Editors' Suggestion

Random Matrix Statistics in Propagating Correlation Fronts of Fermions

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We theoretically study propagating correlation fronts in noninteracting fermions on a one-dimensional lattice starting from an alternating state, where the fermions occupy every other site. We find that, in the long-time asymptotic regime, all the moments of dynamical fluctuations around the correlation fronts are described by the universal correlation functions of Gaussian orthogonal and symplectic random matrices at the soft edge. Our finding here sheds light on a hitherto unknown connection between random matrix theory and correlation propagation in quantum dynamics.

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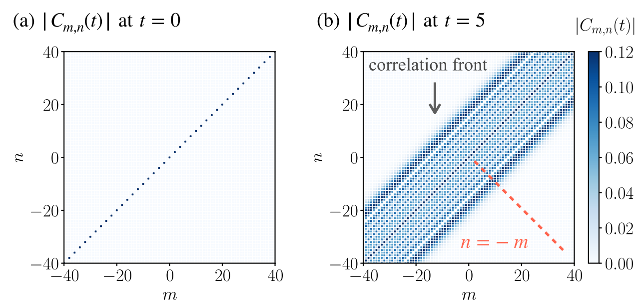
Introduction.—Propagation of correlation has been one of the central topics in quantum many-particle systems [1–3]. It is often associated with emergence of propagating correlation fronts, forming light-cone structures [4–12]. A fundamental result on the subject is the existence of a universal bound for a two-point correlator, known as the Lieb-Robinson bound found in 1972 [13]. Since then, correlation-front dynamics has attracted considerable attention, and the universal aspects have been extensively explored from various viewpoints, such as entanglement entropy [14–17] and operator spreading [18–21]. Currently, the state-of-the-art experiments of cold atoms have observed the light-cone structures [5–7].

Appearance of propagating fronts is not necessarily restricted to correlation dynamics. For instance, particle distribution can develop into propagating particle-density fronts when the particles are initially distributed in a spatially restricted region. One of the typical situations is a one-dimensional system with a domain-wall initial condition, for which previous works [22–44] have investigated the fundamental properties, e.g., the propagating speed of the front and the variance of the integrated particle current. In particular, Eisler and Rácz theoretically studied dynamical fluctuation of the particle density around the front of noninteracting fermions [29], finding that the dynamical fluctuation is characterized by universal eigenvalue distributions of the Gaussian unitary ensemble (GUE) [45,46] of random matrix theory. Subsequent works [31,36–38] have studied the details from various perspectives, such as dependence of initial states and effects of interactions.

Such dynamical fluctuation, however, has yet to be explored well in correlation-front dynamics. So far, previous literature on propagating correlation fronts has focused mainly on two- or four-point correlators, studying the fundamental aspects in terms of the Lieb-Robinson bound and the light-cone structures [1–13,47–50]. With this background, it is intriguing to explore the universal nature of correlation dynamics beyond the conventional light-cone structures captured by low-order correlators, by focusing on the dynamical higher-order fluctuation of correlation.

In this Letter, we study the dynamical fluctuations around propagating correlation fronts in one-dimensional noninteracting fermions starting from an alternating initial state, where the particles occupy every other site. Figures 1(a) and 1(b) display the time evolution of a two-point correlator $C_{m,n}(t)$ [precisely defined after Eq. (1)], from which one can clearly see the propagating correlation



(c) Comparison between the previous and our results

Initial state		Quantity of interest	Random matrix
Domain-wall state		Particle number	GUE (Ref. [29])
Alternating state		Spatial correlation	GOE and GSE **(This Letter)**

FIG. 1. (a),(b) Spatial distributions for modulus of a two-point correlator $C_{m,n}(t) \coloneqq \langle\psi(t)|\hat{a}_m^\dagger \hat{a}_n|\psi(t)\rangle$ with a quantum state $|\psi(t)\rangle$ at time (a) $t = 0$ and (b) $t = 5$. $\hat{a}_m^\dagger$ and $\hat{a}_m$ denote fermionic creation and annihilation operators at a site m. The initial state satisfies $C_{m,n}(0) = \delta_{m,n}[1 + (-1)^n]/2$; no correlations exist for different sites. As time goes by, the correlator grows along a direction parallel to the dashed line corresponding to $n = -m$ in (b) and then the propagating correlation fronts emerge. (c) Table for the previous result [29] (second row) and ours (third row). The first, second, and third columns are initial states with the corresponding schematics, quantities of interest, and classes of random matrices associated with fermionic dynamics in Ref. [29] and this Letter.

