Unveiling the Merger Structure of Black Hole Binaries in Generic Planar Orbits

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The precise modeling of binary black hole coalescences in generic planar orbits is a crucial step to disentangle dynamical and isolated binary formation channels through gravitational-wave observations. The merger regime of such coalescences exhibits a significantly higher complexity compared to the quasicircular case, and cannot be readily described through standard parametrizations in terms of eccentricity and anomaly. In the spirit of the effective one body formalism, we build on the study of the test-mass limit, and introduce a new modeling strategy to describe the general-relativistic dynamics of two-body systems in generic orbits. This is achieved through gauge-invariant combinations of the binary energy and angular momentum, such as a dynamical "impact parameter" at merger. These variables reveal simple "quasi-universal" structures of the pivotal merger parameters, allowing us to build an accurate analytical representation of generic (bounded and dynamically bounded) orbital configurations. We demonstrate the validity of these analytical relations using 311 numerical simulations of bounded noncircular binaries with progenitors from the RIT and SXS catalogs, together with a custom dataset of dynamical captures generated using the Einstein Toolkit, and test-mass data in bound orbits. Our modeling strategy lays the foundations of accurate and complete waveform models for systems in arbitrary orbits, bolstering observational explorations of dynamical formation scenarios and the discovery of new classes of gravitational wave sources.

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Introduction.—Black hole (BH) binary mergers are unique probes of dynamical formation channels in dense environments [1,2], and allow us to push searches of new physics [3,4] into a stronger-field regime. Gravitational-wave (GW) signals emitted by these systems can be detected by interferometric observatories both on the ground [5,6] and in space [7], or by pulsar timing arrays [8–11]. Recently, significant effort has been placed in their modeling and search [12–43], with one signal being already consistent with a noncircular scenario [15,18,44,45]. Crucially, a significant fraction of dynamically formed sources are expected to lie in the high end of the mass distribution, due to hierarchical mergers [46], pushing their inspiral outside the sensitive band of ground-based detectors, and giving rise to a signal dominated by the merger-ringdown portion. While several noncircular inspiral models have been developed, both in bounded and dynamically bounded [47–54] orbits, merger-ringdown waveforms instead still rely on a quasicircular description [55–62], with the exception of a numerical surrogate valid for small eccentricity [63]. Further, analytical models have shown good accuracy also for scattering orbits [64,65], a set of configurations which is crucial in the ongoing effort of connecting quantum scattering amplitude calculations with classical gravity [66–77]. Aside from theoretical considerations, complete models are urgently needed to extend standard template searches based on quasicircular waveforms [78], which exhibit dramatic sensitivity loss to systems in arbitrary orbits [79]. Laying the foundations to go beyond quasicircular merger-ringdown models is the main goal of this Letter.

To this end, appropriate modeling variables capable of capturing the noncircular merger structure are required. For the bounded case, proposed modeling choices in the literature are generalizations of the Newtonian definitions of eccentricity and anomaly parameters. The most recent proposals [63,80–82] are based on waveform-constructed quantities, ensuring gauge invariance. However, these definitions rely on "pericenter" and "apocenter" frequencies constructed from interpolation of the waveform frequency minima and maxima. As a consequence, this method does not apply to situations where only a modest

