Bootstrapping Deconfined Quantum Tricriticality

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The paradigmatic example of deconfined quantum criticality is the Neel to valence bond solid phase transition. The continuum description of this transition is the $N = 2$ case of the CP^{N-1} model, which is a field theory of N complex scalars in 3D coupled to an Abelian gauge field with $\mathrm{SU}(N) \times \mathrm{U}(1)$ global symmetry. Lattice studies and duality arguments suggest the global symmetry of the CP^1 model is enhanced to SO(5). We perform a conformal bootstrap study of SO(5) invariant fixed points with one relevant SO(5) singlet operator, which would correspond to two relevant $\mathrm{SU}(2) \times \mathrm{U}(1)$ singlets, i.e., a tricritical point. We find that the bootstrap bounds are saturated by four different predictions from the large N computation of monopole operator scaling dimensions, which were recently shown to be very accurate even for small N. This suggests that the Neel to valence bond solid phase transition is described by this bootstrap bound, which predicts that the second relevant singlet has dimension ≈2.36.

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Introduction.—Deconfined quantum critical points (DQCPs) are second order phase transitions between one phase with symmetry group H, and a second phase with group H', where H' is not a subgroup of H [1,2]. These phase transitions go beyond the standard Landau-Ginzburg transitions, such as the Wilson-Fisher fixed points, where H would be a subgroup of H'. A striking feature of DQCPs is that they are described by continuum gauge theories in $2+1$ dimensions whose fields are not associated with quasiparticles on either side of the transition, i.e., they are deconfined. Despite many years of work, however, the existence of the simplest DQCP remains controversial.

The paradigmatic example of a DQCP is the transition between the Neel and valence bond solid (VBS) phases of quantum antiferromagnets on a 2D square lattice [3], where the Neel phase breaks an $\mathrm{SU}(2)$ symmetry, while the VBS phase breaks a different $\mathrm{U}(1)$ symmetry. In the continuum limit, the Neel-VBS phase transition is described by the 3D CP^1 model [4] with Lagrangian

$$\mathcal{L} = \sum_{i=1}^{2} [|(\nabla_\mu - iA_\mu)\phi^i|^2 + \lambda(|\phi^i|^2 - 1)], \qquad (1)$$

where ϕ_i are complex scalar fields, A_μ is an Abelian gauge field, and λ is a real scalar. The $\mathrm{SU}(2)$ symmetry rotates the ϕ_i, while the $\mathrm{U}(1)$ symmetry is generated by the current $\epsilon_{\mu\nu\rho}F^{\nu\rho}$, which is conserved due to the Bianchi identity [6]. This theory is strongly coupled, so it is hard to determine if it actually flows to a conformal field theory (CFT), i.e., if it describes a second order phase transition.

Instead of directly analyzing the continuum theory, lattice methods have been applied to various discrete quantum model that are believed to lie in the same universality class. These studies have produced a bewildering array of critical exponents [7–14], while others have claimed the transition is first order [15–18]. One notable lattice study suggested that the $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry is enhanced to SO(5) at the putative critical point [19], which was later attributed to possible quantum dualities [20].

Most lattice studies so far have tried to find a fixed point by tuning one parameter, i.e., they assumed that there was one relevant operator uncharged under $\mathrm{SU}(2) \times \mathrm{U}(1)$ [21,22]. The SO(5) symmetry enhancement would then imply there is no relevant SO(5) singlet [26], but this was also ruled out by the conformal bootstrap [27–29]. This led some to propose the theory is described by a weakly first order phase transition, caused by the merger and annihilation [31,32] of the CP^1 model with a related tricritical model. This scenario received recent support by the study of a theory with microscopic SO(5) symmetry using the fuzzy sphere method [33].

TABLE I. Comparison of scaling dimensions of the lowest dimension scalar operators in the singlet (s), vector (v), rank-2 (t), rank-3 (t_3), and rank-4 (t_4) of SO(5), as determined from the bootstrap study here, the large N expansion, the lattice study that claimed SO(5) symmetry [19], and the recent weakly first order fuzzy sphere study for a certain value of their coupling [33]. The asterisk by Δ_v for bootstrap means we put it in to determine the others.

