Squeezed Ensembles and Anomalous Dynamic Roughening in Interacting Integrable Chains

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(Received 6 April 2023; accepted 28 February 2024; published 25 March 2024)

It is widely accepted that local subsystems in isolated integrable quantum systems equilibrate to generalized Gibbs ensembles. Here, we identify a particular class of initial states in interacting integrable models that evade canonical generalized thermalization. Particularly, we demonstrate that in the easy-axis regime of the quantum XXZ chain, pure nonequilibrium initial states that lack magnetic fluctuations instead locally relax to squeezed generalized Gibbs ensembles governed by nonlocal equilibrium Hamiltonians, representing exotic equilibrium states with subextensive charge fluctuations that violate the self-affine scaling. At the isotropic point, we find exceptional behavior and explicit dependence on the initial state. Particularly, we find that relaxation from the Néel state is governed by extensive fluctuations and a superdiffusive dynamical exponent compatible with the Kardar-Parisi-Zhang universality. On the other hand, there are other nonfluctuating initial states that display diffusive scaling, e.g., a product state of spin singlets. Our predictions provide examples of anomalous quantum transport and fluctuations in strictly quantum states which can be directly tested in state-of-the-art cold atomic experimental settings.

DOI: 10.1103/PhysRevLett.132.130401

Introduction.—The study of nonequilibrium dynamical properties in isolated quantum many-body systems has been at the forefront of theoretical and experimental research in the past decade [1–25]. In particular, the study of thermalization is primarily concerned with the steady-state values of local observables following a quantum quench from a pure initial state. Such protocols have provided a versatile and fruitful tool for understanding the key mechanisms leading to thermalization and quantum scrambling. Despite the global state remaining pure at all times, any large subsystem typically evolves at late times towards an ensemble that maximizes entropy, subject to the constraints of (quasi)local conserved quantities. This paradigm has been examined in a rich variety of systems, including generic chaotic models and free or interacting integrable models. Concurrently, there have been important developments in understanding the eigenstate thermalization hypothesis [3,7,14,26] and its generalization to the integrable cases [23,27,28]. Local, but large, subsystems of size ℓ are said to thermalize whenever the reduced density matrix is described by a canonical Gibbs ensemble or, in the case of integrable models, the generalized Gibbs ensemble [6,13,29]. Such *canonical* ensembles possess by definition extensive and strictly positive fluctuations (static susceptibilities) of (quasi)local charges $\{Q_i\}$ within the subsystem of size ℓ, i.e., the covariance matrix $C_{ij} = \langle Q_i Q_j\rangle^c/\ell$ is positive definite in the $\ell \to \infty$ limit.

In this Letter, we revisit the problem of thermalization in integrable models. We specifically consider interacting quantum spin chains with a global $U(1)$ charge Q, e.g. magnetization in spin chains or electron charge in interacting fermions. We confine our study to quenches from initial states within a specific charge sector by considering superpositions of pure state with the same value of Q. In contrast with a widespread belief, we find that the reduced density matrix emerging at late times is *not* a faithful canonical GGE, and we dub such ensembles as *squeezed* GGEs (SGGEs). Akin to canonical ensembles, GGEs are generically characterized by charge fluctuations (covariances) that scale extensively with system size, yielding finite charge susceptibilities (see, for example, [30]). Squeezed GGEs instead possess subextensive charge fluctuations manifested by *divergent* U(1) chemical potentials and *vanishing charge susceptibility*. Similarly to zero-entropy ensembles, SGGEs can thus represent extremal GGEs with exceptional properties.

To substantiate our claims, we consider the anisotropic (XXZ) Heisenberg chain,

$$H = \sum_j [S^x_j S^x_{j+1} + S^y_j S^y_{j+1} + \Delta S^z_j S^z_{j+1}], \tag{1}$$

where S^α_j are the spin-1/2 generators and Δ is the interaction anisotropy. We shall mostly be interested in the $\Delta > 1$ regime, where we establish that initial states without (global) magnetic fluctuations locally equilibrate to states ρ_ℓ with static spin fluctuations scaling *subextensively* with the subsystem size. This shows that such ensembles cannot be captured by canonical GGEs generated by

