Infinite Quantum Twisting at the Cauchy Horizon of Rotating Black Holes

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We present a numerical calculation of the expectation value of the quantum angular-momentum current flux density for a scalar field in the Unruh state near the inner horizon of a Kerr–de Sitter black hole. Our results indicate that this flux diverges as V_-^{-1} in a suitable Kruskal coordinate such that $V_- = 0$ at the inner horizon. Depending on the parameter values of the scalar field and black hole that we consider, and depending the polar angle (latitude), this flux can have different signs. In the near extremal cases considered, the angle average of the expectation value of the quantum angular momentum current flux is of the opposite sign as the angular momentum of the background itself, suggesting that, in the cases considered, quantum effects tend to decrease the total angular momentum of the spheres away from the extremal value. We also numerically calculate the energy flux component, which provides the leading order divergence of the quantum stress energy tensor, dominant over the classical stress energy tensor, at the inner horizon. Taking our expectation value of the quantum stress tensor as the source in the semiclassical Einstein equation, our analysis suggests that the spheres approaching the inner horizon can undergo an infinite twisting due to quantum effects along latitudes separating regions of infinite expansion and contraction.

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Introduction.—Black hole (BH) spacetimes with non-vanishing charge and/or angular momentum possess inner horizons (IHs). These are Cauchy horizons because they delineate the domain of dependence of the initial value problem for classical or quantum wave-type equations posed on an initial Cauchy surface.

Penrose [1] suggested that perturbations of the BH metric, as well as matter fields on these spacetimes, might diverge at the IH. If his "strong cosmic censorship" (sCC) hypothesis were true, then the regular IHs present in these exact solutions of Einstein equations would be mere mathematical illusions and should get converted to a nontimelike singularity in a full (classical) treatment of the coupled Einstein-matter system. Such a scenario would be an elegant resolution of the issues of determinism raised by IHs, relegating them to the realm of quantum gravity taking over sufficiently near these singularities.

The validity of sCC has been investigated in many works, starting with [2–4] who accumulated suggestive evidence in favor of the hypothesis. One question concerns the exact nature of the singularity. Recent mathematical works on the full Einstein equations [5] show that tidal distortions at the singularity remain finite for sufficiently small initial perturbations [6], but one expects that tidal forces suitably diverge in such a way as to make the metric inextensible as a *weak* solution to the Einstein equations. According to a formulation by [7], this means that first derivatives of the metric ought to fail to be locally square integrable at the IH. In terms of scalar test fields, it means that some component $T_{\mu\nu}$ of the matter stress energy should fail to be locally integrable there. Although a proof of this strong form of sCC is still missing in the context of the full Einstein-matter equations, there is a large body of evidence in the context of the test field approximation of the Einstein equations, and via numerical approaches [8–17].

The charge and angular momentum of BHs have upper (extremal) bounds. Interestingly, it has recently been observed [13,14,18] that sCC is classically violated near the extremal limit of charged, static BHs [Reissner-Nordström (RN)] in a *de Sitter* Universe (i.e., with a positive cosmological constant Λ) (RNdS). Thus, one naturally wonders whether quantum effects could become relevant in such a setting. It was indeed shown by [19,20] (building partly on earlier pioneering work by [21,22]) that the component of the renormalized expectation value $\langle\hat{T}_{\mu\nu}\rangle$ of the quantum stress-energy tensor (RSET) which is relevant for the shear and expansion of a congruence of observers crossing the IH of RNdS has a quadratic divergence in a Kruskal (regular) coordinate. The leading order divergence is independent of the chosen initial Hadamard, i.e., "regular," state and entirely of quantum nature, in the sense that the difference between the RSETs in two states diverges at a (weaker) rate set by the classical theory.

The nature of quantum effects at the IH cannot be naively explained by spontaneous pair production from the vacuum: a charged scalar quantum field would always discharge the IH [23,24] according to Schwinger pair creation [25], whereas a full quantum field theory computation [26,27] of the expectation value of the charge current revealed that the current may increase the charge of the IH of RNdS in a certain parameter region (although near extremality, the charge is decreased).