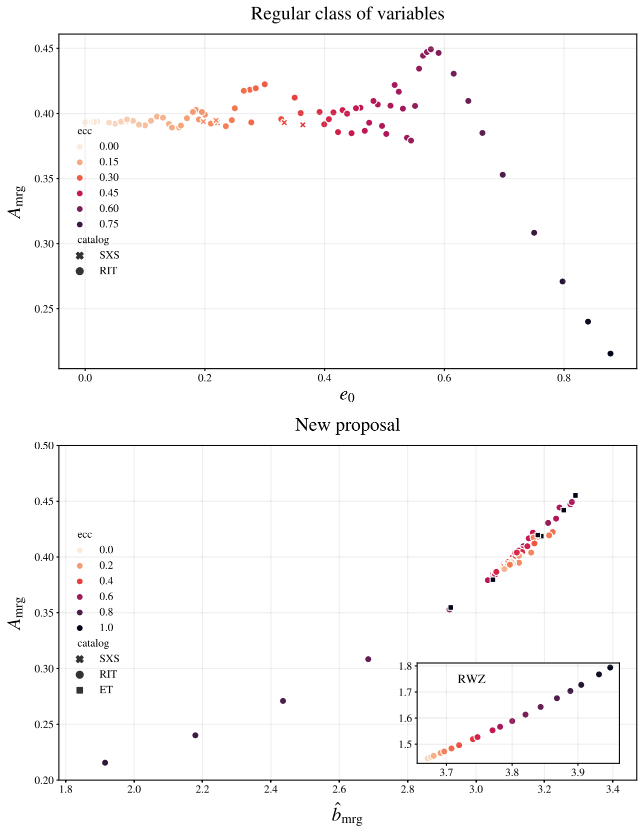

FIG. 1. Top panel: merger amplitude as a function of the initial nominal eccentricity for bounded equal mass nonspinnning binaries. Their relationship is oscillatory and multivalued. Bottom panel: same quantity as above, but as a function of a suitably defined dynamical impact parameter evaluated at merger. The amplitude now displays a simple monotonic dependence, even for hyperboliclike systems (square markers). The inset highlights the same relationship for bounded test-mass data.

number of waveform cycles is available, which include the vast majority of numerical simulations with intermediate to high eccentricity, and dynamical-capture systems. Moreover, an eccentricity defined from frequency minima and maxima is intrinsically ill defined at merger, nor can be readily extended to the merger resulting from hyperbolic initial conditions. It is thus intuitive to expect that these parametrizations do not allow for a simple plunge-merger-ringdown modeling.

This is showcased in the top panel of Fig. 1, displaying the behavior of the merger GW amplitude A_{mrg} (defined below) in the nonspinning equal-mass binary case as a function of the initial orbital eccentricity. The displayed one-dimensional relationship presents a highly complex structure and wide bifurcations: the initial eccentricity parameter does not allow for a smooth merger modeling and, already for intermediate eccentricities, it does not uniquely map the noncircular amplitude value. In the Supplemental Material [83], which includes Refs. [84–89], we show the same result to hold even when using the time-evolved and gauge-invariant eccentricity parameter of Refs. [80–82] for all the reference times available. Additionally, the required interpolation fails for the vast majority of the simulations shown, leaving a much smaller dataset to be considered. The latter point also prevents us from obtaining simple relationships when performing a two-dimensional fit including a gauge-invariant anomaly parameter [82]: the relationship becomes single valued, but it presents a highly complicated oscillatory structure difficult to resolve with the very few points for which such a parameter can be computed. An equivalent behavior has been observed for the late-time ringdown amplitudes, see Fig. 10 of Ref. [90].

Here, we improve on the above parametrizations and introduce a new modeling strategy valid for arbitrary binary planar orbits, relying on results obtained in the test mass limit [91]. The merger amplitude of a particle in bounded eccentric orbits is a smooth function of a suitably defined dynamical "impact parameter," as shown in the inset of the bottom panel in Fig. 1. Now, no bifurcations arise and a simple structure emerges. One of the main results of the Letter is showing how such parametrization can be generalized to the comparable-mass case, previewed in the main bottom panel of Fig. 1. We also consider dynamically bounded systems, showcasing the applicability of our parametrization to comparable-mass binaries in *generic* orbits. This is achieved through a *single* variable representing the two-dimensional space of initial conditions in the noncircular case, a highly nontrivial feature attesting the effectiveness of our modeling strategy. Such a variable thus uncovers a new and nontrivial "quasi-universality" in the merger-remnant structure, deepening the understanding of the two-body dynamics. Our methodology, which leverages the natural variables (energy and angular momentum) describing a generic binary dynamic, readily incorporates BH spins, as we show by including in our study binaries with spins aligned to the orbital angular momentum. Below, we detail the construction and validation of our modeling variables, and of the displayed relationships, similarly built for all key merger parameters, shown in Fig. 2 in the nonspinning case. The constructed relationships provide for the first time the required corner stone for the completion of semi-analytical aligned-spins models, significantly increasing their agreement to numerical solutions (at the ~99% match level even in the challenging late stages of dynamical captures) [92]. Such models will significantly extend the discovery horizon of GW searches for compact binary coalescences.