	Δ_v	Δ_t	Δ_{t_3}	Δ_{t_4}	Δ_s
Bootstrap	0.630*	1.519	2.598	3.884	2.359
Large N	0.630	1.497	2.552	3.770	⋯
Lattice	0.630(3)	1.5	⋯	⋯	⋯
Fuzzy sphere	0.584	1.454	2.565	3.885	2.845

In this Letter, we will instead show evidence that the Neel-VBS phase transition is a tricritical fixed point, with two relevant singlets of $\mathrm{SU}(2)\times\mathrm{U}(1)$, or one relevant singlet of SO(5). We use the fact that the scaling dimension Δ_q of local operators in the CP^1 model with charge $q\in\mathbb{Z}/2$ under the $\mathrm{U}(1)$ symmetry, called monopole operators [34], can be computed to surprising accuracy using the large N expansion in the related CP^{N-1} model [35,36]. We review the evidence for this both from comparison to lattice studies for $q=1/2$ and various N [37,38], as well as for $N=1$ and various q [39] by comparing to the well-studied critical $O(2)$ model via particle-vortex duality [40,41].

We then perform a conformal bootstrap study of SO(5) invariant CFTs whose only relevant operators are the singlet s, rank-1 v, rank-2 t, and rank-3 t_3 scalars, as suggested by large N. The bootstrap rigorously bounds the space of allowed scaling dimensions of these operators, as well as of the irrelevant rank-4 scalar t_4. Physical theories often appear at the boundary of the allowed bootstrap region. In our case, by looking at the point on the boundary given by the large N value of Δ_v and then maximizing Δ_t, we can read off the values of all the other scaling dimensions. As shown in Table I, our results for Δ_t, Δ_{t_3}, and Δ_{t_4} after imposing Δ_v all match the large N estimates, and we also predict that $\Delta_s = 2.36$.

In Table I we also compare our results to other studies that found SO(5) symmetry: the original lattice study [19], and the recent fuzzy sphere paper [33] that starts from an SO(5)-invariant theory. While [19] did not report a relevant SO(5) singlet, since they assumed the $\mathrm{SU}(2)\times\mathrm{U}(1)$ theory was critical, the two scaling dimensions they did predict match ours [42]. Similarly, [33] argued that the theory is weakly first order, such that critical exponents depend on the value of the coupling, but nonetheless we find that their results are similar to ours for a certain value of their coupling [43], except that their singlet has slightly bigger dimension.

The rest of this Letter is organized as follows. In the next section, we review properties of the CP^{N-1} theory, including the large N expansion of monopole operators, and the SO(5) symmetry enhancement for $N=2$. In the "Numerical conformal bootstrap" section, we describe our bootstrap setup, how to numerically implement it using the Skydiving algorithm, and the resulting estimates for CFT data. We end with a discussion of our results.

The CP^1 model.—We now review the CP^1 model, first by generalizing to the CP^{N-1} model at large N, and then by discussing the conjectured SO(5) symmetry enhancement for $N=2$.

The CP^{N-1} model: We start with the Lagrangian of N complex scalar fields ϕ_i coupled to an Abelian gauge field A_μ in 3D:

$$\mathcal{L}=\sum_{i=1}^{N}[|(\nabla_\mu - iA_\mu)\phi^i|^2+m^2|\phi^i|^2]+u\left[\sum_{i=1}^{N}|\phi^i|^2\right]^2+\frac{F^2}{4e^2}, \tag{2}$$

where $F_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu$ is the field strength. At large N, we can tune $m^2=0$ to get a critical theory in the IR with $e,u\to\infty$, or tune both $m^2=u=0$ to get a tricritical theory in the IR with $e\to\infty$ [44]. The critical theory is called the CP^{N-1} model, since the ϕ^4 interaction can be written in terms of a Hubbard-Stratonovich field λ as in the $N=2$ case (1), in which case the theory is equivalent to a nonlinear sigma model with CP^{N-1} target space [5]. Both the critical and tricritical theories have an $\mathrm{SU}(N)$ flavor symmetry that rotates the ϕ_i, as well as a $\mathrm{U}(1)$ topological symmetry whose current $\epsilon_{\mu\nu\rho}F^{\nu\rho}$ is conserved due to the Bianchi identity [45].

Local operators that transform under $\mathrm{SU}(N)$ but not $\mathrm{U}(1)$ can be constructed from the fields in the action ϕ_i and A_μ, as well as λ for the critical theory. The scaling dimension of these operators can be computed at large N using standard Feynman diagrams [44,46–50], but the large N expansion is not very accurate for small N [51].

