Solvable Model of Quantum-Darwinism-Encoding Transitions

Benoît Ferté [1] and Xiangyu Cao [2]
[1] *Université Paris-Saclay, CNRS, LPTMS, 91405 Orsay, France*
[2] *Laboratoire de Physique de l'École normale supérieure, ENS, Université PSL, CNRS, Sorbonne Université, Université Paris Cité, F-75005 Paris, France*

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We propose a solvable model of quantum Darwinism to encoding transitions—abrupt changes in how quantum information spreads in a many-body system under unitary dynamics. We consider a random Clifford circuit on an expanding tree, whose input qubit is entangled with a reference. The model has a quantum Darwinism phase, where one classical bit of information about the reference can be retrieved from an arbitrarily small fraction of the output qubits, and an encoding phase where such retrieval is impossible. The two phases are separated by a mixed phase and two continuous transitions. We compare the exact result to a two-replica calculation. The latter yields a similar "annealed" phase diagram, which applies also to a model with Haar random unitaries. We relate our approach to measurement-induced phase transitions (MIPTs), by solving a modified model where an environment eavesdrops on an encoding system. It has a sharp MIPT only with full access to the environment.

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Introduction.—A pillar of modern quantum statistical mechanics [1–3] is the idea that unitary dynamics in a many-body system generically scrambles local quantum information. Eventually, it becomes highly nonlocal and impossible to retrieve, unless the observer has access to more than half of the system: The information has been encoded [4–7]. Information scrambling and encoding have far-reaching consequences, for example, on the quantum physics of black holes [8–13].

Meanwhile, a basic premise of quantum Darwinism (QD) [14–19] is that a macroscopic environment, e.g., a measurement apparatus, *duplicates* some classical information. Hence, the latter becomes retrievable in multiple small fractions of the environment. It is important to view the environment itself as a many-body quantum system. Indeed, the theory of QD aims to deduce the properties of the classical world from the core principles of quantum physics. According to QD, the duplication of information underlies the emergence of classical *objectivity* [20–23]: Being objective is being known to many.

Quantum Darwinism and encoding are distinct ways of many-body quantum information spreading. Both behaviors emerge from the microscopic laws of quantum mechanics, just like both ferro- and paramagnetism can emerge from the Ising model. Ferro- and paramagnetism are distinct phases of matter, separated by a continuous phase transition. Can we view QD and encoding as stable phases of quantum information, and are they separated by some transition [24,25]? In this Letter, we propose a solvable model of sharp phase transitions from QD to encoding. Our model is a random Clifford unitary circuit on an expanding tree, whose root forms a maximally entangled pair with a reference qubit [Fig. 1(a)]. It has one parameter, analog of the temperature in the Ising model. We then ask whether it is possible to retrieve information about the reference bit from a small fraction $f < 1/2$ of the tree's leaves (output qubits). We determine exactly the model's

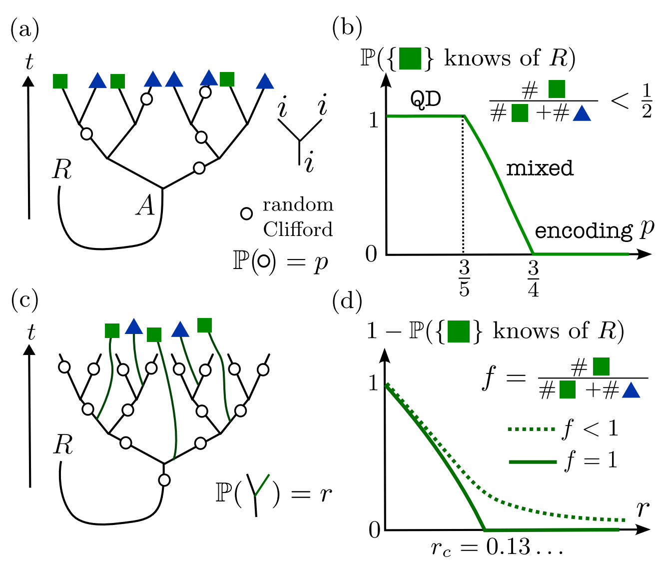
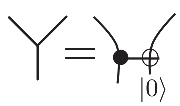