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quasilocal effective equilibrium Hamiltonians, which is supported by the observation that the rescaled (Hilbert–Schmidt) norm of an effective Hamiltonian $\|\log \varrho_\ell\|/\ell$ *diverges* with ℓ (see Ref. [31]). Such anomalous behavior is no longer present in the gapless regime $\Delta < 1$, where the spectrum of quasiparticles comprises only finitely many magnon species, consistently with thermalization to canonical GGEs. Curiously, at the isotropic point $\Delta = 1$ we encounter a qualitatively different behavior. While there exist initial states, e.g., the antiferromagnetic Néel state, that comply with canonical GGE description, exhibiting finite magnetic susceptibility and superdiffusive spin transport, there are other nonfluctuating pure states (e.g., the product state of spin singlets) that reveal very distinct, unorthodox properties.

Given a pure initial state $|\Psi\rangle$, we probe magnetic fluctuations within finite sublattices Λ_ℓ of size ℓ, $Q_\ell = \sum_{j\in\Lambda_\ell} S^z_j$, and investigate the dynamical scaling properties of the local second moment

$$W^2(\ell, t) \equiv \langle\Psi(t)|Q_\ell^2|\Psi(t)\rangle. \tag{2}$$

Drawing an analogy with the interface roughness in stochastic models of interface growth, we can, in general, expect $W(\ell, t)$ to exhibit a (self-affine) Family-Vicsek (FV) scale-invariant law [32–37],

$$W(\ell, t) \sim \ell^\zeta \Phi(t/\ell^z), \tag{3}$$

with $\Phi(y) \sim y^\beta$ for $y \ll 1$ and $\Phi(y) \to 1$ for $y \gg 1$, roughness (Hurst) exponent ζ, and growth exponent $\beta = \zeta/z$. While in the case of, e.g., Néel state we confirm the above scaling, both at the isotropic point, with Kardar-Parisi-Zhang (KPZ) exponents $\zeta = 1/2$, $z = 3/2$ [38], and for $\Delta < 1$ (with ballistic exponents $\zeta = 1/2$, $z = 1$), we observe violation in the diffusive regime ($z = 2$), where $W^2(\ell, t)$ scales subextensively with ℓ, with an estimated (fitted) exponent is $\zeta \approx 0.22 < 1/2$ at $\Delta = 3$, see Fig. 1. While at present we have no theory to predict the values of roughness exponents $\zeta \leq 1/2$, we have verified that they are dependent on anisotropy and, possibly, also on the type of initial state, see additional plots in [31].

GGEs for interacting integrable systems.—In the scope of the standard quantum quench protocol, we consider integrable interacting quantum spin chains with an internal (charge) degree of freedom. For simplicity, we assume the system possesses a single U(1) charge (i.e., no nesting) and consider only a class of product pure initial states $|\Psi\rangle$ of the form $|\Psi\rangle = |\psi\rangle^{\otimes n}$, with system length L and $n = L/b \in \mathbb{N}$, where $|\psi\rangle$ is a "block state" involving b adjacent lattice sites. In the thermodynamic limit, the main object of interest is the reduced density matrix on a sublattice Λ_ℓ of size ℓ, $\varrho_\ell(t) = \lim_{L\to\infty} \mathrm{Tr}_{\bar{\Lambda}_\ell}|\Psi(t)\rangle\langle\Psi(t)|$, where the trace is over the complementary lattice $\bar{\Lambda}_\ell$. At late times, $\varrho_\ell(t)$ is expected to approach a GGE, $\lim_{\ell\to\infty}\lim_{t\to\infty}\varrho_\ell(t) = \varrho_{\mathrm{GGE}} = \mathcal{Z}_L^{-1}\exp(-\sum_i \lambda_i I_i + hQ)$ involving the global U(1) charge Q and all the (quasi)local conserved quantities I_i of the model [39,40] (coupling to chemical potentials β_i, cf. [31] for a precise definition). Generalized Gibbs ensembles admit several equivalent descriptions [41,42]. One can, for instance, employ various state functions of the thermodynamic Bethe ansatz (TBA) enumerated by (integer) quantum numbers s, e.g., the macrostate densities $\rho_s(u)$ of quasiparticles with (bare) momenta $k_s(u)$ with rapidity u, or Fermi occupation (filling) functions $n_s(u) = \rho_s(u)/\rho_s^{\mathrm{tot}}(u)$, where the total density of states $\rho_s^{\mathrm{tot}}(u)$ are related to dressed momenta p_s via $\rho_s^{\mathrm{tot}}(u) = p'_s(u)/2\pi$. Crucially, the coarse-grained information stored in state functions $\rho_s(u)$ is sufficient to uniquely fix all local correlation functions in a GGE [43,44]. There exist a class of initial states with $b = 2$, [45,46], where analytic closed-form computation of the n_s is possible. The first successful demonstration of this program has been achieved in Ref. [47] (see also [41,48–52]), however, Ref. [47] inappropriately identifies some of the ensembles as canonical GGE, while, as we clarify in turn, the noncanonical character of such states is revealed by the vanishing of static spin susceptibility and the finite occupations of giant bound states.

