Propagator Zeros and Lattice Chiral Gauge Theories

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Symmetric mass generation (SMG) has been advocated as a mechanism to render mirror fermions massive without symmetry breaking, ultimately aiming for the construction of lattice chiral gauge theories. It has been argued that in an SMG phase, the poles in the mirror fermion propagators are replaced by zeros. Using an effective Lagrangian approach, we investigate the role of propagator zeros when the gauge field is turned on, finding that they act as coupled ghost states. In four dimensions, a propagator zero makes an opposite-sign contribution to the one-loop beta function as compared to a normal fermion. In two dimensional Abelian theories, a propagator zero makes a negative contribution to the photon mass squared. In addition, propagator zeros generate the same anomaly as propagator poles. Thus, gauge invariance will always be maintained in an SMG phase, in fact, even if the target chiral gauge theory is anomalous, but unitarity of the gauge theory is lost.

Introduction.—Chiral gauge theories play an important role in particle physics. The standard model is a chiral gauge theory, and many theoretical models for physics beyond the standard model are based on chiral gauge theories as well. At present, no nonperturbative gauge invariant definition of (anomaly free) non-Abelian chiral gauge theories exists. Unlike the case of vectorlike asymptotically free gauge theories such as QCD, for which the lattice provides a nonperturbative definition, the physics of chiral gauge theories is understood mostly in a perturbative framework, supplemented by incomplete and rather qualitative arguments based on large-N expansions, anomaly matching, duality relations, and other phenomenological approaches.

The nonperturbative construction of chiral gauge theories on the lattice is a long-standing challenge, because of the fermion species-doubling problem. The deep reasons underlying species doubling were first discussed by Karsten and Smit [1], tying the phenomenon to the chiral anomaly, and then generalized by Nielsen and Ninomiya [2]. In its simplest form, the doubling problem arises when one considers a local free lattice Hamiltonian in one spatial dimension. The dispersion relation near $p = 0$ will be $E = +p$ for one chirality. Because of the periodicity of

the Brillouin zone, unless there is a nonanalyticity in the dispersion relation, there must exist another point p_c such that, for small $\delta p = p - p_c$, the dispersion relation is $E = -\delta p$, which signifies a fermion of the opposite chirality, the doubler.

During the 1980s and 1990s there was much activity in this field, which lead to a better understanding of the fundamental obstacles, as well as to some successes. For reviews, see Refs. [3,4]. Building on the Ginsparg–Wilson relation [5] as well as on earlier work by Kaplan [6] and by Narayanan and Neuberger [7–10], Lüscher achieved the construction of (anomaly-free) Abelian chiral gauge theories [11]. He also constructed non-Abelian chiral gauge theories to all orders in lattice perturbation theory [12,13]. An alternative approach, where the chiral gauge invariance is explicitly broken on the lattice, and is only restored in the continuum limit, is the gauge-fixing approach. The inclusion of a suitable gauge-fixing lattice action ensures the existence of a novel critical point, where the target chiral gauge theory emerges in the continuum limit [14–19]. For another approach, see Refs. [20,21]. While these proposals are based on nontrivial insights into the nature of the problem, it is still an open question whether any of them will lead to a complete nonperturbative definition of asymptotically free chiral gauge theories on the lattice.

The last decade saw renewed interest in the *mirror fermion* approach. One starts from a vectorlike fermion spectrum containing both LH (left-handed) and RH (right-handed) fields. For each irreducible representation of the gauge group, the fermions of one chirality are included in the target chiral gauge theory, while the fermion fields of

the opposite chirality are unwanted "mirror" fermions. Originally, it was proposed to take the continuum limit in the broken (Higgs) phase [22,23]. However, this approach does not allow for full decoupling on the mirrors. The reason is that full decoupling requires the Higgs vacuum expectation value v to diverge in physical units when taking the continuum limit. But since the mass of the gauge bosons in the Higgs phase is $\sim gv$, this would imply that the gauge bosons decouple as well. Keeping v physical instead would imply that mirrors are also physical. An interesting question is whether such an approach is phenomenologically viable, but the underlying gauge theory remains vectorlike, and not chiral.

Since then, the focus has shifted to attempts to decouple the mirror fermions using some strong nongauge interaction while at the same time keeping the chiral gauge symmetry unbroken. In principle, this would allow for a complete decoupling, with the mirrors obtaining a mass of the order of the ultraviolet (lattice) cutoff of the theory. For a recent review, see Ref. [24]. In an asymptotically free theory, the gauge interaction itself is controlled by the Gaussian fixed point, and turning it on or off is not expected to change the elementary fermion spectrum. The elementary fermion spectrum is thus controlled by the *reduced model*, obtained by turning off the gauge interaction. The reduced model contains the fermion fields and, possibly, additional scalar fields. The original (target) chiral gauge symmetry G turns into a global symmetry of the reduced model.

The dynamical question is what the phase diagram of the reduced model looks like [3,4,24]. There is always a free-fermion limit containing both LH and RH massless fermions in the same (in general, reducible) representation of the symmetry group G to be gauged, which constitutes a vectorlike spectrum. By turning on specially designed multifermion or Yukawa interactions, one hopes to achieve *symmetric mass generation* (SMG). In this Letter, we use the term SMG for a strong-coupling phase in the reduced model where (a) the mirror fermions develop a mass gap of the order of the lattice cutoff, (b) the target chiral fermions remain massless, and (c) the symmetry G is unbroken. The low-energy limit then consists of a chiral fermion spectrum in the desired representation of the (unbroken) symmetry G. If the SMG paradigm is successful, one would hope to recover the target chiral gauge theory when the gauge field is turned back on.

