Erratum: Linear Optical Quantum Computation with Frequency-Comb Qubits and Passive Devices [Phys. Rev. Lett. 130, 200602 (2023)]

Tomohiro Yamazaki, Tomoaki Arizono, Toshiki Kobayashi, Rikizo Ikuta, and Takashi Yamamoto

(Received 18 December 2023; published 8 March 2024)

DOI: 10.1103/PhysRevLett.132.109902

In our Letter, we proved that our tool box [time-frequency Gottesman-Kitaev-Preskill (TFGKP) qubit generators, time-resolving detectors, optical interleavers, and beam splitters] is sufficient for universal quantum computation. However, one component in the proof, "type-I' fusion gate" needs to be corrected. In this Erratum, we introduce another linear optical circuit that performs the required operation correctly, that is, enables us to generate a three-qubit cluster state from two Bell states. Therefore, the proof is recovered by redefining the new circuit shown in Fig. 1 as the type-I' fusion gate. The corrected type-I' fusion gate requires additional two ancilla photons, and its success probability is $1/8$ [1]. This replacement does not affect the error analysis because all the detectors used in the corrected type-I' fusion gate are time-resolving detectors. The advantages of our scheme, such as high error robustness and ease of operations due to the use of time-frequency degree of freedom and passive devices, are not compromised with this correction.

As a consequence, Fig. 3(e) is replaced with Fig. 1 in this erratum and, in the caption of Fig. 3, the sentence "Type-I' fusion gate, which succeeds when a detector detects a photon with a probability of $1/4$" is replaced with "Type-I' fusion gate, which succeeds when one of the two left detectors, one of the two right detectors, and the bottom center detector detect a photon, respectively, with a probability of $1/8$." In addition, the sentence "Thus, we additionally introduce type-I' gate… with a success probability of $1/4$." in the main text is replaced with "Thus, we additionally introduce type-I' gate… with a success probability of $1/8$."

In the following, we explain how the corrected fusion gate works in detail. The corrected type-I' fusion gate is the composition of two types of fusion gates in Figs. 2(a) and 2(b) which we call frequency-path fusion gate and path-frequency fusion gate, respectively. We represent states as Fock states with the frequency index (in terms of frequency basis of the

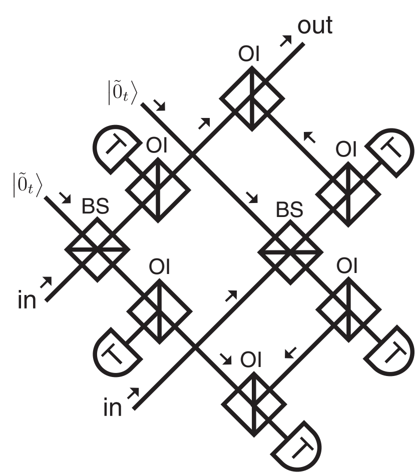

FIG. 1. Type-I' fusion gate, which succeeds when one of the two left detectors, one of the two right detectors, and the bottom center detector detect a photon, respectively, with a probability of $1/8$.