To provide a few instructive examples, we subsequently focus on the $\Delta \geq 1$ regime of the Hamiltonian (1), where the excitation spectrum (above the ferromagnetic vacuum) comprises an infinite tower of magnon bound states with $s = 1, 2, \ldots$ quanta of magnetization. The density of free energy $f = -\lim_{L\to\infty} L^{-1} \log \mathcal{Z}_L$ in *canonical* GGEs can be split as $f = h/2 - \mathfrak{f}$, with [31], $\mathfrak{f} \equiv -\sum_{s=1}^{\infty} \int \mathrm{d}u k'_s(u) \log(1 - n_s(u))/2\pi$. Noticing that the kernel $k'_s(u)$ in the integrand tends to a constant at large s, the convergence of the infinite sum is fully predicated on the large-s behavior of n_s.

Squeezed GGEs.—We consider quantum quenches from a class of pure initial states $|\Psi\rangle$ with vanishing charge cumulants, $c^{(n)} = (\mathrm{d}/\mathrm{d}\lambda)^n F_Q(\lambda)|_{\lambda=0} = 0$, with the scaled cumulant generating function $F_Q(\lambda) \equiv \lim_{L\to\infty} L^{-1} \log \langle\Psi|e^{\lambda Q}|\Psi\rangle$. In spite of $c^{(n)}$ remaining globally conserved at all times, any subsystem of length ℓ will, in general, possess positive time-dependent cumulants $W^{(n)}(\ell, t) = \langle\Psi(t)|Q_\ell^n|\Psi(t)\rangle$. Accordingly, one expects the emergent local equilibrium state to exhibit strictly positive cumulants, namely, $\chi^{(n)} = \lim_{\ell\to\infty}\lim_{t\to\infty}\ell^{-1}W^{(n)}(\ell, t) > 0$. Surprisingly, however, this is not what happens in the easy axis regime $\Delta > 1$. As we clarify in the following, there $W^2(\ell, t)$ behaves anomalously, scaling subextensively with ℓ.

We now discuss how such a dynamical suppression of magnetic fluctuations in local equilibrium states is subtly related to "*freezing*" of the mode occupations n_s; instead of diminishing with increasing s, n_s are found to converge toward nontrivial limiting functions (attractors) depending on whether s is even or odd, see, Fig. 2. As a corollary, the infinite sum over the quasiparticle spectrum becomes