Charged BHs can be considered as toy models for the more complicated, and astrophysically relevant, rotating ones: Kerr when $\Lambda = 0$ and Kerr–de Sitter (KdS) when $\Lambda > 0$. Even though in Kerr and KdS, sCC is expected to hold already at the classical level [4,15,28], the calculation of quantum fluxes is nevertheless interesting because one would like to see whether they can dominate and/or be qualitatively different at the IH even in such a situation. Quantum energy fluxes have recently been computed numerically by [29] in Kerr, who found that this is indeed the case. Another interesting question is how quantum matter would influence the angular momentum of spheres approaching the IH.

In this Letter, we compute the energy flux $\langle \hat{T}_{vv} \rangle_{\rm U}$ and angular momentum current-density $\langle \hat{T}_{v\varphi_-} \rangle_{\rm U}$ (see Sec. "Geometric setup" for the azimuthal coordinate φ_- and Eddington-type coordinate v) components of the RSET for a real scalar quantum field in the Unruh state at the IH of KdS. The Unruh state is the relevant one since it models the late-time behavior in gravitational collapse to a BH.

Our numerical evidence suggests that $\langle \hat{T}_{V_-V_-} \rangle_{\rm U} \sim V_-^{-2}$ as $V_- \to 0$, where $V_- \equiv -e^{-\kappa_- v}$ is a Kruskal coordinate such that $V_- = 0$ on the IH and κ_- is the surface gravity of the IH. For classical scalars on KdS, the divergence is weaker: $T_{V_-V_-} \sim V_-^{-(2-2\beta)}$, where $\beta \in (0, 1/2)$ [15,30–32]; similarly, changing the Unruh state to other initial Hadamard states would result in a correction of the same size as in the classical case [33]. We thus find the *leading* irregularity [34] of the RSET at the IH. We also find that $\langle \hat{T}_{V_-V_-} \rangle_{\rm U}$ can change sign with the polar angle θ, differently from the classical case.

In its turn, we find that the angular momentum current density behaves as $\langle \hat{T}_{V_-\varphi_-} \rangle_{\rm U} \sim V_-^{-1}$, which is still divergent at the IH but subdominant to $\langle \hat{T}_{V_-V_-} \rangle_{\rm U}$. We find that either sign is possible depending on the values of the BH parameters and that, sufficiently close to extremality, the sign changes once between the poles and equatorial plane as the polar angle θ varies. However, we also find that the angle average of $\langle \hat{T}_{V_-\varphi_-} \rangle_{\rm U}$ near extremality is positive,

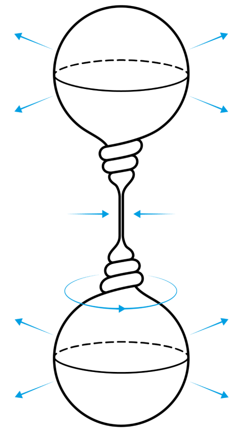

FIG. 1. Infinite twisting between regions of diverging expansion and contraction on a cross section of the IH.

suggesting in view of standard flux-balance relations for the Komar angular momentum that quantum effects tend to decrease the total angular momentum of 2-spheres approaching the IH. Extrapolating from our results on the fixed KdS background to a (hypothetical) solution of the semiclassical Einstein equations with backreaction, we find that the divergence of the angular momentum current-density will lead to diverging geometric twist of these 2 spheres along latitudes separating infinite expansion and infinite contraction, see Fig. 1.

The rest of this Letter is organized as follows. Sections "Geometric setup" and "The scalar quantum field," respectively, introduce KdS spacetime and scalar field quantum field theory on KdS. The formula for the RSET at the IH is derived in Sec. "The stress-energy tensor." Section "Numerical results" contains our numerical results for the RSET. We summarize the results in Sec. "Conclusions." Throughout, we set $\hbar = c = G_N = 1$.