fronts. To explore universal aspects of the dynamical fluctuations, we introduce a cumulative correlation operator, which can capture the fluctuations around the fronts. Then, exactly solving the Schrödinger equation, we analytically show that, in the long-time dynamics, all the moments of the cumulative correlation operator are determined by universal correlation functions of the Gaussian orthogonal ensemble (GOE) and the Gaussian symplectic ensemble (GSE) at the soft edge in random matrix theory [45,46]. Our main result is summarized in Table of Fig. 1(c), where we emphasize differences between our findings and the previous results of Ref. [29]. The major contribution of this Letter to the research field of correlation dynamics is to uncover that the dynamical higher-order fluctuation around the correlation front can exhibit the universal behaviors featuring random matrix theory.

Setup.—We consider noninteracting fermions on a one-dimensional lattice and denote the fermionic annihilation and creation operators at a site $m \in \mathbb{Z}$ by $\hat{a}_m$ and $\hat{a}_m^\dagger$. Then, the Hamiltonian is defined by $\hat{H} := -\sum_{m=-\infty}^{\infty}(\hat{a}_{m+1}^\dagger \hat{a}_m + \hat{a}_m^\dagger \hat{a}_{m+1})$. Under this setup, we can compute a quantum state $|\psi(t)\rangle$ at time t by solving the Schrödinger equation $\mathrm{i}d|\psi(t)\rangle/dt = \hat{H}|\psi(t)\rangle$ with a given initial state $|\psi(0)\rangle$. Here we set $\hbar = 1$. The initial state used in this Letter is the alternating state $|\psi_{\mathrm{alt}}\rangle$, where the fermions occupy only the even sites [see the third row of Fig. 1(c)],

$$|\psi_{\mathrm{alt}}\rangle := \prod_{m=-\infty}^{\infty} \hat{a}_{2m}^\dagger |0\rangle \tag{1}$$

with a vacuum $|0\rangle$.

The quantity of our interest is the dynamical fluctuations around the propagating correlation fronts. Figures 1(a) and 1(b) show the time evolution for the modulus of the two-point correlator $C_{m,n}(t) := \langle \hat{a}_m^\dagger \hat{a}_n \rangle_t$ with $\langle \bullet \rangle_t := \langle \psi(t)| \bullet |\psi(t)\rangle$. One can see that the correlation fronts propagate along a direction parallel to the line $n = -m$ [see the dashed line in Fig. 1(b)]. To investigate the dynamical fluctuations around the fronts, we focus on the $2N$-point correlator on the line $n = -m$ defined by

$$\left\langle \prod_{j=1}^{N} \hat{a}_{m_j}^\dagger \hat{a}_{-m_j} \right\rangle_t \tag{2}$$

with an integer $m_j (j \in \{1, \ldots, N\})$. This quantity captures the dynamical fluctuation around the correlation front when we appropriately choose $\{m_1, \ldots, m_N\}$ corresponding to the propagating front. As we will see shortly, the fluctuation of the correlation front is related to random matrix theory. To show this, it is convenient to consider a cumulative correlation operator $\hat{F}_l$ defined by

$$\hat{F}_l := \sum_{m=l}^{\infty} \hat{a}_m^\dagger \hat{a}_{-m}, \tag{3}$$

where $l > 0$ is a positive integer. The moments of $\hat{F}_l$ capture the dynamical fluctuation around the front by appropriately choosing l. For the following calculations, we define a generating function for the moments by

$$Q(\lambda, t, l) := \langle e^{\lambda \hat{F}_l} \rangle_t \tag{4}$$

with a real number λ. As shown in the following, the generating function includes the multipoint correlator (2).

Appearance of the GOE Tracy-Widom distribution.—Focusing on the propagating correlation front, we shall analytically show that $Q(\lambda, t, l)$ is related to the universal eigenvalue distribution function of random matrix theory, namely the GOE Tracy-Widom distribution [46,51].