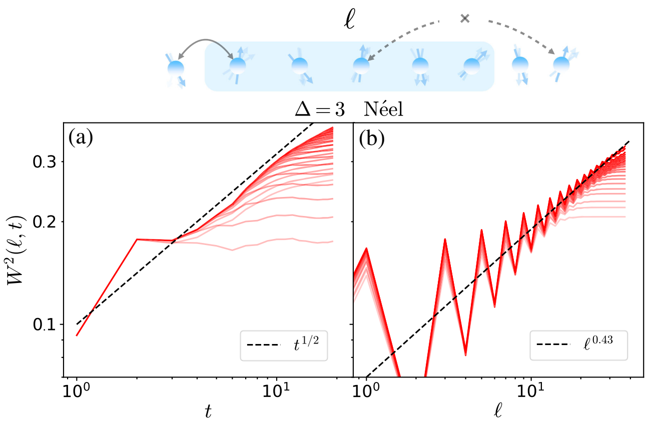

FIG. 1. Top: nonequilibrium time evolution from an initial state. Spatial fluctuations of local charge Q_ℓ produced during the time evolution in the subsystem of length ℓ grow subextensively with for large ℓ. Bottom: temporal and spatial scaling of magnetic fluctuations after the quench from the Néel state in the Heisenberg XXZ chain with $\Delta = 3$: (a) double log-plot of $W^2(\ell, t)$ as a function of time for different $\ell \in [4, 40]$ (increasing from light to dark); (b) double log-plot of $W^2(\ell, t)$ as function of ℓ for different times $t \in [5, 16]$ with $\delta t = 0.5$ (from light to dark), with the asymptotic scaling $W^2(\ell, t) \sim \ell^{2\zeta}$ and fitted exponent $2\zeta \approx 0.43$. Analogous results for the dimer initial state are reported in [31].

divergent, $\mathfrak{f} \to \infty$. The physical density of free energy f nonetheless remains finite. Indeed, infinitely many contributions can be resumed using certain kernel identities (cf. [31] for details), signifying that f is manifestly finite in both canonical GGEs and squeezed ensembles. Importantly, however, a *divergent* $\mathfrak{f}$ implies $h = \infty$. There are several key remarks in order: (i) although the employed TBA formulas are strictly applicable only for canonical GGEs [41,42], one can always regularize a divergent $\mathfrak{f}$ by introducing an appropriate twist (say τ) that renders $\mathfrak{f}$ and hence also $\chi^{(n)}$ finite, removing the twist only at the end; (ii) we emphasize that $\langle Q \rangle = 0$ does not generally imply $h = 0$ in a GGE, cf. [31]. In fact, for $\Delta > 1$ and $\tau > 0$ we find instead $\mathfrak{f}(\tau) < \infty$, but with $\mathfrak{f}(\tau)$ and $h = h(\tau)$ both diverging as $\tau \to 0$, see Ref. [31]; curiously, such a divergence is also exhibited by the Lagrange multiplier in the quench action [53,54]. (iii) the peculiar freezing phenomenon cannot take place in integrable systems with a finite number of bound states since $\mathfrak{f}$ cannot grow unboundedly. Hence, freezing is not present in the gapless regime with $|\Delta| < 1$ and, for the same reason, this effect is genuinely due to attractive interaction (see also [31]); (iv) our conclusions apply likewise to nonfluctuating magnetized states with $c^{(1)} \neq 0$ upon subtracting the first moment in Eq. (2), $Q_\ell \to Q_\ell - \langle Q_\ell \rangle$.

Proceeding now to explicit examples, we focus our analysis on simple initial valence-bond product states of two-site ($b = 2$) blocks [46]. We consider specifically the Néel state and the "dimer" state,

$$|\Psi_N\rangle = |\uparrow\downarrow\rangle^{\otimes L/2}, \qquad |\Psi_D\rangle = \left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right]^{\otimes L/2}, \tag{4}$$

allowing for explicit analytic computation of state functions in a recursive manner [31]. Our main conclusions nevertheless hold very generally, i.e., are valid for other initial product states that only involve eigenstates with the same value of Q. We have verified that the observed anomalous relaxation is not an artifact of coherent pair production associated with integrable quenches [55,56].

Static spin susceptibility.—Dynamical suppression of magnetic fluctuations in SGGEs implies a vanishing static spin susceptibility, namely, $\chi = 0$, given by the exact formula [57]

$$\chi = \sum_{s \geq 1} \int \mathrm{d}u \chi_s(u) [m_s^{\mathrm{dr}}(u)]^2, \tag{5}$$