FIG. 1. (a) Model for quantum-Darwinism-encoding transitions on an expanding tree with $t = 3$ generations. (b) Information on R is accessible to a small subsystem (squares) in the QD phase and inaccessible in the encoding phase. In the mixed phase, the information is accessible in a fraction of random realizations. (c) A tree model of an environment eavesdropping on an encoding dynamics. (d) A transition is possible only with full access to the environment $f = 1$.

phase diagram [Fig. 1(b)]. It has a stable QD (encoding, respectively) phase, where one may (may not, respectively) extract a classical bit of information about the reference bit. Unlike the Ising model, the encoding and QD phases are separated by an intermediate mixed phase and two continuous transitions.

Another inspiration for this work is the measurement-induced phase transitions (MIPTs) [26–37], which are also "quantum information transitions." In the standard setup, a generic many-body unitary evolution is continually interrupted by local measurements. By tuning the measurement rate, one obtains a transition between a phase with volume-law entanglement entropy and one with area law. The MIPTs concern entanglement properties of random states drawn from the Born rule and are delicate to study and observe [38–40]. Here, we consider a "Darwinian" MIPT setup; see Figs. 1(c) and 1(d). We amend our model in the encoding phase with eavesdropping qubits [41] and ask whether they can extract a classical bit of information about the reference [34,42–45]. We show that a sharp transition occurs at a critical rate of eavesdropping, if and only if one has access to all the eavesdropping bits.

Model for QD-encoding transition.—Consider a maximally entangled pair $(|0\rangle_R|0\rangle_A + |1\rangle_R|1\rangle_A)/\sqrt{2}$ between a reference qubit R that will be kept intact and the qubit A that will be the root of an expanding binary tree unitary circuit; see Fig. 1. The edges of the tree represent the world lines of the qubits constituting a growing system [46,47]. At each branching, we recruit a new qubit with state $|0\rangle$ and apply a CNOT gate to it and the input qubit:



(1)

Equivalently, the branching acts on the input qubit as an isometry $\sum_{i=0,1} |ii\rangle\langle i|$. In addition, we apply a random one-body Clifford unitary (drawn uniformly) to each edge of the tree with probability p, which is the parameter that interpolates between the QD ($p = 0$) and encoding limits ($p = 1$). After t time steps, there are $N = 2^t$ output qubits, from which we draw the subsystem F randomly: Each output qubit belongs to F with probability f. We denote by U the resulting unitary from A and $N - 1$ recruits to the N output qubits. By construction, U is a Clifford unitary, which can be efficiently simulated [48,49]. Here, we can analyze the knowledge of F on R analytically [50].

For this, we recall the defining property of a Clifford unitary: It transforms any Pauli operator to a *single* product of Paulis, known as a Pauli string. For example, a one-body Clifford unitary permutes X, Y, and Z, and choosing a random one-body Clifford amounts to picking one among the six permutations (here and below, a Pauli string will be always considered modulo a phase ± 1, $\pm i$). Now, let us fix a realization of our model and consider a Pauli string P acting on the subsystem F. By definition, our Clifford unitary U will pull it back to $Q = U^\dagger P U$, a Pauli string acting on A and the $N - 1$ recruits. We then contract it with the recruit states $(|0\rangle\langle 0|)^{\otimes N-1}$ to obtain a Pauli operator O_A acting on A. There are two possibilities: (i) If Q contains an X or Y acting on some recruit bit, O_A vanishes. (ii) Otherwise, $O_A \in \{I, Z, X, Y\}$ is identity or a Pauli.