Geometric setup.—In Boyer-Lindquist coordinates (t, r, θ, φ), the KdS metric describing a BH of mass M with angular momentum parameter a in the presence of a positive cosmological constant Λ is

$$
\begin{aligned}
g = {} & \frac{\Delta_\theta a^2 \sin^2\theta - \Delta_r}{\rho^2 \chi^2} \mathrm{d}t^2 + \frac{\rho^2}{\Delta_r} \mathrm{d}r^2 + \frac{\rho^2}{\Delta_\theta} \mathrm{d}\theta^2 \\
& + \left[\Delta_\theta (r^2 + a^2)^2 - \Delta_r a^2 \sin^2\theta\right] \frac{\sin^2\theta}{\rho^2 \chi^2} \mathrm{d}\varphi^2 \\
& + 2 \frac{a \sin^2\theta}{\rho^2 \chi^2} \left[\Delta_r - \Delta_\theta (r^2 + a^2)\right] \mathrm{d}t \mathrm{d}\varphi, \qquad (1)
\end{aligned}
$$

where

$$
\Delta_r \equiv (1 - \Lambda r^2/3)(r^2 + a^2) - 2Mr, \qquad \chi \equiv 1 + a^2 \Lambda/3,
$$
$$
\Delta_\theta \equiv 1 + (a^2 \Lambda/3) \cos^2\theta, \qquad \rho^2 \equiv r^2 + a^2 \cos^2\theta.
$$

Henceforth, we set $M = 1$ and restrict the parameters Λ and a to the subextremal range in which $\Delta_r(r)$ has three distinct

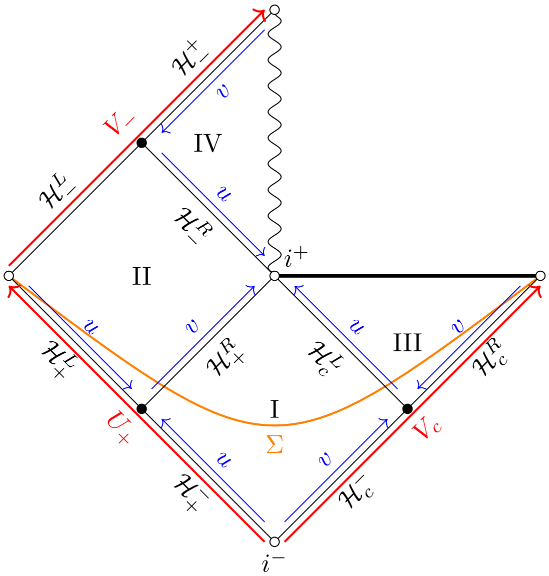

FIG. 2. Carter-Penrose diagram of KdS. The orange line Σ is a Cauchy surface for regions I, II, and III. The red lines indicate the ranges of different Kruskal coordinates, while the blue lines illustrate u and v in the corresponding block. The wavy line represents the timelike ring singularity of KdS (at $r = 0$ and $\theta = \pi/2$).

positive roots defining the locations of the cosmological (r_c), event (r_+), and inner (r_-) horizons. The near-extremal regime is for a close to its maximally allowed value, $a_{\max}(\Lambda)$. In that case, the Boyer-Lindquist coordinates cover the regions $\mathbb{R}_t \times (r_i, r_j) \times \mathbb{S}^2_{\theta,\varphi}$, with r_i and r_j two subsequent zeros of Δ_r (or $\pm\infty$). Each horizon r_j, $j \in \{-, +, c\}$, has associated an angular velocity Ω_j, a surface gravity κ_j and an azimuthal coordinate via $d\varphi_j \equiv d\varphi - \Omega_j dt$. We also define a new radial coordinate via $dr_* = \chi(r^2 + a^2)dr/\Delta_r$ and Eddington-type coordinates $v \equiv t + r_*$ and $u \equiv t - r_*$. Analytic continuation of the metric across $r = r_j$ is achieved in Kruskal coordinates (U_j, V_j) [35,38], from which the Carter-Penrose diagram is constructed—see Fig. 2. The region I ∪ II ∪ III, together with the horizons $\mathcal{H}_c^L$ and $\mathcal{H}_+^R$ constitute a globally hyperbolic spacetime [39]: there exists a Cauchy surface (i.e., a smooth, spacelike hypersurface which is crossed exactly once by every inextendible causal curve) for this region.