where $\chi_s(u) \equiv \rho_s(u)[1 - n_s(u)]$ are the single-mode susceptibilities and m_s^{dr} denote the dressed magnetizations of quasiparticles (computed by solving the dressing equations [31]). Despite m_s^{dr} all vanish upon approaching an unmagnetized state, $q \equiv \lim_{\ell\to\infty} \langle Q_\ell \rangle / \ell = 0$, absence of uniform convergence requires regularization when evaluating Eq. (5). In order to reinstate finite magnetization density and finite fluctuations we employ *twisted* initial states. For instance, we use the twisted Néel state $|\Psi_N(\tau)\rangle = |\psi_N(\tau)\rangle^{\otimes L/2}$, where $|\varphi_N(\tau)\rangle \simeq \sum_{\sigma,\sigma' \in \{\uparrow,\downarrow\}} \varphi_{\sigma,\sigma'} |\sigma, \sigma'\rangle$, with amplitudes $\varphi_{\uparrow\uparrow}(\tau) = -\varphi_{\downarrow\downarrow}^{-1}(\tau) = e^{\tau}$, $\varphi_{\uparrow\downarrow}(\tau) = -\varphi_{\downarrow\uparrow}^{-1}(\tau) = \cot(\tau/2)$ depending on "twist" parameter $\tau > 0$. Such twisting in particular ensures that the mode occupation functions experience exponential decay for large s, while $m_s^{\mathrm{dr}} \sim q s^2$ for small or intermediate s, mirroring thermal states. Unlike in thermal (Gibbs) states, where the density decay algebraically as $\rho_s(u) \sim s^{-3}$, which gives a finite limit for the susceptibility $\lim_{q\to 0} \chi > 0$, SGGEs (in $\Delta > 1$ regime) instead generically exhibit exponential falloff $\rho_s(u) \sim e^{-\xi s}$. Consequently, the zero-twist limit can be interchanged with the infinite sum over s, yielding $\chi = 0$. Such exponential suppression of the densities is not incompatible with the observed freezing of the n_s at large s since the effective Brillouin zone [i.e., the Jacobian $k_s'(u)$] available for quasiparticles with large s shrinks exponentially in SGGEs. Indeed, in this limit only the giant quasiparticles that carry finite effective magnetization, telling that they become effectively pinned locally in space and consequently preclude the distribution of magnetic fluctuations through the system.

Spin diffusion.—We now examine the diffusion constant $\mathfrak{D}$, using the following exact mode resolution [58–60]

$$\mathfrak{D} = \sum_{s \geq 1} \int \mathrm{d}u \chi_s(u) |v_s^{\mathrm{eff}}(u)| \mu_s^2, \tag{6}$$

with effective velocities $v_s^{\mathrm{eff}}(u)$ (computed from the dressed dispersion relations, see Ref. [31]) and magnetic moments

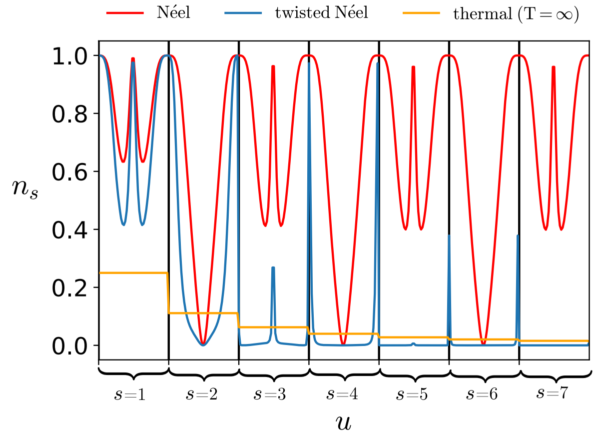

FIG. 2. Freezing of the quasiparticle mode occupations $n_s(u)$ in the SGGE emerging from the Néel quench in the XXZ chain with $\Delta = 2$, showing the Brillouin zones with $u \in [-\pi/2, \pi/2]$ (delimited by black vertical lines) for the few initial s, and compared to canonical behavior in GGEs [shown for the twisted Néel state $|\Psi_N(\tau)\rangle$] and thermal equilibrium values in the high-temperature limit.