Repeating this for all Pauli strings acting on F, we construct a set $\mathbf{s} \subset \{I, X, Y, Z\}$ of all the nonzero operators O_A thus obtained. It is not hard to see that $\mathbf{s}$ is a subgroup of $\{I, X, Y, Z\}$ (modulo phase); i.e., $\mathbf{s}$ must equal one of these:

$$\mathbf{n} = \{I\}, \qquad \mathbf{z} = \{I, Z\}, \qquad \mathbf{x} = \{I, X\}, \\ \mathbf{y} = \{I, Y\}, \qquad \mathbf{a} = \{I, Z, X, Y\}. \tag{2}$$

Since RA is initially a maximally entangled pair, $\mathbf{s}$ tells us exactly what information about R is accessible from F. If $\mathbf{s} = \mathbf{n}$, F is uncorrelated with R. If $\mathbf{s} = \mathbf{z}$, $\mathbf{x}$, or $\mathbf{y}$, F contains one classical bit of information on R: Some Pauli string O_F on F is perfectly correlated with $O_R = Z, X$, or Y on R. More precisely, $O_F O_R$ is a stabilizer of the output state Ψ_t: $O_F O_R|\Psi_t\rangle = \pm|\Psi_t\rangle$. If $\mathbf{s} = \mathbf{a}$, one may distill from F a qubit maximally entangled with R [50].

Phase diagram.—The "order parameter" of our model is, thus, the probability distribution of $\mathbf{s}$:

$$\pi := (\pi_\mathbf{n}, \pi_\mathbf{z}, \pi_\mathbf{x}, \pi_\mathbf{y}, \pi_\mathbf{a}), \tag{3}$$

where $\pi_\mathbf{n}$ is the probability that $\mathbf{s} = \mathbf{n}$, and so on. We can compute π of a tree with t generations from one with $(t - 1)$ using a "backward recursion" relation. The phase diagram of the model is determined by iterating this relation and analyzing the $t \to \infty$ limit of π as a function of p (and f) [50]. As a result, we find three phases; see Fig. 1(b) for a sketch and Fig. 2 for plots. When $p < 3/5$, we have a QD phase, where, for any $f \in (0, 1)$, we have $\pi_\mathbf{a} \to 0$, $\pi_\mathbf{n} \to 0$, and

$$\pi_\mathbf{z} \to \frac{3 - 6p + \sqrt{24(p-1)p + 9}}{6 - 6p}, \quad \pi_{x,y} \to \frac{1 - \pi_\mathbf{z}}{2}. \tag{4}$$

($\pi_\mathbf{z} \to 1$ as $p \to 0$). When $p > 3/4$, we have an *encoding* phase, where $\pi_\mathbf{n} \to 1$ if $f < 1/2$ and $\pi_\mathbf{a} \to 1$ if $f > 1/2$. Finally, when $3/5 < p < 3/4$, we have a mixed phase. For any $f < 1/2$, we have $\pi_\mathbf{a} \to 0$ while

$$(\pi_\mathbf{n}, \pi_\mathbf{z}, \pi_\mathbf{x}, \pi_\mathbf{y}) \xrightarrow{f < \frac{1}{2}} \left(1 - u, \frac{u}{2}, \frac{u}{4}, \frac{u}{4}\right), \qquad u = \frac{6 - 8p}{3 - 3p}. \tag{5}$$

Here, u is probability that we can retrieve one classical bit from the subsystem F, and it decreases from 1 to 0 as p varies from $3/5$ to $3/4$. The solution for $f > 1/2$ is obtained from (5) by swapping $\pi_\mathbf{n}$ and $\pi_\mathbf{a}$.

The existence of the two transitions, at $p = 3/5$ and $p = 3/4$, respectively, where π is nonanalytical, can be associated to the breaking or restoration of two symmetries of the model. First, a $\mathbb{Z}_2$ symmetry acts by exchanging $\pi_\mathbf{n} \leftrightarrow \pi_\mathbf{a}$ or swapping the subsystem F and its complement (without R) [51]. This symmetry is preserved by the circuit dynamics, weakly broken by the "boundary condition" (the choice of F), and restored only in the QD phase. Second, a $\mathcal{S}_3$ symmetry acts by permuting $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ (while leaving $\mathbf{n}$ and $\mathbf{a}$ invariant). This symmetry is preserved by the random one-body Clifford unitary, broken by the branching (1), and restored only in the encoding phase. The mixed phase breaks both symmetries. We numerically explored a few other Clifford variants of our model and found the above two-stage scenario to be rather general [52].