Local operators that transform under $\mathrm{U}(1)$ with charge $q\in\mathbb{Z}/2$ are not built from fields in the action, but instead these monopole operators are defined as inserting magnetic flux $q=(1/4\pi)\int F$ [34]. The lowest dimension monopoles are scalars and singlets under $\mathrm{SU}(N)$ that we denote as M_q. Their scaling dimensions Δ_q are identified via the state-operator correspondence with the ground state energies in the Hilbert space on $S^2\times\mathbb{R}$ with $4\pi q$ magnetic flux through the S^2, which can be computed at large N using a saddle point expansion [35,55]. This calculation was carried out to subleading order in [36], and the results were found to be extremely accurate even at small N. For instance, for $q=1/2$ we compare the large N estimates to lattice studies for $N=3$, 4, 5, 6 [37,38] in Table II. For $N=1$, the theory is dual to the O(2) Wilson-Fisher fixed point [40,41], so following [39] we can compare very precise bootstrap estimates of rank $2q$ operators in the

TABLE II. Comparison of lowest charge monopole scaling dimension $\Delta_{1/2}$ between large N and lattice studies for $N=3$, 4 [37] (JQ model) and $N=5$, 6 [38] (J_1-J_2 model).

$\Delta_{1/2}$	$N=3$	$N=4$	$N=5$	$N=6$
Lattice	0.785	0.865	1.00(5)	1.1(1)
Large N	0.755	0.880	1.01	1.13

critical O(2) model to M_q for low q [56], as we review in Table III.

SO(5) symmetry enhancement: We now specialize to $N=2$ and discuss the conjectured enhancement of the $\mathrm{SU}(2)\times\mathrm{U}(1)$ global symmetry to SO(5). In general, the rank-$2q$ symmetric traceless irrep of SO(5) includes charge $\pm q, \pm(q-1), \ldots$ irreps after decomposition to $\mathrm{SU}(2)\times\mathrm{U}(1)$. For instance, the vector 5 of SO(5) decomposes as

$$\mathbf{5}\to\mathbf{3}_0\oplus\mathbf{1}_{\pm 1/2}, \tag{3}$$

where $\mathbf{d}_q$ denotes the dimension d (isospin $[(d-1)/2]$) irrep of SU(2) with charge q under U(1), and $\mathbf{d}_{\pm q}$ means both $\mathbf{d}_q$ and $\mathbf{d}_{-q}$ appear. As discussed in [19,20], we thus see that the VBS order parameter $M_{1/2}$ combines with the Neel order parameter $\phi^i\phi_j^\dagger$, which is the lowest dimension scalar operator in the adjoint of SU(2), to form the lowest dimension vector operator v of SO(5).

Similarly, the rank-2 **14** of SO(5) decomposes as

$$\mathbf{14}\to\mathbf{5}_0\oplus\mathbf{1}_0\oplus\mathbf{3}_{\pm 1/2}\oplus\mathbf{1}_{\pm 1}. \tag{4}$$

Thus, the lowest dimension $\mathrm{SU}(2)\times\mathrm{U}(1)$ singlet scalar $\phi^i\phi_i^\dagger$ combines with M_1, the composite monopole operator $\phi^i\phi_j^\dagger M_{1/2}$, and a nonmonopole operator with isospin 2 to form the lowest dimension rank-2 operator t of SO(5). This also implies that $\Delta_{\phi^i\phi_i^\dagger}=\Delta_1$, where $\Delta_1\approx 1.497$ from large N.

The rank-3 **30** of SO(5) decomposes as

$$\mathbf{30}\to\mathbf{7}_0\oplus\mathbf{3}_0\oplus\mathbf{5}_{\pm 1/2}\oplus\mathbf{1}_{\pm 1/2}\oplus\mathbf{3}_{\pm 1}\oplus\mathbf{1}_{\pm 3/2}, \tag{5}$$

which implies that $M_{3/2}$ joins with a $q=1/2$ scalar monopole as well as other operators to form the lowest

TABLE III. Comparison of charge q monopole scaling dimension Δ_q computed at large N extrapolated to $N=1$, to values of the dual rank-$2q$ operators in the critical O(2) model as computed from the conformal bootstrap in [63].