We are interested in the angular momentum of the IH. Considering that, under backreaction, the BH becomes dynamic and the spacetime is no longer a vacuum solution of the Einstein equation, it seems reasonable to investigate a quasilocal notion of angular momentum. Assuming that the spacetime remains axisymmetric, one plausible choice is the Komar angular momentum

$$J[\mathcal{S}] = \frac{1}{16\pi}\int_{\mathcal{S}} \nabla_\alpha \psi_\beta d\Sigma^{\alpha\beta}, \tag{2}$$

of a topological sphere $\mathcal{S}$, where $\psi^\mu \equiv \delta^\mu_\varphi$ is the rotational Killing field and $d\Sigma^{\alpha\beta}$ an appropriately oriented covariant surface integration element. We choose orientations such that, in KdS, $J_{\mathrm{KdS}} = (aM/\chi^2)$, which is independent of $\mathcal{S}$. However, in a dynamical axisymmetric spacetime, by Gauss's theorem and C.3.6 of [40]

$$J[\mathcal{S}_2] - J[\mathcal{S}_1] = -\int_\Delta \left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T^\sigma_\sigma - \Lambda g_{\alpha\beta}\right)\psi^\alpha d\Sigma^\beta. \tag{3}$$

Here, Δ is a three-dimensional surface tangent to ψ^α bounding $\mathcal{S}_1$ and $\mathcal{S}_2$, and $d\Sigma^\alpha$ is an appropriately oriented covariant integration element on Δ. We are interested in the case where $\mathcal{S}_2$ approaches the future IH, $\mathcal{H}_-^R$, at constant value of u, while $\mathcal{S}_1$, located at the same constant u, stays at some constant value $V_- < 0$. We will assume that, to estimate quantum effects, the stress tensor can be replaced in the small backreaction regime by the RSET $\langle \hat{T}_{\alpha\beta} \rangle_{\mathrm{U}}$; we denote its value at the IH by $\langle \hat{T}_{\alpha\beta}(\theta) \rangle_{\mathrm{U}}^{\mathrm{IH}}$.

To evaluate Eq. (3), we require the RSET component $\langle \hat{T}_{\varphi_- v} \rangle_{\mathrm{U}}$. We present evidence in the following sections that this quantity approaches a finite value at the IH. Using this result and Eq. (1), we obtain

$$J[\mathcal{S}(u, v)] \approx -v \langle\langle \hat{T}_{v\varphi_-} \rangle\rangle_{\mathrm{U}}^{\mathrm{IH}} \tag{4}$$

as $v \to \infty$ at constant u, where $\mathcal{S}(u, v)$ is a 2 sphere of constant values u, v. The angle-averaged expectation value of the RSET at the IH in Eq. (4) is defined by [41]

$$\langle\langle \hat{T}_{v\varphi_-} \rangle\rangle_{\mathrm{U}}^{\mathrm{IH}} \equiv 2\pi \frac{r_-^2 + a^2}{\chi} \int_0^\pi d\theta \sin\theta \left\langle \hat{T}_{v\varphi_-}(\theta) \right\rangle_{\mathrm{U}}^{\mathrm{IH}}. \tag{5}$$

The scalar quantum field.—Consider a minimally coupled scalar field ϕ of mass $\mu = \sqrt{2\Lambda/3}$, which satisfies the same equations of motion as the massless, conformally coupled scalar field,

$$(g^{\alpha\beta}\nabla_\alpha\nabla_\beta - \mu^2)\phi = 0. \tag{6}$$

On a single Boyer-Lindquist block, this differential equation reduces to a set of ordinary differential equations via

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \mathcal{N}_{\omega\ell m} e^{-i\omega t} e^{im\varphi} S^\omega_{\ell m}(\theta) R_{\omega\ell m}(r). \tag{7}$$

Here, $m \in \mathbb{Z}$, $\ell \in \mathbb{N}_{\geq |m|}$, $S^\omega_{\ell m}(\theta)$ are the spheroidal wave functions [42] and $R_{\omega\ell m}(r)$ obey a Schrödinger-like equation [42,43].