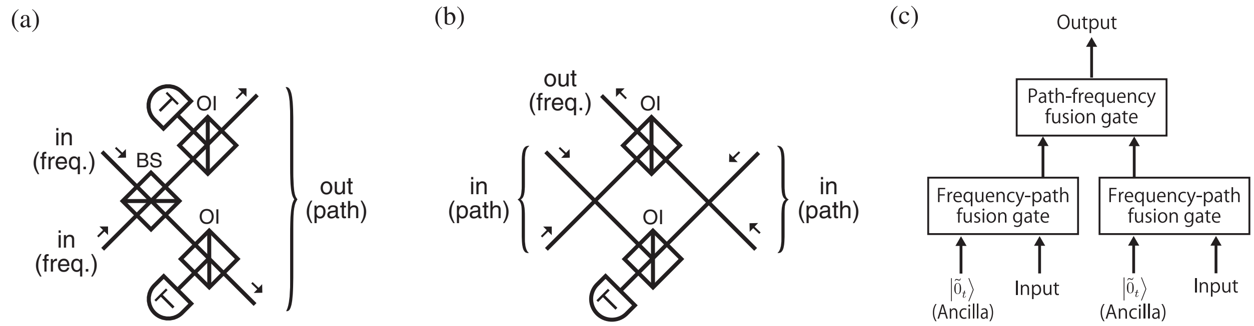

FIG. 2. Configuration of corrected type-I' fusion gate. All detectors, beam splitters (BSs), and optical interleavers (OIs) are time-resolving detectors, 50∶50 BSs, and 2∶2 OIs, respectively. (a) Frequency-path fusion gate, which succeeds when one of the detectors detect a photon with probability 1/2. (b) Path-frequency fusion gate, which succeeds when the detector detects a photon with probability 1/2. (c) Construction of corrected type-I' fusion gate from two frequency-path fusion gates, a path-frequency fusion gate, and two single photon ancillae.

TFGKP qubit) and the path index, where the $2k + r$th mode represents the rth frequency mode ($r = 0, 1$) and the kth path mode ($k = 0, 1, \ldots$). We assume that the transfer matrix of the 50∶50 beam splitter is the Hadamard matrix and, as in Fig. 2 in the Letter, the optical interleavers reflect the zeroth frequency mode and transmit the first frequency mode. We represent the states of the path-encoded qubit with the zeroth (first) frequency mode by $|0_p^0\rangle = |1000\rangle$ and $|1_p^0\rangle = |0010\rangle$ ($|0_p^1\rangle = |0100\rangle$ and $|1_p^1\rangle = |0001\rangle$).

In the frequency-path fusion gate shown in Fig. 2(a), the inputs are two TFGKP qubits, and the output is a path-encoded qubit with the first frequency mode (where path index is $k = 0, 1$). Especially, we consider the input states $|\tilde{0}_f\rangle|\tilde{0}_f\rangle$, $|\tilde{1}_f\rangle|\tilde{1}_f\rangle$, and $(|\tilde{0}_f\rangle|\tilde{1}_f\rangle \pm |\tilde{1}_f\rangle|\tilde{0}_f\rangle)/\sqrt{2}$. After the 50∶50 beam splitter, these states are transformed as

$$|\tilde{0}_f\rangle|\tilde{0}_f\rangle = |1010\rangle \to (|2000\rangle - |0020\rangle)/\sqrt{2}, \qquad |\tilde{1}_f\rangle|\tilde{1}_f\rangle = |0101\rangle \to (|0200\rangle - |0002\rangle)/\sqrt{2}, \tag{1}$$

$$(|\tilde{0}_f\rangle|\tilde{1}_f\rangle + |\tilde{1}_f\rangle|\tilde{0}_f\rangle)/\sqrt{2} = (|1001\rangle + |0110\rangle)/\sqrt{2} \to (|1100\rangle - |0011\rangle)/\sqrt{2}, \tag{2}$$

$$(|\tilde{0}_f\rangle|\tilde{1}_f\rangle - |\tilde{1}_f\rangle|\tilde{0}_f\rangle)/\sqrt{2} = (|1001\rangle - |0110\rangle)/\sqrt{2} \to (|1001\rangle - |0110\rangle)/\sqrt{2}. \tag{3}$$

Then, each path mode is spatially separated by a 2∶2 optical interleaver, and the zeroth frequency mode is detected by a time-resolving detector. When one of the detectors detects two or no photons in Eq. (1), this fusion gate fails because the resulting quantum operation is represented by the Kraus operator $\langle\tilde{0}_f|\langle\tilde{0}_f|$ or $\langle\tilde{1}_f|\langle\tilde{1}_f|$. On the other hand, when one of the detectors detects a photon in Eqs. (2) and (3), this fusion gate succeeds. The resulting quantum operations by the frequency-path fusion gate are represented by the Kraus operators