Δ_q	$q=1/2$	$q=1$	$q=3/2$	$q=2$
O(2)	0.519 130 434	1.236 489 71	2.1086(3)	3.115 35(73)
Large N	0.506 09	1.1856	2.0087	2.9546

dimension rank-3 operator t_3 of SO(5). This $q=1/2$ scalar monopole cannot be the lowest dimension monopole $M_{1/2}$, because that was already used to form v, so it must be at least the second lowest dimension monopole $M'_{1/2}$ with $\Delta'_{1/2}=\Delta_{3/2}$. Since the large N estimate gives $\Delta_{3/2}\approx 2.55$, this implies that the third lowest dimension $q=1/2$ scalar monopole that would be used to form the second lowest SO(5) vector v' according to (3) must have an even bigger dimension, which strongly suggests that its irrelevant. An analogous argument suggests that the second lowest rank-2 t'_2 and rank-3 t'_3 must have dimensions bigger than Δ_2 and $\Delta_{5/2}$ with large N estimates 3.77 and 5.12 [36], respectively, which shows that t and t_3 are the only relevant operators in their irreps.

The rank-4 **55** of SO(5) decomposes as

$$\mathbf{55}\to\mathbf{9}_0\oplus\mathbf{5}_0\oplus\mathbf{1}_0\oplus\mathbf{7}_{\pm 1/2}\oplus\mathbf{3}_{\pm 1/2}\oplus\mathbf{5}_{\pm 1}\oplus\mathbf{1}_{\pm 1}\oplus\mathbf{3}_{\pm 3/2}\oplus\mathbf{1}_{\pm 2}, \tag{6}$$

which implies that M_2 joins with M'_1, a $\mathrm{SU}(2)\times\mathrm{U}(1)$ singlet, and other operators to form the lowest dimension rank-4 operator t_4 of SO(5). Large N gives $\Delta_2\approx 3.77$, so this singlet is irrelevant, while a similar argument suggests that singlets that appear in the decomposition of higher rank SO(5) operators are also irrelevant. If a second relevant $\mathrm{SU}(2)\times\mathrm{U}(1)$ singlet exists, then it must form the lowest dimension SO(5) singlet scalar s. In terms of fields, this second lowest singlet would be some linear combination of F^2 and λ^2, which have dimension 4 at $N\to\infty$.

The mixed irrep **35**$'$ of SO(5) decomposes as

$$\mathbf{35}'\to\mathbf{5}_0\oplus\mathbf{3}_0\oplus\mathbf{1}_0\oplus\mathbf{5}_{\pm 1}\oplus\mathbf{5}_{\pm 1/2}\oplus\mathbf{3}_{\pm 1/2}, \tag{7}$$

which includes an $\mathrm{SU}(2)\times\mathrm{U}(1)$ singlet. Since the lowest two such singlets were already used to form t and s, this means the lowest dimension operator $\mathcal{O}_{\mathbf{35}'}$ must have at least $\Delta_{\mathcal{O}_{\mathbf{35}'}}\geq\Delta_s$, and probably much higher. The last irrep we consider is the **35**, which decomposes as

$$\mathbf{35}\to\mathbf{5}_0\oplus\mathbf{3}_0\oplus\mathbf{3}_0\oplus\mathbf{5}_{\pm 1/2}\oplus\mathbf{3}_{\pm 1}\oplus\mathbf{3}_{\pm 1/2}\oplus\mathbf{1}_{\pm 1/2}, \tag{8}$$

which includes a $q=1/2$ scalar monopole. Since the lowest two such operators were already used to form v and t_3, this means the lowest dimension operator $\mathcal{O}_{\mathbf{35}}$ must have at least $\Delta_{\mathcal{O}_{\mathbf{35}}}\geq\Delta_{t_3}$, which suggests it is probably irrelevant.

Numerical conformal bootstrap.—We will now describe our numerical bootstrap study of the SO(5) invariant CFT. We consider four-point functions of the lowest dimension singlet s, vector v, and rank-2 t scalar operators of an SO(5) invariant CFT. Imposing that these correlators are invariant under permuting the four operators leads to crossing equations, which constrain the scaling dimensions and operator product expansion (OPE) coefficients that appear in the OPEs of these correlators. In our case, we computed the crossing equations using the general $O(N)$ code [64] from the project of [65], which yields 29 crossing equations. More details about the derivation of these crossing equations can be found in the Supplemental Material [66], while the explicit crossing vectors can be found in the attached Mathematica file.

We bootstrap this system by truncating the 29 crossing equations, rephrasing them as a semidefinite program as in [67], which crucially assumes unitarity, and then solving these constraints efficiently using SDPB [68]. We assume that s, v, t, and t_3 are the only relevant scalar operators in any of the irreps that appear in the OPEs of the correlators we consider, which is supported by the large N analysis of the previous section. For Δ_s and $\Delta_{\mathbf{35}'}$, we put the slightly stronger gap 4 to improve numerical stability. We also impose that s, v, t are unique [69] by scanning over the four ratios of their OPE coefficients as in [63]. The output of the bootstrap is an allowed region in the eight-dimensional space $\{\Delta_v, \Delta_s, \Delta_t, \Delta_{t_3}, (\lambda_{sss}/\lambda_{vvt}), (\lambda_{tts}/\lambda_{vvt}), (\lambda_{vvs}/\lambda_{vvt}), (\lambda_{ttt}/\lambda_{vvt})\}$.