Consider a complete set $\{\phi_I\}_I$, of mode solutions to the Klein-Gordon equation (6) which is orthonormal with respect to the usual Klein-Gordon inner product. Then one can quantize the field by expanding it as

$$\hat{\phi}(x) = \sum_I \left(\phi_I(x)\hat{b}_I + \overline{\phi_I}(x)\hat{b}_I^\dagger\right). \tag{8}$$

The coefficients $\hat{b}_I$ and $\hat{b}_I^\dagger$ are creation and annihilation operators, and the vacuum state $|\mathrm{U}\rangle$ of the Fock space on which these operators act satisfies $\hat{b}_I|\mathrm{U}\rangle = 0$ for all I.

The Unruh state can be constructed in this way by choosing ϕ_I to be the Unruh modes: (i) modes which are positive frequency with respect to U_+ on $\mathcal{H}_+^L \cup \mathcal{H}_+^-$ and vanish on $\mathcal{H}_c^- \cup \mathcal{H}_c^R$; together with (ii) modes which are positive frequency with respect to V_c on $\mathcal{H}_c^- \cup \mathcal{H}_c^R$ and vanish on $\mathcal{H}_+^- \cup \mathcal{H}_+^L$.

However, the Boulware modes $\phi_{\omega\ell m}^{\mathrm{in/up,I}}$ (with reflection and transmission coefficients $\mathcal{R}_{\omega\ell m}^{\mathrm{in/up,I}}$ and $\mathcal{T}_{\omega\ell m}^{\mathrm{in/up,I}}$) and $\phi_{\omega\ell m}^{\mathrm{in/up,II}}$, defined in [35], are easier to calculate than the Unruh modes. By expressing the Unruh modes in terms of Boulware modes similarly as in [26,44], we obtain for the anticommutator in II:

$$\begin{aligned}\left\langle\hat{\phi}(x)\hat{\phi}(y)\right\rangle_{\mathrm{U}}^{(s)} &\equiv \frac{1}{2}\left\langle\left(\hat{\phi}(x)\hat{\phi}(y)+\hat{\phi}(y)\hat{\phi}(x)\right)\right\rangle_{\mathrm{U}} = \frac{1}{2}\sum_{\ell,m}\int_0^\infty \mathrm{d}\omega\left[\coth\left(\frac{\pi\omega_c}{\kappa_c}\right)\left|\mathcal{T}_{\omega\ell m}^{\mathrm{in,I}}\right|^2\left|\frac{\omega_+}{\omega_c}\right|\left\{\phi_{\omega\ell m}^{\mathrm{in,II}}(x),\overline{\phi_{\omega\ell m}^{\mathrm{in,II}}}(y)\right\}\right.\\ &+\coth\left(\frac{\pi\omega_+}{\kappa_+}\right)\left[\left\{\phi_{\omega\ell m}^{\mathrm{up,II}}(x),\overline{\phi_{\omega\ell m}^{\mathrm{up,II}}}(y)\right\}+\left|\mathcal{R}_{\omega\ell m}^{\mathrm{up,I}}\right|^2\left\{\phi_{\omega\ell m}^{\mathrm{in,II}}(x),\overline{\phi_{\omega\ell m}^{\mathrm{in,II}}}(y)\right\}\right]\\ &\left.+2\mathrm{csch}\left(\frac{\pi\omega_+}{\kappa_+}\right)\mathrm{Re}\left[\mathcal{R}_{\omega\ell m}^{\mathrm{up,I}}\left\{\phi_{\omega\ell m}^{\mathrm{in,II}}(x),\overline{\phi_{\omega\ell m}^{\mathrm{up,II}}}(y)\right\}\right]\right], \qquad (9)\end{aligned}$$

where $\{f(x),g(y)\}\equiv f(x)g(y)+f(y)g(x)$ for two functions f and g and $\omega_j(\omega,m)\equiv\omega-m\Omega_j$ for $j=-,+,c$.