First, we derive a determinantal formula for $Q(\lambda, t, l)$. A straightforward calculation leads to

$$Q(\lambda, t, l) = 1 + \sum_{n=1}^{\infty} \sum_{\substack{m_1 < m_2 < \cdots < m_n \\ m_k \in \{l, l+1, \ldots\}}} \lambda^n \left\langle \prod_{j=1}^{n} \hat{a}_{m_j}^\dagger \hat{a}_{-m_j} \right\rangle_t. \tag{5}$$

The Wick theorem enables us to decompose the $2n$-point correlator of Eq. (5) into products of $C_{m,n}(t)$,

$$\left\langle \prod_{j=1}^{n} \hat{a}_{m_j}^\dagger \hat{a}_{-m_j} \right\rangle_t = \det \left[C_{j,-k}(t) \right]_{j,k \in \{m_1, \ldots, m_n\}}. \tag{6}$$

Substituting Eq. (6) into Eq. (5), we obtain

$$Q(\lambda, t, l) = \det \left[\delta_{m,n} + \lambda C_{m,-n}(t) \right]_{\ell^2\{l, l+1, \ldots\}} \tag{7}$$

with the Kronecker delta $\delta_{m,n}$ and $\ell^2\{l, l+1, \ldots\}$, the square-summable sequence space on $\{l, l+1, \ldots\}$. The correlator $C_{m,n}(t)$ in Eq. (7) is written as

$$C_{m,n}(t) = \frac{1}{2}\delta_{m,n} + \frac{1}{2}\mathrm{i}^{n+m} J_{n-m}(4t) \tag{8}$$

with the nth-order Bessel function of the first kind $J_n(x)$. The expression of the two-point correlator $C_{m,n}(t)$ in Eq. (8) already appeared in Ref. [52] for noninteracting bosons. For completeness, we give its derivation in Sec. I of Supplemental Material [53]. Combining Eqs. (7) and (8), we obtain

$$Q(\lambda, t, l) = \det \left[\delta_{m,n} + \frac{\lambda}{2} J_{n+m}(4t) \right]_{\ell^2\{l, l+1, \ldots\}}, \tag{9}$$

where we use $J_{-n}(x) = (-1)^n J_n(x)$ and then eliminate the trivial factor $\mathrm{i}^{m-n}(-1)^{n+m}$ by expanding the determinant.

Second, we derive the spatial form of the propagating correlation front by applying asymptotic analysis to Eq. (8) for studying the dynamical fluctuation around the front. We here focus on the front along the line

$n = -m < 0$, namely, $C_{m,-m}(t)$. Introducing a variable x by $m = \lfloor 2t + x(2t)^{1/3}/2 \rfloor$ with the floor function $\lfloor \bullet \rfloor$, we can show, for $t \gg 1$,

$$C_{m,-m}(t) \simeq \frac{1}{2(2t)^{1/3}} \mathrm{Ai}(x), \tag{10}$$

where we use the asymptotic formula $J_{\lfloor 4t+(2t)^{1/3}x \rfloor}(4t) \simeq \mathrm{Ai}(x)/(2t)^{1/3} (t \gg 1)$ with the Airy function $\mathrm{Ai}(x)$ [54] (see Sec. II of the Supplemental Material [53] for the derivation). This shows that the peak of $C_{m,-m}(t) (m > 0)$ is approximately given by $(m, n) \simeq (2t, -2t)$, consistent with the fact that the front ballistically propagates with the maximal group velocity $\max_k \{d\epsilon(k)/dk\} = 2$, as shown in Fig. 1(b). Here, $\epsilon(k) = -2\cos(k)$ is the energy eigenvalue of our model.

Finally, we study the generating function $Q(\lambda, t, l)$ around the propagating correlation front. Taking into account the fact that the front moves with the velocity 2, we choose $l = l_{t,s} \coloneqq \lfloor 2t + s(2t)^{1/3}/2 \rfloor$ with a rescaled coordinate s and introduce two rescaled coordinates x and y by $n = \lfloor 2t + x(2t)^{1/3}/2 \rfloor$ and $m = \lfloor 2t + y(2t)^{1/3}/2 \rfloor$ in Eq. (9). Then, the moments of $\hat{F}_{l_{t,s}}$ characterize the fluctuation around the front. Under this setup, we take $\lambda = -2$ and use the asymptotic formula employed in the derivation of Eq. (10), getting

$$Q(-2, t, l_{t,s}) \simeq \det \left[1 - \frac{1}{2} \mathrm{Ai}\left(\frac{x+y}{2} \right) \right]_{L^2(s,\infty)} \tag{11}$$

for $t \gg 1$. This form is identical to a determinantal formula for the GOE Tracy-Widom distribution [55,56], which is a universal cumulative distribution function for the largest eigenvalue for GOE.

