection loophole in future experiments [58,59]. While heralded entanglement generation has previously been realized only for bipartite states [60,61], our experiment marks a steady progression towards multipartite systems. This opens up a plethora of quantum photonics protocols that were previously experimentally inaccessible and removes the constraints of outcome postselection.

Note added.—Recently, we became aware of related works [62,63].

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