The stress-energy tensor.—The quantum observable corresponding to the classical stress-energy tensor in some quantum state Ψ is the RSET $\langle\hat{T}_{\mu\nu}\rangle_\Psi$. We wish to see if, and how, the RSET in the Unruh state ($|\Psi\rangle=|\mathrm{U}\rangle$) diverges.

At least for sufficiently small a or Λ, the Unruh state is Hadamard up to but not including the IH [39]. To understand the divergent behavior of $\langle\hat{T}_{\mu\nu}\rangle_{\mathrm{U}}^{\mathrm{IH}}$, it suffices to calculate the offset to the expectation value, $\langle\hat{T}_{\mu\nu}\rangle_{\mathrm{C}}$, in some comparison state $|\mathrm{C}\rangle$ which is Hadamard in an open two-sided neighborhood of the IH. Then—see, e.g., [45]—$\langle\hat{T}_{\mu\nu}\rangle_{\mathrm{C}}$ is smooth in a two-sided neighborhood of the IH in Kruskal coordinates, and so we must have $\langle\hat{T}_{vz}\rangle_{\mathrm{C}}^{\mathrm{IH}}=0$, for $z\in\{v,\varphi_-\}$. Thus,

$$\begin{aligned}\left\langle\hat{T}_{vz}(x)\right\rangle_{\mathrm{U}}^{\mathrm{IH}} = \lim_{x\to\mathcal{H}_-^{L,R}}\lim_{x'\to x}D_{vz'}&\left(\left\langle\hat{\phi}(x)\hat{\phi}(x')\right\rangle_{\mathrm{U}}^{(s)}\right.\\ &\left.-\left\langle\hat{\phi}(x)\hat{\phi}(x')\right\rangle_{\mathrm{C}}^{(s)}\right), \qquad (10)\end{aligned}$$

where $D_{\alpha\beta'}$ is a bidifferential operator given in [35] and x is any point where *both* states are defined.

We construct the comparison state similarly to [19] for RN: we modify the metric in $r<\delta<r_-$, with $0<\delta\ll r_-$, replacing the singularity at $r=0$. Combined with Eqs. (9) and (10), this yields [35]

$$\begin{aligned}\left\langle\hat{T}_{vz}(\theta)\right\rangle_{\mathrm{U}}^{\mathrm{IH}} &= \frac{\chi}{4\pi(r_-^2+a^2)}\left\{\sum_{\ell=0}^\infty\int_0^\infty\frac{\mathrm{d}\omega_-}{\omega_-}F_{\ell 0}^z(\omega_-,\theta)\right.\\ &+\sum_{\ell\geq m>0}\int_0^\infty\frac{\mathrm{d}\omega_-}{\omega_-}\left(F_{\ell m}^z(\omega_-,\theta)\right.\\ &\left.\left.-F_{\ell m}^z(-\omega_-,\theta)\right)\right\}, \qquad (11)\end{aligned}$$