Our result of Eq. (11) strongly suggests that the dynamical fluctuation around the propagating correlation front is related to universal eigenvalue distributions of random matrix theory. In the rest of the Letter, we further explore the detailed connection of the fluctuation to random matrix theory, by focusing on the nth moment $M_n(t, l_{t,s}) \coloneqq \langle (\hat{F}_{l_{t,s}})^n \rangle_t$.

Connection of random matrix theory to the moments.—Employing the analytical method used in the derivation of Eq. (11), we shall show that universal correlation functions of GOE and GSE asymptotically determine all the nth moments $M_n(t, l_{t,s})$.

We first introduce notations of random matrix theory before the detailed analysis. Let us denote n-point eigenvalue correlation functions for GOE and GSE by $R_n^{\mathrm{GOE}}(x_1, \dots, x_n)$ and $R_n^{\mathrm{GSE}}(x_1, \dots, x_n)$. We suppose that $G_{\mathrm{GOE}}(\lambda, s)$ and $G_{\mathrm{GSE}}(\lambda, s)$ represent the generating functions of gap probabilities in the region (s, ∞). By definition, they satisfy

$$\left. \frac{d^n G_\alpha(\lambda, s)}{d\lambda^n} \right|_{\lambda=0} = (-1)^n \int_s^\infty dx_1 \cdots \int_s^\infty dx_n R_n^\alpha(x_1, \dots, x_n) \tag{12}$$

with α taking GOE or GSE [45,46]. We summarize their detailed definitions and properties in Sec. III of the Supplemental Material [53]. In Ref. [57], Bornemann showed that, under the soft-edge scaling limit, the generating functions have determinantal formulas,

$$G_{\mathrm{GSE}}(\lambda, s) = \frac{1}{2} H\left(\sqrt{\lambda}, s\right) + \frac{1}{2} H\left(-\sqrt{\lambda}, s\right), \tag{13}$$

$$G_{\mathrm{GOE}}(\lambda, s) = \frac{1}{2}\left(1 + \sqrt{\frac{\lambda}{2-\lambda}}\right) H\left(\sqrt{\lambda(2-\lambda)}, s\right) + \frac{1}{2}\left(1 - \sqrt{\frac{\lambda}{2-\lambda}}\right) H\left(-\sqrt{\lambda(2-\lambda)}, s\right) \tag{14}$$

with $H(z, s) \coloneqq \det\{1 - (z/2)\mathrm{Ai}[(x+y)/2]\}_{L^2(s,\infty)}$ and real variables λ and s.

To see the connection between our correlation-front dynamics and random matrix theory, let us define two functions for the cumulative correlation operator $\hat{F}_l$ as

$$G_1(\lambda, t, l) \coloneqq \left\langle \cosh\left(2\sqrt{\lambda} \hat{F}_l \right) \right\rangle_t, \tag{15}$$

$$G_2(\lambda, t, l) \coloneqq \left\langle \cosh\left(2\sqrt{\lambda(2-\lambda)} \hat{F}_l \right) \right\rangle_t - \sqrt{\frac{\lambda}{2-\lambda}} \left\langle \sinh\left(2\sqrt{\lambda(2-\lambda)} \hat{F}_l \right) \right\rangle_t. \tag{16}$$

Expanding them with λ, we find that $G_1(\lambda, t, l)$ consists of the even-order moments $M_{2n}(t, l)$, while $G_2(\lambda, t, l)$ does of all the moments $M_{2n}(t, l)$ and $M_{2n+1}(t, l)$. Note that $G_1(\lambda, t, l)$ and $G_2(\lambda, t, l)$ are the power series with λ and thus we can differentiate them at $\lambda = 0$. Using the same asymptotic analysis used in the derivation of Eq. (11), we can show, for $t \gg 1$,

$$G_1(\lambda, t, l_{t,s}) \simeq G_{\mathrm{GSE}}(\lambda, s), \tag{17}$$

$$G_2(\lambda, t, l_{t,s}) \simeq G_{\mathrm{GOE}}(\lambda, s). \tag{18}$$

These are the fundamental relations, establishing that the dynamical fluctuation of the propagating correlation front in the long-time regime is described by the universal correlation functions of GOE and GSE at the soft edge, as shown in the subsequent paragraphs. The detailed derivation of Eqs. (17) and (18) is given in Secs. IV and V of the Supplemental Material [53].