Conventions.—We use geometric units $c = G = 1$. The gravitational-wave strain is decomposed in spin-weighted spherical harmonics modes, $h_{\ell m}(t)$, split in amplitude and phase as $h_{\ell m}(t) = A_{\ell m}(t)e^{i\phi_{\ell m}(t)}$, with GW frequency, $\omega_{\ell m}(t) \equiv 2\pi f_{\ell m}(t) = \dot{\phi}_{\ell m}(t)$. Individual ADM masses of the two BHs are denoted as $m_{1,2}$, with $M = m_1 + m_2$, the mass ratio $q \equiv m_1/m_2 \geq 1$, the symmetric mass ratio $\nu = m_1 m_2/M^2$, individual BH spin components aligned to

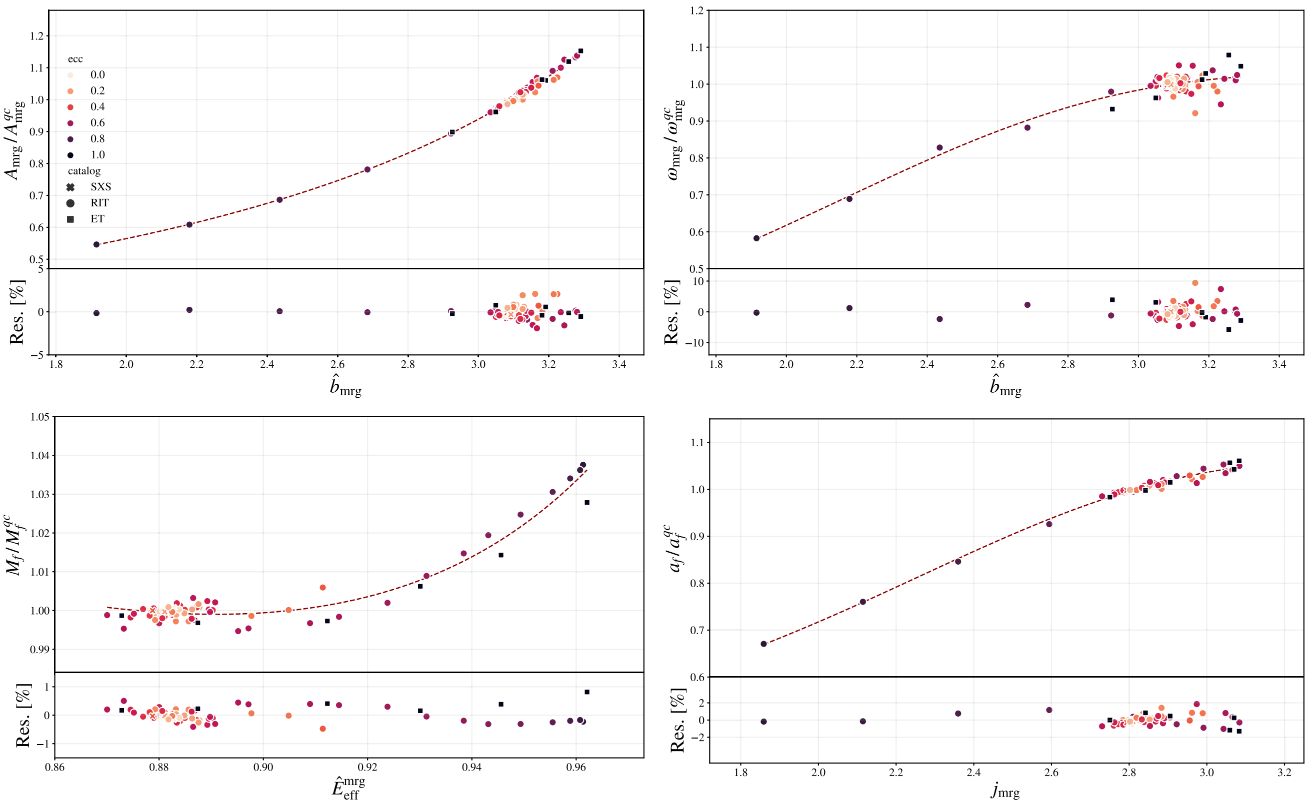

FIG. 2. Quasi-universal relationships of the merger quantities in terms of dimensionless evolved variables, for nonspinning configurations. For visual purposes, ET data are assigned eccentricity value 1. Note the smaller variation range of M_f compared to the other quantities. Smaller panels indicate residuals as defined in the text.

the orbital angular momentum are denoted as $\chi_{1,2}$, and the effective binary spin $\chi_{\rm eff} = (m_1\chi_1 + m_2\chi_2)/M$. Since we will be focusing on the dominant $(\ell, m) = (2, \pm 2)$ mode, we will drop the mode subscript. All available modes are, however, used in the fluxes computations.