$$\begin{aligned}F_{\ell m}^z(\omega_-,\theta)\equiv\omega_-^2\frac{c_z}{\omega_+}|S_{\ell m}^\omega(\theta)|^2&\left[\frac{-\omega_+}{\omega_-}\coth\left(\frac{\pi\omega_-}{\kappa_-}\right)\right.\\ &+\coth\left(\frac{\pi\omega_+}{\kappa_+}\right)\left(\left|\mathcal{R}_{\omega\ell m}^{\mathrm{in,II}}\right|^2+\left|\mathcal{R}_{\omega\ell m}^{\mathrm{up,I}}\right|^2\left|\mathcal{T}_{\omega\ell m}^{\mathrm{in,II}}\right|^2\right)\\ &+2\mathrm{csch}\left(\frac{\pi\omega_+}{\kappa_+}\right)\mathrm{Re}\left[\mathcal{R}_{\omega\ell m}^{\mathrm{up,I}}\mathcal{R}_{\omega\ell m}^{\mathrm{in,II}}\mathcal{T}_{\omega\ell m}^{\mathrm{in,II}}\right]\\ &\left.+\coth\left(\frac{\pi\omega_c}{\kappa_c}\right)\left(1-\left|\mathcal{R}_{\omega\ell m}^{\mathrm{up,I}}\right|^2\right)\left|\mathcal{T}_{\omega\ell m}^{\mathrm{in,II}}\right|^2\right], \qquad (12)\end{aligned}$$

where $z\in\{v,\varphi_-\}$, $c_v\equiv\omega_-$, and $c_{\varphi_-}\equiv -m$.

Numerical results.—We plot in Fig. 3 the angular momentum current at the IH as a function of $\cos\theta$ for $\Lambda=1/30$ and $a=0.95$, 0.975, 1, and $a_{\max}-1/200$, where $a_{\max}\approx 1.012$. In [35] we also plot the angular momentum current at $\cos\theta=0.9$ as a function of a for

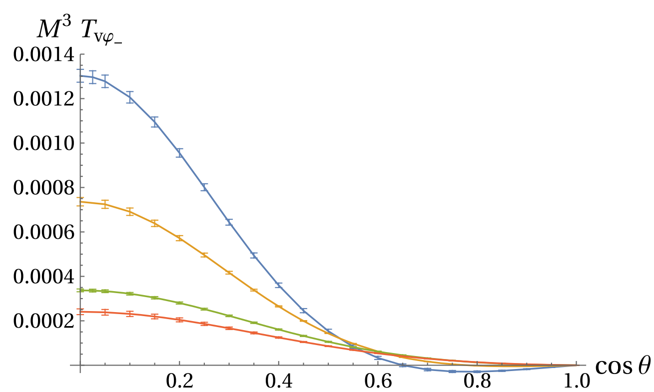

FIG. 3. The $\cos\theta$ dependence of $\langle T_{v\varphi_-}\rangle_{\mathrm{U}}^{\mathrm{IH}}$ for $\Lambda=1/30$ and $a=0.95$ (blue), 0.975 (orange), 1 (green), and 1.007 (red). Note that $a_{\max}\approx 1.012$.

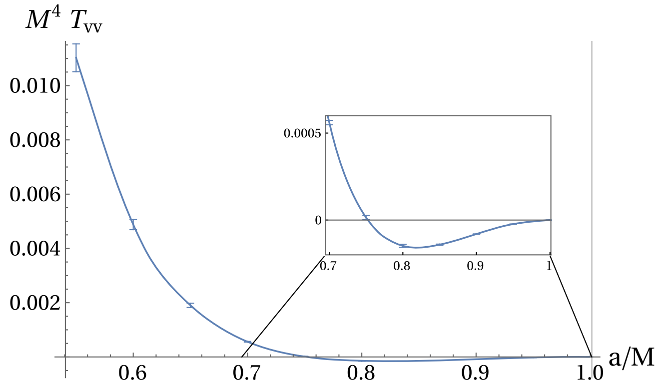

FIG. 4. $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}^{\mathrm{IH}}$ for $\theta = 0$, $\Lambda = 1/270$ as a function of a. The vertical line indicates $a_{\max}$. The inset is a closeup near extremality.

different values of Λ. Our first main result is that $\langle T_{v\varphi_-} \rangle_{\mathrm{U}}^{\mathrm{IH}}$ can have different signs depending on the latitude θ.