We next differentiate $G_1(\lambda, t, l_{t,s})$ with respect to λ, getting

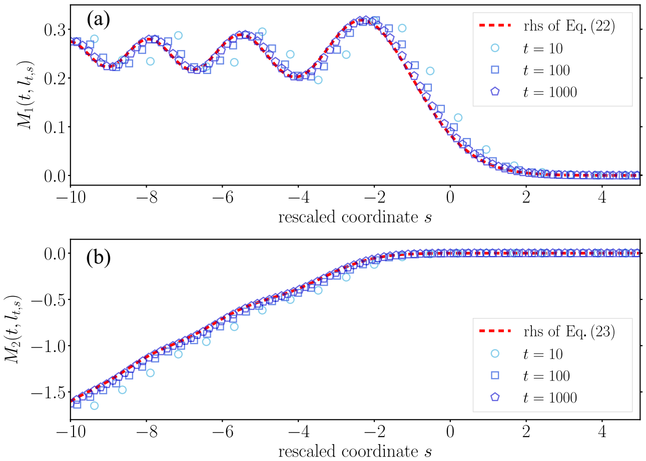


FIG. 2. Numerical verification of Eqs. (22) and (23). The circle, square, and pentagon markers denote (a) $M_1(t, l_{t,s})$ and (b) $M_2(t, l_{t,s})$ at $t = 10$, 100, and 1000, respectively. The rescaled coordinate s is defined through $l_{t,s} = \lfloor 2t + s(2t)^{1/3}/2 \rfloor$. The dashed lines in (a) and (b) represent the right-hand sides of Eqs. (22) and (23), respectively.

$$\left.\frac{d^n G_1(\lambda, t, l_{t,s})}{d\lambda^n}\right|_{\lambda=0} = \frac{4^n n!}{(2n)!} M_{2n}(t, l_{t,s}) \tag{19}$$

as derived in Sec. VI of the Supplemental Material [53]. Using Eqs. (12), (17), and (19), we obtain

$$M_{2n}(t, l_{t,s}) \simeq \frac{(-1)^n (2n)!}{4^n n!} \int_s^\infty dx_1 \cdots \int_s^\infty dx_n R_n^{\mathrm{GSE}}(x_1, \ldots, x_n) \tag{20}$$

for $t \gg 1$. Thus we elucidate that all the even-order moments $M_{2n}(t, l_{t,s})$ are asymptotically determined by the universal correlation function $R_n^{\mathrm{GSE}}(x_1, \ldots, x_n)$ for GSE.

Following the same procedure just above, we differentiate $G_2(\lambda, t, l_{t,s})$ with respect to λ and then obtain

$$\begin{aligned}&\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^{k+n} 2^{3n-4k} (n-k)! n!}{(2n-2k)!(n-2k)!k!} M_{2n-2k}(t, l_{t,s}) \\ &\quad - \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(-1)^{k+n} 2^{3n-4k-2} (n-k-1)! n!}{(2n-2k-1)!(n-2k-1)!k!} M_{2n-2k-1}(t, l_{t,s}) \\ &\simeq \int_s^\infty dx_1 \cdots \int_s^\infty dx_n R_n^{\mathrm{GOE}}(x_1, \ldots, x_n)\end{aligned} \tag{21}$$

for $t \gg 1$ (see Sec. VII of the Supplemental Material [53] for the detailed derivation). Using Eqs. (20) and (21), we can recursively demonstrate that all the odd-order moments $M_{2n+1}(t, l_{t,s})$ are expressed by combining the universal correlation functions $R_n^{\mathrm{GOE}}(x_1, \ldots, x_n)$ and $R_n^{\mathrm{GSE}}(x_1, \ldots, x_n)$ for GOE and GSE. For example, putting $n = 1$ into Eqs. (20) and (21), we obtain

$$M_1(t, l_{t,s}) \simeq \frac{1}{2} \int_s^\infty dx \left(R_1^{\mathrm{GOE}}(x) - 2R_1^{\mathrm{GSE}}(x) \right), \tag{22}$$

$$M_2(t, l_{t,s}) \simeq -\frac{1}{2} \int_s^\infty dx R_1^{\mathrm{GSE}}(x). \tag{23}$$