Mutual information and discord.—It is useful to consider the mutual information between F and R, defined as $I(R,F) = H(R) + H(F) - H(RF)$, where $H(X) = -\mathrm{Tr}[\rho_X \log_2 \rho_X]$ is the von Neumann entropy. In our model, it is not hard to see that $I(R,F) = \log_2 |\mathbf{s}|$ is the dimension of $\mathbf{s}$ as a vector space over $\mathbb{Z}_2$ [50]. So, in the QD phase,

$$I(R,F) \to 1 \qquad (0 < f < 1) \qquad (\text{QD}) \tag{6}$$

with probability one [Fig. 2(c)]. The independence of I on the fraction size f, sometimes called the "objectivity plateau," is a hallmark of QD [16]. Meanwhile, in the encoding phase,

$$I(R,F) \to \begin{cases} 0 & f < 1/2 \\ 2 & f > 1/2 \end{cases} \qquad (\text{encoding}) \tag{7}$$

with probability one [Fig. 2(d)], as expected from the Page curve [53]. In the mixed phase, we may wonder what the I-f curve looks like in a *single* realization (with large t), where we increase f by gradually adding random qubits into F. To address this question, we computed the joint distribution of $(\mathbf{s},\mathbf{t})$ corresponding to two random subsystems $F \subset G$ and a same unitary U [50]. As a result, we found that a single-realization I-f curve is exactly the QD one (6) with probability u defined in (5) and exactly the encoding curve (7) with probability $1-u$. In other words, the intermediate-phase ensemble is a mixture of QD and encoding realizations, both occurring with nonzero probability in the $t \to \infty$ limit.

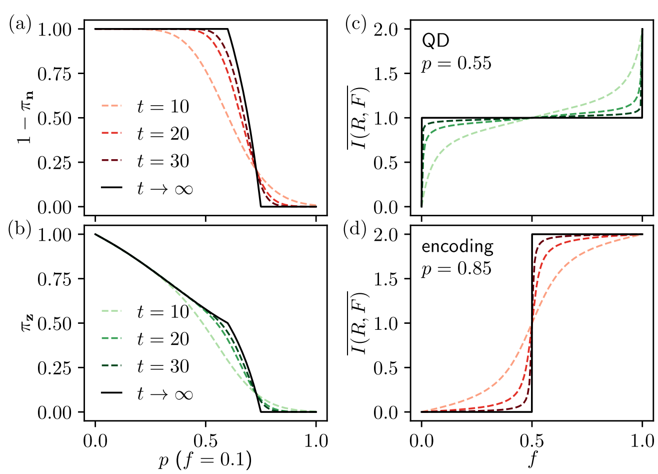


FIG. 2. (a),(b) p dependence of the order parameters $1-\pi_\mathbf{n}$ [identical to Fig. 1(b)] and $\pi_\mathbf{z}$. (c),(d) Averaged mutual information $\overline{I(R,F)}$ as a function of the size fraction f in the QD and encoding phase (respectively). The average is over the random Clifford unitary and the random subset F. The finite t data are from numerical iteration of the backward recursion, and the $t = \infty$ curves are the exact prediction [50].

In general, the mutual information between F and R does not correspond exactly to the amount of information that one can learn about R by observing F [54,55]. The discrepancy is known as "quantum discord." Here, the discord vanishes whenever $I(R,F) = 1$, given the knowledge of the unitary circuit: We can construct the observable on F which reveals the classical bit of information on F. Moreover, we can show that, in the QD phase, one may still retrieve a bit of information from R even with access to only the Z operators on F.