The integral (5) determines the change of the quasilocal angular momentum near the IH (Sec. "Geometric setup"). Our second main result is that, for all near-extremal values of a in Fig. 3, we find that this integral is positive: $\langle\langle \hat{T}_{v\varphi_-} \rangle\rangle_{\mathrm{U}}^{\mathrm{IH}} = 0.006\,67 \pm 0.000\,17$, $0.004\,769 \pm 0.000\,074$, $0.002\,974 \pm 0.000\,048$, and $0.002\,398 \pm 0.000\,039$ for, respectively, $a = 0.95$, 0.975, 1, and $a_{\max} - 0.005$. Substituting these positive values into Eq. (4), we conclude that the angular momentum $J[\mathcal{S}(u, v)]$ of the quantum field, associated with a sphere of constant v and u, is going to $-\infty$ linearly in v as the IH is approached, $v \to \infty$. Thus, it is of the *opposite* sign as the angular momentum of such a sphere in the background ($\equiv Ma/\chi^2$).

In Fig. 4 we plot $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}^{\mathrm{IH}}$ on the rotation axis as a function of a. To achieve better comparability with the corresponding results obtained in RN(-dS) [20,22] and Kerr [29] for a minimally coupled scalar field of mass $\mu^2 = 2\Lambda/3$ (which vanishes in RN and Kerr), we have set $\Lambda = 1/270$. We see qualitatively very similar results. In particular, $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}^{\mathrm{IH}}$ begins positive for small a, changes sign at an intermediate value, and then approaches zero from below as a approaches $a_{\max}$. For the same parameter values as the orange curve in Fig. 3, we have also checked that $\langle \hat{T}_{vv} \rangle_{\mathrm{U}}^{\mathrm{IH}}$ changes sign with θ, qualitatively similar to [29].

Conclusions.—We have computed the vv and $v\varphi_-$ components of the RSET in the Unruh state on the IH of a KdS BH. These components constitute, respectively, the leading divergence of the V_-V_- and $V_-\varphi_-$ components of the RSET on the IH under a tensorial transformation with $\partial V_-/\partial v \sim V_-$.

The divergence of $\langle \hat{T}_{V_-V_-} \rangle_{\mathrm{U}}$ on the IH ($V_- = 0$) is proportional to V_-^{-2} with a proportionality factor that is generically nonzero, at least on the axis of rotation. The sign of this factor can change with a and θ, similarly as in Kerr [29].

The divergence of $\langle \hat{T}_{V_-\varphi_-} \rangle_{\mathrm{U}}$ on the IH is proportional to V_-^{-1}, again with a generically nonzero proportionality factor. Interestingly, the sign of this quantity can change with the latitude on the sphere θ near extremality. Nonetheless, we find that the *angle average* of $\langle \hat{T}_{V_-\varphi_-} \rangle_{\mathrm{U}}$ diverges near extremality to $-\infty$ for our chosen parameters. This indicates that, if backreaction effects were taken into account, the *total* angular momentum of such a sphere would suffer an $O(1)$ decrease relative to the background value $J_{\mathrm{KdS}} = Ma/\chi^2$ in our conventions where $a > 0$, see Eq. (4), by which time the background underlying our calculation would have to be updated. This resembles the behavior of the total scalar charge of a charged quantum scalar field near the IH of a near-extremal RN BH [27], and indicates the absence of "runaway" behavior for the charge and angular momentum.

Combined with the results of [33], the leading divergence of the RSET that we find is not special for the Unruh state but true for an *arbitrary* initial (Hadamard) state, and stronger than the divergence of the classical stress energy, at least in the parameter regime where KdS is mode stable [46].

As we argue in [35], if we extrapolate the qualitative features of our findings to a setting with backreaction, then topological spheres approaching the IH undergo infinite expansion in some parts, and contraction in others. On a latitude separating such regions, we have a diverging relative twisting of neighboring latitudes and/or a twisting of perpendicular incident light rays toward or against the φ_--direction, Fig. 1.

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