Variables construction.—To model systems in arbitrary orbits, we start by considering the initial ADM energy and angular momentum $(E_0^{\rm ADM}, J_0^{\rm ADM})$. Their values at time t, $[E(t), J(t)]$ are obtained by subtracting from $(E_0^{\rm ADM}, J_0^{\rm ADM})$ the gravitational wave losses (see the Supplemental Material [83] for details), $E(t) = E_0^{\rm ADM} - \int_{t_0}^{t} \dot{E}(t')dt'$, $J(t) = J_0^{\rm ADM} - \int_{t_0}^{t} \dot{J}(t')dt'$, where t_0 is the simulation starting time and $[\dot{E}(t), \dot{J}(t)]$ are the radiated fluxes of energy and angular momentum. We define the merger quantities $E_{\rm mrg} \equiv E(t_{\rm mrg})$, $J_{\rm mrg} \equiv J(t_{\rm mrg})$, where $t_{\rm mrg}$ is the merger time. In the generic parameter space under study, a robust and physically meaningful definition of $t_{\rm mrg}$ can be obtained selecting the time corresponding to the peak of the emission immediately before a quasinormal-driven regime (ringdown) begins. For all the configurations under consideration here, this time simply corresponds to the *last* peak of the *quadrupolar* waveform amplitude A_{22}. It is important to note that for generic initial data, this last peak does not coincide with the largest maximum of A_{22}, which can occur at the periastron passage previous to the plunge merger, after which the system still displays a two-body orbital dynamics incompatible with a merger definition. For more extreme eccentricities (very close to the head-on limit) and larger mass ratios the above definition can be generalized by considering the peak of the total amplitude, including higher harmonics, or by simultaneously fitting for quasinormal modes, still guaranteeing the applicability of the above definition.

Even when considering merger-evolved quantities, a key point to obtain accurate fits is to use dimensionless variables factoring out the appropriate mass scaling. In particular, we use the mass-normalized energy $h_{\rm mrg} \equiv E_{\rm mrg}/M$ and angular momentum $j_{\rm mrg} \equiv J_{\rm mrg}/(\nu M^2)$. The effective-one-body (EOB) approach to the two-body general relativistic dynamics [93] is a powerful analytical method that maps the dynamics of a two-body system into the dynamics of an effective body of mass $\mu = m_1 m_2/M$ moving into an effective external potential. The map between the real energy h and the effective energy (per unit mass) $\hat{E}_{\rm eff} \equiv E_{\rm eff}/\mu$ is given by $\hat{E}_{\rm eff}(h, \nu) = 1 + (h^2 - 1)/(2\nu)$. For scattering configurations, an impact parameter of the form $b_{\rm EOB}/M = jh/\sqrt{\hat{E}_{\rm eff}^2 - 1}$ can be defined, see Eq. (2.29) in Ref. [68] or Eq. (2.5) in Ref. [94]. In the test-mass limit, i.e., the case of a particle moving on a Schwarzschild BH, we have $h \to 1$, while $\hat{E}_{\rm eff}$ becomes the real energy of the particle. The parameter $b_{\rm EOB}/M$ is well defined only when $\hat{E}_{\rm eff} > 1$ and thus it cannot be straightforwardly applied as it is our intention to characterize the dynamics of bound configurations, where $\hat{E}_{\rm eff} < 1$. Thus, inspired by recent results in the test-mass limit [91], we use as dynamical impact parameter at merger the quantity $\hat{b}_{\rm mrg} = b_{\rm mrg}/M \equiv j_{\rm mrg} h(\hat{E}_{\rm eff}^{\rm mrg}, \nu)/\hat{E}_{\rm eff}^{\rm mrg}$; that is, the one used in the right panel of Fig. 1. As fitting variables, we will employ functions of $\{\hat{E}_{\rm eff}^{\rm mrg}, j_{\rm mrg}, \hat{b}_{\rm mrg}, \nu\chi_{\rm eff}\}$, depending on whether we discuss quasi-universal relations, or consider the full dimensionality of the parameter space. This choice of parameters is key to obtain simple and accurate relationships in arbitrary orbits, shifting the focus from orbit-based quantities, to evolved dynamical ones, more naturally incorporating radiation-reaction effects.