Two-replica analysis.—A valuable tool to compute quantum information quantities is the "replica trick" [56–61]. Yet, results of replica calculations can be subtle to interpret, especially if one is not able to take the appropriate replica number limit. Here, we perform a two-replica analysis of our model and compare the result with the exact phase diagram.

In the replica approach, the accessible quantity is the "annealed" mutual information

$$I^{(2)}(F,R) \coloneqq \log_2 \mathrm{Tr}\left[\overline{\rho_{FR}^2}\right] - \log_2 \mathrm{Tr}\left[\overline{\rho_F^2}\right] + 1, \tag{8}$$

where $\overline{[\ldots]}$ denotes an average over U and F. Note that $I^{(2)}$ would equal to the average von Neumann mutual information if $\mathrm{Tr}[\overline{\rho_X^2}]$ were equal to $2^{-\overline{H(X)}}$ (which is wrong). The annealed mutual information can be computed by random unitary circuit techniques [37,51,62–64]; indeed, since the Clifford group is a 2-design [65], $I^{(2)}(F,R)$ will not change if we replace a random one-body Clifford unitary with a Haar-random one in $U(2)$. We find [50]

$$I^{(2)}(F,R) \to \begin{cases} 0 & f < 1/2, p > p_c(f), \\ 2 & f > 1/2, p > p_c(f), \\ 1 & p < p_c(f). \end{cases} \tag{9}$$

Here, $p_c(f) = p_c(1-f)$ is a threshold function that increases from $p_c(0) = 3/4$ to $p_c(1/2) = \frac{3}{7}(2\sqrt{2}-1) = 0.783\ldots$; see Fig. 3.

The "annealed phase diagram" of $I^{(2)}$ is similar to the exact one, with, however, differences: $I^{(2)}(F,R) = 1$ in both QD and mixed phases as well as a small part of the encoding phase. So, the annealed phase diagram is biased

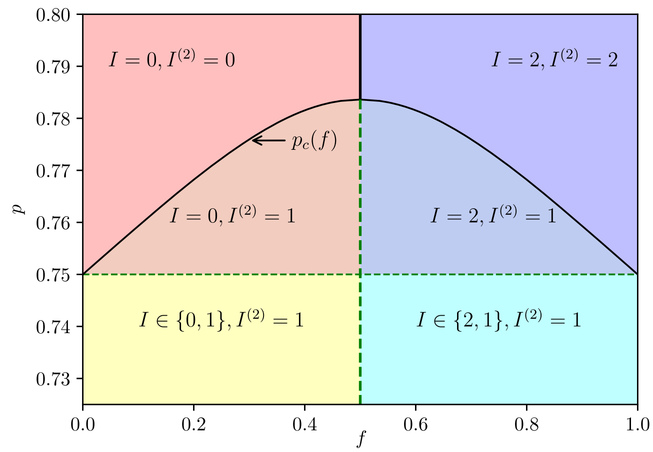


FIG. 3. Comparing the annealed mutual information $I^{(2)}(F, R)$ (9) with the genuine one $I(F, R)$. They disagree in the mixed phase ($3/5 < p < 3/4$) and in part of the encoding phase where $3/4 < p < p_c(f)$. $p_c(f)$ (solid curves) is determined numerically using the recursion relation for $I^{(2)}$ [50].

toward QD, which we qualitatively explain as follows. Both purity averages in (8) are dominated by realizations with small entanglement entropy in F. Now, QD states tend to have low entanglement; indeed, the "perfect" QD state (produced at $p = 0$) is the Greenberger-Horne-Zeilinger (GHZ) state [66]:

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0_R\underbrace{0\ldots0}_{F}0\ldots0\rangle + |1_R\underbrace{1\ldots1}_{F}1\ldots1\rangle).$$

It has one bit of entanglement entropy for any bipartition. In comparison, an encoding state has a volume law entropy. Hence, in both QD and mixed phases, QD realizations will dominate $I^{(2)}$, which fails to distinguish them. In the encoding phase, a QD realization occurs with an exponentially small (in t) probability, yet its $\mathrm{Tr}[\rho_F^2]$ and $\mathrm{Tr}[\rho_{FR}^2]$ can be exponentially large compared to the typical encoding states. Hence, rare QD states in the encoding phase can dominate the annealed mutual information.