$$|0_p^1\rangle(\langle\tilde{0}_f|\langle\tilde{1}_f| + \langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2} + |1_p^1\rangle(\langle\tilde{0}_f|\langle\tilde{1}_f| - \langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2} = |+_p^1\rangle\langle\tilde{0}_f|\langle\tilde{1}_f| + |-_p^1\rangle\langle\tilde{1}_f|\langle\tilde{0}_f|, \tag{4}$$

$$-|1_p^1\rangle(\langle\tilde{0}_f|\langle\tilde{1}_f| + \langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2} - |0_p^1\rangle(\langle\tilde{0}_f|\langle\tilde{1}_f| - \langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2} = -|+_p^1\rangle\langle\tilde{0}_f|\langle\tilde{1}_f| + |-_p^1\rangle\langle\tilde{1}_f|\langle\tilde{0}_f|, \tag{5}$$

where $|\pm_p^1\rangle = (|0_p^1\rangle \pm |1_p^1\rangle)/\sqrt{2}$. In the same way, we can define the frequency-path fusion gate whose output is a path-encoded qubit with the zeroth frequency mode. Therefore, the frequency-path fusion gate enables us to generate a three-qubit cluster state from two Bell states, except for the difference in the encodings of the input and output qubits.

In the path-frequency fusion gate shown in Fig. 2(b), the inputs are two path-encoded qubits whose frequency modes are different (where path index is $k = 0, 1, 2, 3$), and the output is a TFGKP qubit (where path index is $k = 0$). Letting the frequency mode of the first (second) qubit be 0 (1), we consider the input states $|0_p^0\rangle|0_p^1\rangle$, $|1_p^0\rangle|1_p^1\rangle$, $(|0_p^0\rangle|1_p^1\rangle \pm |1_p^0\rangle|0_p^1\rangle)/\sqrt{2}$. They are transformed by the two optical interleavers as

$$|0_p^0\rangle|0_p^1\rangle = |10000100\rangle \to |11000000\rangle, \qquad |1_p^0\rangle|1_p^1\rangle = |00100001\rangle \to |00110000\rangle \tag{6}$$

$$(|0_p^0\rangle|1_p^1\rangle + |1_p^0\rangle|0_p^1\rangle)/\sqrt{2} = (|10000001\rangle + |00100100\rangle)/\sqrt{2} \to (|10010000\rangle + |01100000\rangle)/\sqrt{2} \tag{7}$$

$$(|0_p^0\rangle|1_p^1\rangle - |1_p^0\rangle|0_p^1\rangle)/\sqrt{2} = (|10000001\rangle - |00100100\rangle)/\sqrt{2} \to (|10010000\rangle - |01100000\rangle)/\sqrt{2}. \quad (8)$$

Then, the zeroth path mode is measured by the time-resolving detector. The transformation from the frequency basis to the time basis corresponds to virtually applying the 50:50 beam splitter as

$$|11000000\rangle \to (|20000000\rangle - |02000000\rangle)/\sqrt{2}, \qquad |00110000\rangle \to (|00200000\rangle - |00020000\rangle)/\sqrt{2}, \quad (9)$$

$$(|10010000\rangle + |01100000\rangle)/\sqrt{2} \to (|10100000\rangle - |01010000\rangle)/\sqrt{2}, \quad (10)$$

$$(|10010000\rangle - |01100000\rangle)/\sqrt{2} \to (|01100000\rangle - |10010000\rangle)/\sqrt{2}, \quad (11)$$

where the left-hand sides are in the frequency basis while the right-hand sides are in the time basis. The time-resolving detector measures the first two modes in the right-hand sides. When it detects no (two) photon in Eq. (9), this gate fails with the Kraus operator $\langle 0_p^0|\langle 0_p^1|$ $(\langle 1_p^0|\langle 1_p^1|)$. When the time-resolving detector detects one photon in Eqs. (10) and (11), this gate succeeds with the corresponding Kraus operators

$$|\tilde{0}_t\rangle(\langle 0_p^0|\langle 1_p^1| + \langle 1_p^0|\langle 0_p^1|)/\sqrt{2} - |\tilde{1}_t\rangle(\langle 0_p^0|\langle 1_p^1| - \langle 1_p^0|\langle 0_p^1|)/\sqrt{2} = |\tilde{1}_f\rangle\langle 0_p^0|\langle 1_p^1| + |\tilde{0}_f\rangle\langle 1_p^0|\langle 0_p^1| \quad (12)$$