The key quantities determining the merger emission are $\{A_{\rm mrg}, \omega_{\rm mrg}, M_f, a_f\}$, where $a_f \equiv J_f/M_f^2$, (M_f, J_f) denote the mass and spin of the remnant BH, $A_{\rm mrg} \equiv A(t = t_{\rm mrg})$ and $\omega_{\rm mrg} \equiv \omega(t = t_{\rm mrg})$. We decide to focus on modeling these quantities ratio compared to the quasicircular case. This allows us to scale leading order dependences (e.g., on ν) observed independently of the orbital configuration. We thus obtain a factorized fit, straightforwardly applicable on top of quasicircular values, maintaining the accuracy of the quasicircular limit, where more simulations are available. Both astrophysical inference and searches for new physics rely on merger-remnant predictions in the quasicircular case, well studied both for remnant mass and spin [3,78,95–102], and for merger quantities [103–105]. As a fitting template, we consider $Y = Y_0(1 + p_1 Q + p_2 Q^2) \cdot (1 + p_3 Q + p_4 Q^2)^{-1}$, where Q is the dimensionless fitting variable, Y the ratio of the modeled quantity with its quasicircular value, and $p_k = b_k(1 + c_k X)$, with $X \equiv 1–4\nu$, $b_k, c_k \in \mathbb{R}$. We found this function sufficiently simple and flexible to capture the structure of our dataset [106]. Residuals are defined by $\Delta Y \equiv (Y - Y_{\rm NR})/Y_{\rm NR}$. Technical details of the fits are provided in the Supplemental Material [83]. There, we also discuss how to incorporate these relationships in an waveform template. It is important to note that the above relationships only enter the merger and postmerger portions of the waveform, while fluxes are derived only through the premerger waveform.

Dataset.—We restrict to binaries with progenitors spins aligned to the orbital angular momentum and employ 311 publicly available noncircular bounded simulations. The vast majority of them is contained in the RIT catalog [107], complemented by available simulations from the SXS Collaboration [108–120]. The RIT catalog spans the ranges $q = [1, 32]$, $e_0 = [0, 1]$, $\chi_{1,2} = [-0.8, 0.8]$, while the SXS one has ranges $q = [1, 3] e_0 = [0, 0.2]$, $\chi_{1,2} = [-0.5, 0.85]$. Here, e_0 is the nominal gauge-dependent eccentricity of the simulation. Additionally, we employ a custom dataset of the latest stages of nonspinning dynamical capture simulations generated with the Einstein Toolkit (ET) package [121–125], with a range $q = [1.0, 2.15]$, see Ref. [92]. These additional simulations allow us to show that the quasi-universal behavior under investigation is not restricted to bound orbits, but is a generic feature in planar orbits. Finally, we consider also the test-mass data of Ref. [91]. The latter are generated by eccentric inspirals of a nonspinning test particle around a Schwarzschild BH, driven by a EOB-based radiation reaction. They span a large range of eccentricities, and have been computed by numerically solving the Regge-Wheeler and Zerilli (RWZ) inhomogeneous equations [126–129] with `RWZHyp` [130–132]. Details of simulations used and our selection criterion based on the quality of the numerical data (quantified by balance laws), are provided in the Supplemental Material [83].

Results.—We start by considering the equal mass nonspinning case, with five free parameters in our template. The results are shown in Fig. 2. The merger parameters are well described by $Q = \hat{b}_{\rm mrg}$ in our template, as expected from perturbation theory, while the remnant BH properties $M_f/M_f^{\rm qc}$ $(a_f/a_f^{\rm qc})$ are naturally expressed in terms of $Q = \hat{E}_{\rm mrg}^{\rm eff}$ $(j_{\rm mrg})$. This level of discrepancy in $A_{\rm mrg}/A_{\rm mrg}^{\rm qc}$ is fully compatible with the expected numerical error of the simulations (1%) for the vast majority of the cases [107]. Note the nonmonotonic behavior of the amplitude as a function of eccentricity: a stronger merger emission is observed for moderate eccentricities, while larger eccentricities display a highly suppressed merger amplitude. The nonmonotonic behavior of the amplitude as a function of eccentricity is smoothly accounted for by our variable. In the case of $\omega_{\rm mrg}/\omega_{\rm mrg}^{\rm qc}$, the residuals stay below 3% for the majority of the cases (83/91 simulations), with a few outliers that reach up to 10%. As discussed in Ref. [91], the $\omega_{\rm mrg}$ variation with eccentricity is suppressed compared to the amplitude for intermediate eccentricities, hence the impact of numerical noise on this quantity is larger (compare the patterns regularity between top left and right panels of Fig. 3). The impact on noncircular corrections on $M_f/M_f^{\rm qc}$ is the smallest among all considered quantities. Our parametrization captures its variation with an accuracy better than 0.5% for almost all simulations (89/91 cases). The visual spread of the datapoints with respect to our model is apparently larger compared to the one encountered for other quantities, but fully within numerical accuracy $O(0.1\%)$, Ref. [107], and consistent with our flux-based analysis discussed above. The noncircular behavior of a_f is very well captured by the evolved variable $j_{\rm mrg}$, with better than 1% accuracy for almost all simulations considered (88/91 cases). Note the large variation (up to 35%) compared to the quasicircular case.