Relating to MIPT.—The QD-encoding transitions (QDETs) differ from the MIPTs in two ways. First, MIPTs result from the competition between a scrambling system and its environment (the measurement apparatus). Meanwhile, QDETs take place within a *structured* environment [67]. In the QD phase, the environment behaves as a macroscopic apparatus that "measures" the reference spin in some direction (the direction is Z with probability $\pi_{\mathbf{z}}$, and so on) and broadcasts the outcome. As we tune the apparatus into the encoding phase, it becomes dysfunctional and fails to broadcast any information on the reference system.

Second, QDETs are about the information available in small environment fractions, while MIPTs are observable only with full access to the environment. To support this claim, we consider a variant of our model that mimics the MIPT setup. We take the above model at $p = 1$ (in the encoding phase) and let every qubit in the tree be subject to an eavesdropping event with probability r. The eavesdropping consists again as a branching (1), of which one output bit is then emitted to the "environment"; see Fig. 1(c). After t generations, we have a system with $N = 2^t$ bits and an environment E of average size $|E| = (2N - 1)r$.

Then we ask: Can we retrieve information on R from a fraction F of the *environment*, with $|F|/|E| = f$? Moreover, we allow access only to Z operators on F (allowing access to all operators results in an entirely different phase diagram [52]). Then, the order parameter (3) obeys a modified recursion relation [50]. In particular, $\pi_{\mathbf{a}} = 0$, and the probability of retrieving one classical bit equals $1 - \pi_{\mathbf{n}}$. We find that, when $f = 1$, there is a transition:

$$\pi_{\mathbf{n}} \xrightarrow{f=1} \begin{cases} \frac{4r^2-8r+1}{1-r} & r < r_c \\ 0 & r > r_c, \end{cases} \tag{10}$$

where $r_c = \frac{1}{2}(2 - \sqrt{3}) \approx 0.134$. This transition is equivalent to the standard MIPT. Indeed, consider projectively measuring Z on all the qubits of F. If $\mathbf{s} = \mathbf{n}$, the measurements reveal nothing about R, which remains entangled with unmeasured bits. Otherwise, if, say, $\mathbf{s} = \mathbf{x}$, the measurements will project the qubit R to an eigenstate of X, disentangling it. Therefore, $r > r_c$ is the area-law (purified) phase and $r < r_c$ the volume-law (encoded) phase [34,42,44,45]. Note that the transition exists *only* at $f = 1$, where almost all the environment is accessible. For any $f < 1$, $\pi_{\mathbf{n}}(t \to \infty)$ depends smoothly on r and never vanishes. This is, after all, reasonable from the MIPT point of view: We need all the measurement outcomes to construct the quantum trajectory state.

Outlook.—We introduced a solvable model for QDETs. They are a new type of quantum information phase transitions under unitary evolution, where the different phases are characterized by whether information about the reference qubit is retrievable from small fractions of the environment. It will be interesting to identify QDETs in finite-dimensional ($d < \infty$) systems and characterize their universality classes; our tree model is equivalent to an all-to-all ($d = \infty$) circuit and has simple mean-field critical exponents [68]. In particular, it may be nontrivial to establish a QD phase in a $d < \infty$ geometry, which hinders the fast spread of information [69–71]; an expanding (de Sitter) geometry could be necessary. Another important question concerns QDETs in non-Clifford models [24,41,72], in particular, whether the mixed phase is generic. Indeed, the knowledge of F on R is, in general, not "quantized" as in a Clifford model. This will affect the nature of the order parameter and make even the mean-field theory more involved [46,47,73]. Finally, encoding is proper to the quantum realm, and quantum Darwinism is a theory of the emergence of the classical. Thus, we hope to shed light on the quantum-classical transition through the lens of dynamical critical phenomena.

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