Since this is a very large space, it would be computationally infeasible to map out the entire region, and we do not find any evidence that the allowed region is a small island. Instead, our hope is that the physical theory lies on the boundary of the allowed region, as was the case for bootstrap studies of many theories such as the critical $O(N)$ models [70,71] and QED_3 with four fermions [72]. Our strategy is to look at the point on the boundary of this eight-dimensional space given by imposing the value $\Delta_v \approx 0.63$ that was predicted from the large N analysis, maximizing Δ_t, and then reading off the CFT data given by the approximate solution to the bootstrap equations at that boundary point [70]. We can do this using the recently developed Skydive method [73], which is currently the most efficient way of finding the boundary of the allowed region. Even with this method, it takes over a week for the bootstrap to converge with bootstrap accuracy $\Lambda = 19$ (See Supplemental Material [66] for more details). The resulting scaling dimensions of all lowest dimension scalar operators up to rank 4, which is the highest rank we can access from our setup, are given in Table I. Other CFT data, such as ratios of OPE coefficients, are summarized in the Supplemental Material [66].

Discussion.—In this Letter, we showed that a point on the boundary of the allowed region of CFTs with SO(5) symmetry corresponds to the large N estimate of scaling dimensions of monopole operators in the CP^1 model. In particular, by inputting Δ_v, we found that Δ_t, Δ_{t_3}, and Δ_{t_4} matched their large N values. We also made a prediction for a relevant SO(5) singlet $\Delta_s \approx 2.34$, which implies that the CP^1 theory is a tricritical fixed point in terms of $SU(2) \times U(1)$ with relevant singlets: Δ_s and $\Delta_t \approx 1.499$. It would be interesting to understand what happens to this putative SO(5) invariant CFT when it is perturbed by this scalar, and to determine the resulting phase diagram [74].

Curiously, aside from Δ_s our results are very similar to those of the recent fuzzy sphere model [33], which claimed to observe a weakly first order transition. Recent lattice studies has also suggested the CP^{N-1} model stops being critical below $N_c \approx 7$ [15] or $4 \leq N_c < 10$ [18], perhaps because the critical and tricritical theories merge and go off into the complex plane. This is in some tension with the previous match between large N and lattice studies for $N = 3, 4, 5, 6$ [37,38], as well as the fact that the $N = 1$ theory is widely believed to be critical due to particle-vortex duality. A possible resolution is that these theories become tricritical below N_c. Indeed, a recent lattice study suggested that the $N = 2$ theory is tricritical [23], but reported some different critical exponents than us and did not discuss an enhanced SO(5) [75]. A possible resolution is that the critical and tricritical theories reemerge from the complex plane to become unitary CFTs below N_c. This might also explain why the large N results for Δ_q in the CP^{N-1} model, which is critical at large N, seem to match a tricritical theory at small N as the analytic continuation in N for Δ_q might switch between the critical and tricritical theories below N_c [77]. It would be interesting if the bootstrap could be used to see the merger of the critical and tricritical theories as a function of real N, just as the merger and annihilation of the critical and tricritical three-state Potts model was recently seen using the bootstrap as a function of dimension $2 < d < 3$ [78].

Looking ahead, we would like to improve our bootstrap study so that we can find a rigorous island around the large N values. One way might be to bootstrap a system of correlators including the relevant rank-3 scalar t_3, as bootstrapping all relevant operators drastically improved bounds in other cases such as the critical $O(2)$ and $O(3)$ models [63,79]. This would also give us access up to rank-6 operators, which could then be compared to the large N predictions corresponding monopole operators.

It would also be interesting to resolve a related tension between the lattice results for critical QED_3 with $N = 2$ fermions [80–82], whose $SU(2) \times U(1)$ symmetry was conjectured to enhance to $O(4)$ [20,83–85], and bootstrap bounds that ruled out these estimates [86]. As in the SO(5) case discussed here, the conflict with bootstrap can be avoided if we assume that $N = 2$ QED_3 is tricritical. The large N estimate for monopole operators has also been shown to be accurate at least for $N = 4$, where there are independent bootstrap results [72,87].

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