Although our strategy allows for an accurate representation with a *single* effective parameter (quasi-universality), residuals show a subdominant trend. This is expected, since initial conditions in the noncircular case are determined by

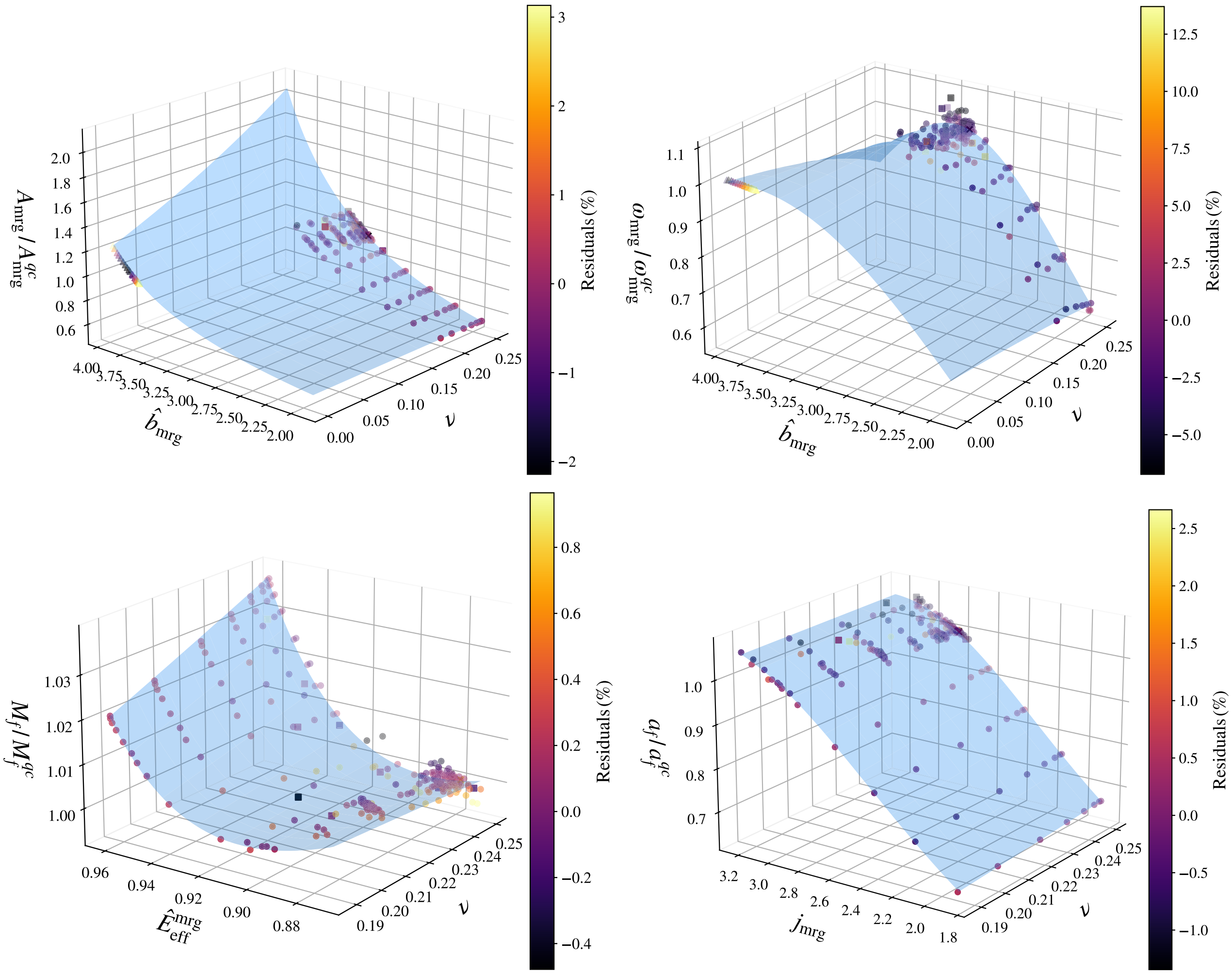

FIG. 3. Quasi-universal relationships for the nonspinning unequal mass dataset (SXS, ET, RIT catalogs, with markers matching the previous figures, and triangles indicating RWZ data). The same structure observed in the equal mass case holds for arbitrary mass ratios, including the test mass limit.

two parameters, not one. The generalization of the above relationship beyond quasi-universality, including two parameters, is discussed in the Supplemental Material [83]. Such generalizations naturally provide yet smaller residuals (given the larger dimensionality of the parameter space employed), allowing us to construct even more accurate models.

Our description maintains a comparable accuracy also in the nonspinning unequal mass case, where ν-dependent corrections are folded in through the p_k factors in our template. The number of free parameters is now nine, and results are shown in Fig. 3. Even in this extended parameter space, the data points lie on a surface, and display smooth changes in terms of the fitting variables all the way to the test mass case. In the Supplemental Material [83] we discuss the extension of these fits beyond quasi-universality. The inclusion of multiple variables has a more pronounced impact on the unequal-mass dataset, bringing the accuracy to the remarkable level already achieved in the equal mass case, with the bulk of the residuals of $O(1\%)$ on $(A_{\rm mrg}, a_f)$, $O(3\%)$ on $\omega_{\rm mrg}$, and $O(0.1\%)$ on M_f.

Finally, our generic strategy can be straightforwardly applied to the case of progenitors with spin. Since this application is conceptually identical to the nonspinning case, we defer these results to the Supplemental Material [83]. There, we show how the same relationships are naturally extended to this case, with an accuracy at the same level of the nonspinning one.