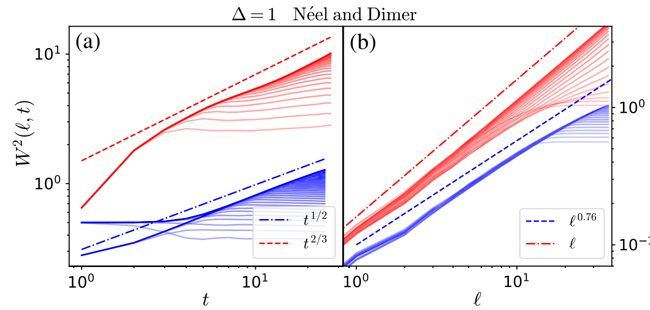

FIG. 3. Local magnetic fluctuations $W^2(\ell, t)$ in the isotropic Heisenberg chain ($\Delta = 1$): double log-plot of $W^2(\ell, t)$ as a function of t for $\ell \in [2, 29]$ for (a) the Néel (multiplied by 10 to improve readability) and dimer initial states, compared to $t^{2\beta}$ asymptotics, with growth exponents $\beta = 1/3$ and $\beta = 1/4$, respectively. (b) Double log-plot of $W^2(\ell, t)$ as a function of ℓ for $t \in [10, 38]$ with $\Delta t = 1.0$ for the Néel (red curves) and ℓ for $t \in [5, 16]$ with $\Delta t = 0.5$ for dimer (blue curves) states, compared to the linear slope in ℓ (Néel) and a subextensive scaling $\ell^{2\zeta}$ with approximate (fitted) exponent $\zeta \approx 0.38$ (dimer).

$\mu_s \equiv \partial_q m_s^{\mathrm{dr}}|_{q=0}$. Using the general scaling $\mu_s \sim s^2$, alongside $|v_s^{\mathrm{eff}}| \sim e^{-\kappa s}$ for $\Delta > 1$, we readily conclude that $\mathfrak{D} > 0$ in SGGEs. However, as we explain shortly, we find a clear signature of anomalous diffusive behavior. We also note that dc spin conductivity σ vanishes identically, that is $\sigma = \mathfrak{D}\chi = 0$.

Numerical simulations.—To confirm our theoretical predictions, and to additionally infer the scaling properties of finite subsystems, we carry out numerical simulations using the matrix product states (MPS) with the iTensor library [61]. We simulate the time evolution of chains of length $L = 100$ up to maximal times $t \approx 15$ (using the maximal bond dimension of 1024). We compute the time dependence of charge variance in a local subsystem $W^2(\ell, t)$ by time-evolving the initial state $|\Psi\rangle$ with TEBD [62], for subsystems of length ℓ ranging from 2 to 40. For both Néel (see Fig. 1) and dimer states (see Ref. [31]), we observe diffusive temporal growth $W^2(\ell, t) \sim t^{1/2}$ followed by saturation to a value $\sim \ell^{2\zeta}$ with the approximate fitted exponent $\zeta \approx 1/4$. This value would only be consistent with the FV scaling hypothesis (3) in the case of ballistic dynamical exponent $z = 1$, but not with $z = 2$ associated with diffusive processes. This leads us to rule out the self-affine structure of the magnetic fluctuations in SGGEs, implying the absence of normal spin diffusion. Lastly, we also verify that the norm of the equilibrium Hamiltonian $\log \varrho_\ell$ grows superextensively [31], contrasting the extensive behavior of (quasi)local charges in canonical GGEs (as found, e.g., in the XXZ Hamiltonian with $\Delta = 0.5$).

Isotropic chain and KPZ fluctuations.—The isotropic limit $\Delta \to 1$ requires special attention due to an enhanced non-Abelian symmetry [63]. It is by now well established that in thermal equilibrium with $q = 0$ magnetization exhibits anomalous transport characterized by a superdiffusive dynamical exponent $z = 3/2$ associated with the KPZ physics [59,60,64–68]. On the other hand, hydrodynamic relaxation from pure states is much less explored, and our work partially fills this void. Indeed, it turns out, somewhat surprisingly, that in the case of isotropic interaction the nonfluctuating initial states can exhibit different qualitative behavior. For example, the Néel state relaxes to a GGE with regular (i.e., decaying) occupation functions, enabling restoration of fluctuations with a finite $\chi \approx 0.6$. In contrast, the dimer state again yields $\chi = 0$. Using the exact result for $n_s^{\mathrm{D}}(u)$, the divergence of $\mathfrak{f} = \sum_{s\geq 1} \mathfrak{f}_s \to \infty$ follows rigorously from the large-s behavior $\mathfrak{f}_s = 3\log(s) - 4\log(s+1) + \log(s+2) \sim s^{-1}$, causing a logarithmic divergence of $\mathfrak{f}$ with the cutoff $s_{\max}$. In the case of Néel state, the spin diffusion constant is found to diverge; the terms in Eq. (6) tend to constant at large s, mirroring the thermal states (exhibiting $\chi_s(u) \sim s^{-3}$ decay, and $|v_s^{\mathrm{eff}}(u)| \sim s^{-1}$ (see the additional numerical data in [31]), stipulated by the "superuniversality" of spin superdiffusion [69] with dynamical exponent $z = 3/2$). As shown in Fig. 3, spin fluctuations grow as $\sim t^{2/3}$, i.e., $\beta = 1/3$. In the dimer quench, however, the equilibrium state reveals distinctly nonthermal features despite preservation of the SU(2) symmetry ($q = 0$), with scaling $\chi_s(u) \sim s^{-5}$ and $|v_s^{\mathrm{eff}}(u)| \sim s^0$, indicating a logarithmic divergence of $\mathfrak{D}$ with $s_{\max}$. Together with the vanishing of the spin susceptibility, this implies finite spin conductivity $\sigma = \chi\mathfrak{D}$, i.e., normal spin transport.