We numerically verify Eqs. (22) and (23) by solving the Schrödinger equation. Figure 2 displays the time evolution of $M_1(t, l_{t,s})$ and $M_2(t, l_{t,s})$. We find that Eqs. (22) and (23) hold well for $t \gg 1$.

Discussion.—We discuss (i) dependence on initial states, (ii) experimental possibility, and (iii) interaction effect for our results.

Let us first consider the topic (i). As described in Sec. VIII of the Supplemental Material [53], we analytically and numerically investigate the dependence on initial states, showing that GOE and GSE can characterize all the moments under appropriate rescaling even when using a modified alternating state with a filling factor different from 1/2. We also find several exceptional initial states, spatial configurations of which are similar to the alternating state but the GOE and GSE behaviors do not appear. We identify a condition for this absence of the GOE and GSE behaviors, which is given by Eq. (S-67) in the Supplemental Material [53]. We find that the number of such exceptional states is smaller than that of the initial states exhibiting the GOE and GSE behaviors (see the details in Sec. VIII C of the Supplemental Material [53]).

As to the topic (ii), we need to observe the multipoint correlator (2) to verify our theoretical prediction, but its observation is generally difficult. To our knowledge, in Ref. [12], Takasu *et al.* experimentally obtained the two-point correlator of bosonic dynamics in optical lattices by observing the momentum distribution via time-of-flight images, while there are no such works for fermions. In the future, it may be possible to observe the fermionic correlator in cold atom experiments, which will enable us to explore our theoretical prediction.

We discuss the topic (iii). In a previous work [36], Collura *et al.* theoretically studied the interaction effect on the result of Ref. [29], where Eisler and Rácz reported that GUE characterizes the propagating particle-density front in noninteracting fermions. Using the density-matrix renormalization-group method, they numerically found that the GUE behavior disappeared in the interacting system. Thus, it is interesting to discuss our results in interacting systems. We, however, will leave it as a future work since it will be pretty demanding to numerically access the long-time dynamics starting from the alternating state.

Conclusions and prospects.—We theoretically considered the noninteracting fermions on the one-dimensional lattice, studying the dynamics starting from the alternating initial state, where the particles are on every other site. In this case, the two-point correlator $C_{m,n}(t)$ formed the propagating correlation front, as shown in Fig. 1(b).

Focusing on the dynamical fluctuation around the front in the late stage of the dynamics, we analytically showed that the universal correlation functions of GOE and GSE asymptotically determine all the moments $M_n(t, l_{t,s})$ for the cumulative correlation operators capturing the fluctuation around the propagating correlation front. We further studied the dependence of our results on initial states, finding that the GOE and GSE behaviors can survive for the modified alternating states. Thus our result is universal in that the behaviors emerge when systems are mapped into the noninteracting fermions with initial states similar to the alternating state.

As a prospect, it will be intriguing to study dynamical fluctuations around propagating fronts for other physical quantities. Thanks to the light-cone propagation, propagating fronts often emerge even for spins, particle numbers, and local energy. Connections of random matrix theory to such unexplored dynamical fluctuations around the fronts are of great interest.

Another interesting direction is to explore relations between classical stochastic processes and our results. Universal distributions of random matrix theory have been intensively investigated in classical stochastic processes, examples of which include the Kardar-Parisi-Zhang equation, asymmetric simple exclusion processes, and polynuclear growth models [58–64]. For example, in a totally asymmetric exclusion process starting from the alternating state, the particle transport features the GOE Tracy-Widom distribution. This classical dynamics may be related to our result. Thus, it is fundamentally interesting to pursue relations between classical stochastic processes and correlation dynamics in quantum systems from the unified perspective of random matrix theory.

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