$$-|\tilde{1}_t\rangle(\langle 0_p^0|\langle 1_p^1| + \langle 1_p^0|\langle 0_p^1|)/\sqrt{2} + |\tilde{0}_t\rangle(\langle 0_p^0|\langle 1_p^1| - \langle 1_p^0|\langle 0_p^1|)/\sqrt{2} = |\tilde{1}_f\rangle\langle 0_p^0|\langle 1_p^1| - |\tilde{0}_f\rangle\langle 1_p^0|\langle 0_p^1|, \quad (13)$$

where $|\tilde{0}_t\rangle = (|\tilde{0}_f\rangle + |\tilde{1}_f\rangle)/\sqrt{2}$ and $|\tilde{1}_t\rangle = (|\tilde{0}_f\rangle - |\tilde{1}_f\rangle)/\sqrt{2}$. Therefore, the path-frequency fusion gate enables us to generate a three-qubit cluster state from two two-qubit cluster states, except for the difference in the encodings of the input and output qubits.

Finally, by combining two frequency-path fusion gates and a path-frequency gate as shown in Fig. 2(c), we can construct the quantum gate closed in the TFGKP encoding. Let the output states from two frequency-path fusion gates be the input of frequency-path fusion gates. (Thus, we use the both types of frequency-path fusion gates, where the frequency mode of the output state is zeroth or first.) Then, if all three fusion gates succeed, for example, with the operation determined by Eqs. (4) and (12), the whole quantum operation is represented by the Kraus operator

$$|\tilde{0}_t\rangle(\langle\tilde{0}_f|\langle\tilde{1}_f|\langle\tilde{0}_f|\langle\tilde{1}_f| - \langle\tilde{1}_f|\langle\tilde{0}_f|\langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2} + |\tilde{1}_t\rangle(\langle\tilde{1}_f|\langle\tilde{0}_f|\langle\tilde{0}_f|\langle\tilde{1}_f| - \langle\tilde{0}_f|\langle\tilde{1}_f|\langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2}. \quad (14)$$

On the corrected type-I' fusion gate shown in Fig. 1, the input states in the first and third modes are $|\tilde{0}_t\rangle$. Letting the frequency basis be the computational basis, $|0\rangle \sim |\tilde{0}_f\rangle$ and $|1\rangle \sim |\tilde{1}_f\rangle$, (that is, $|+\rangle \sim |\tilde{0}_t\rangle$ and $|-\rangle \sim |\tilde{1}_t\rangle$,) one of the Kraus operators of the type-I' fusion gate is

$$|\tilde{1}_t\rangle(\langle\tilde{1}_f|\langle\tilde{0}_f| - \langle\tilde{0}_f|\langle\tilde{0}_f|)/\sqrt{2} + |\tilde{1}_t\rangle(\langle\tilde{0}_f|\langle\tilde{1}_f| - \langle\tilde{1}_f|\langle\tilde{0}_f|)/\sqrt{2} \sim |+\rangle(\langle 1|\langle 1| - \langle 0|\langle 0|)/\sqrt{2} + |-\rangle(\langle 0|\langle 1| - \langle 1|\langle 0|)/\sqrt{2}. \quad (15)$$

Thus, it enables us to generate a three-qubit cluster state from two Bell states. Adding up all success cases, the total success probability of the corrected type-I' fusion gate is 1/8.

[1] The corrected type-I' fusion gate is less efficient than the conventional type-I fusion gate in the aspects of both the success probability and the necessity of ancilla photons. However, each component of the type-I' fusion gate, that is, the frequency-path and path-frequency fusion gates, has the same success probability as that of the type-I fusion gate and needs no ancilla photons. Thus, further considerations would allow our toolbox to do universal quantum computation with little or no reduction in the efficiency compared with the linear optical quantum computation schemes using other encoding such as polarization encoding.