These findings are well supported by our numerical simulations. In the Néel quench, the data are well compatible with the anticipated scaling form with $\zeta = 1/2$ and $\beta = 1/3$,

$$W^2(\ell, t) \sim \ell \Phi_{\mathrm{iso}}(t/\ell^{3/2}), \tag{7}$$

consistently $\chi > 0$ and singular spin diffusion constant in the associated GGE. In the dimer case, we observe (on the accessible times) an algebraic growth $W^2(\ell, t) \sim t^{2\beta}$ with $\beta = 1/4$, whereas the exponent ζ appears to be slightly smaller than the extensive value $\zeta = 1/2$ (compatibly with the theoretically predicted freezing of n_s).

Conclusions.—By considering a class of nonfluctuating initial product states, we demonstrated that interacting integrable systems hosting infinitely many bound states can sometimes evade thermalization to canonical GGEs. Here we discuss quench scenarios in which local subsystems equilibrate to unorthodox states called squeezed GGEs, featuring subextensive magnetic fluctuations [signaled by a divergent U(1) chemical potential]. In addition, we find the approach to equilibrium violates the Family-Vicsek scaling hypothesis. Another distinguished property of SGGEs are non-decaying, so-called "frozen," mode occupations of giant quasiparticles, causing an emergent large-scale semi-classical description [70] to break down. This phenomenon is thus a genuine effect of interaction and cannot take place in free systems.

We are hopeful that the state-of-the-art quantum simulators [19,65,71] and modern quantum processors [72,73] can provide a test bed and an ideal opportunity to verify our predictions, as they do not suffer from the rapid growth of entanglement generated by the quantum quench. We emphasize that unlike the recently studied quantum transport in canonical classical ensembles [65], the outlined phenomena are genuine properties of *quantum states* with no equivalent in classical spin systems.

Several recent studies reported anomalous behavior of macroscopic fluctuating quantities, such as the full counting statistics (FCS), Rényi entropy [74,75]), in quantum and classical Heisenberg chains [67,76,77] and related models [78–80] featuring fragmentation. It currently remains unclear whether anomalous FCS bears any connection to the observed anomalous dynamic roughening which, according to our simulations, appears to be intimately tied to integrability. Upon breaking integrability, generic Hamiltonians with equidistant energy levels in the limit $\Delta \to \infty$ involve quasilocal quantities conserved up to exponential time $\sim e^{\kappa\Delta}$ for any $\Delta > 1$, being a corollary of Ref. [81]. In this view, integrability guarantees the exact conservation of magnetic fluctuations for large subsystems $\ell \gg 1$ at arbitrary times. We currently lack any deeper mathematical insight behind this mechanism and how (weak) integrability-breaking perturbations influence the picture, which we plan to investigate in future works.

We thank B. Doyon, L. Zadnik, and M. Žnidarič for insightful discussions and S. Gopalakrishnan, K. Takeuchi, and R. Vasseur for discussions and collaborations on related subjects. This work has been partially funded by the ERC Starting Grant No. 101042293 (HEPIQ) (J. D. N. and G. C.) and ANR-22-CPJ1-0021-01 (J. D. N.). E I. is supported by project N1-0243 of the Slovenian Research Agency.

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