Discussion.—We have identified the existence of nontrivial order and a simple structure in all public NR simulations of noncircular binaries with aligned-spin progenitors. This structure relies on evolved dynamical variables, such as the dynamical impact parameter $\hat{b}_{\rm mrg}$, uncovering quasi-universal relationships yielding highly accurate models of the merger properties. Such construction, inspired from the behavior observed in the test-mass limit, avoids the issues arising from eccentricity-based parametrizations and frequency peaks interpolations. Our results apply equally well to bounded and hyperboliclike orbits from multiple NR catalogs (including test-mass data), unifying these *a priori* different regimes into a unique framework. The presented relationships yield the necessary building blocks required to construct highly accurate, semi-analytical full-waveform models, valid for generic planar orbits with mismatches smaller than 1% [92]. This strategy will allow us to include numerical information into phenomenological waveform models (see Supplemental Material [83]) even beyond what has been considered here. For example, a natural application will be to generalize the templates of Refs. [133,134], incorporating their coefficients dependence on $\hat{b}_{\rm mrg}$. These results enable new exciting discoveries of binaries in nonstandard configurations, with dramatic implications on our understanding of binary formation in chaotic astrophysical settings.

Our parametrization will also aid parameter estimation of GW signals emitted by noncircular binaries. The correlations shown in Fig. 2 of Ref. [44] can now be seen to indicate the impact parameter as a leading-order measurable quantity. The same reasoning applies to the bandlike structure observed in the number of encounters as a function of the binary initial conditions [44,51]. We thus expect our new variables to allow for a much easier sampling convergence (due to the higher degree of smoothness of our parametrization), unlocking new observational investigations of GW signals sourced by binaries in generic orbits, previously hindered by computational cost.

We release the dataset behind the figures and a PYTHON implementation of our fits in a repository [135]. This study made use of the open-software PYTHON packages: CORE-WATPY, GW_ECCENTRICITY, H5PY, JSON, LAL, MATPLOTLIB, NUMPY, PANDAS, PYRING, SCIPY, SEABORN, SXS [82,136–147].

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