Starting from the Smit-Swift [25] and Eichten-Preskill [26] models, many unsuccessful attempts were made over the years to find an SMG phase, and the dynamical reasons underlying this failure were investigated [27–29]. The last decade or so has seen renewed interest in the *3450 model*. This is an anomaly free, two-dimensional Abelian chiral gauge theory containing fermions with charges 3 and 4 of one handedness, and fermions with charges 5 and 0 of the opposite handedness. (The zero charge fermion does

not couple to the gauge field, but it participates in the multifermion interactions that gap the mirrors in the reduced model.) The focus on two-dimensional models is motivated by the relative simplicity of gauge theories in two dimensions, with the hope that lessons learned generalize to the more interesting case of four-dimensional chiral gauge theories.

An attempt to gap the mirrors in the reduced version of the 3450 model was made in Refs. [30,31]. The vacuum polarization was calculated numerically, but the result indicated that the mirrors did not decouple. A possible reason for this failure was discussed in Ref. [32]. Recently, building on developments in condensed matter physics [33], an SMG phase was reported in a different reduced version of the 3450 model, with the SMG phase induced by especially designed multifermion interactions [34–36].

The availability of a concrete construction allows us to reexamine the question of whether the desired chiral gauge theory will indeed be recovered when the gauge interaction is turned back on. The question is nontrivial for the following reason. As already noted, the desired low-energy limit of the reduced model is a theory of free, undoubled, massless chiral fermions. This is essentially the domain of the no-go theorems. Indeed, it was argued in Ref. [37] that, under some very general assumptions, the Nielsen-Ninomiya theorem will be applicable to an effective low-energy Hamiltonian of an unspecified underlying theory, thus excluding a chiral spectrum in any reduced model unless some of these assumptions are not satisfied [38].

A caveat, already noted in Ref. [37], is that the fermion propagator may contain a zero in the mirror channel. Equivalently, the effective Hamiltonian associated with the inverse propagator has a mirror pole, and thus is nonlocal. While to our knowledge the issue was not directly investigated in the SMG phase of the 3450 model, there are strong general arguments that an SMG phase will always be accompanied by the appearance of a zero in the propagator [33,39–44]. This zero takes the place of the original massless mirror pole, and its essential features are captured by the phenomenological expression

$$P_{R,L} \frac{i\not{p}}{p^2 + m^2}, \tag{1}$$

for the mirror fermion propagator valid near $p = 0$, where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ are the chiral projectors. The large mass scale m characterizes the mass gap in the mirror sector after SMG has taken place. In the generic case, it is presumably on the order of the lattice cutoff.

The goal of this Letter is to examine the consequences of a propagator zero of a massive charged fermion in the reduced model when the gauge field is turned back on. We will first discuss the contribution of a propagator zero to the

vacuum polarization, and then its impact on anomalies, in both two and four dimensions.

Propagator zero and the vacuum polarization.—Before discussing the physics of a propagator zero, let us briefly recall the calculation of the vacuum polarization for normal RH or LH fields. Starting from the gauge invariant Lagrangian (in Euclidean space)

$$\mathcal{L} = \bar{\psi}(\slashed{\partial} + ig\slashed{A})P_{R,L}\psi, \tag{2}$$

the vacuum polarization diagram is

$$\Pi^{R,L} = \frac{1}{2}\langle(-ig\bar{\psi}\slashed{A}P_{R,L}\psi)^2\rangle = \frac{g^2}{2}\mathrm{tr}\left(\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\slashed{A}P_{R,L}\right)$$
$$= \Pi^e \pm \Pi^o, \tag{3}$$

where Π^e and Π^o are the parity-even and parity-odd parts. In $d = 4$ the parity-odd part vanishes, whereas in $d = 2$ it gives rise to the anomaly, which we discuss later.

Moving on, we consider a RH fermion field $\psi_R = P_R\Psi_R$, $\bar{\psi}_R = \bar{\Psi}_R P_L$, for which the propagator pole has been replaced by a propagator zero. The momentum space RH propagator takes the form [compare Eq. (1)]

$$\int d^d x e^{-ipx}\langle\psi_R(x)\bar{\psi}_R(0)\rangle = P_R\frac{i\slashed{p}}{m^2}\left(1 + \mathcal{O}(p^2/m^2)\right). \tag{4}$$

The (even) dimension of spacetime is d. For simplicity, we assumed that the propagator zero occurs at $p = 0$. Our results are unchanged if the zero occurs at a different location in the Brillouin zone. The corrections to the leading behavior are suppressed by powers of p^2/m^2 near the zero, and do not affect our results either. [We assume that the (Euclidean) lattice theory regains full rotational invariance at large distances, and thus that the zeros of the propagator are relativistic. Relaxing this assumption is likely to lead to yet worse problems than those we find in this Letter.]

Keeping only the leading behavior, in operator language the RH propagator is

$$\langle\psi_R\bar{\psi}_R\rangle = P_R\frac{\slashed{\partial}}{m^2}, \tag{5}$$

and the corresponding free effective lagrangian has a RH pole,

$$\mathcal{L}_0 = m^2\bar{\psi}_R\frac{1}{\slashed{\partial}}\psi_R. \tag{6}$$

This effective Lagrangian is nonlocal. As we will see, this has consequences for the gauged theory at low energy, even though the mirror fermion has a mass gap.

We introduce the gauge field via minimal coupling, as usual. The effective Lagrangian becomes

$$\mathcal{L} = m^2\bar{\psi}_R\frac{1}{\slashed{\partial} + ig\slashed{A}}\psi_R$$
$$= m^2\bar{\psi}_R\left(\frac{1}{\slashed{\partial}} - ig\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}} - g^2\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}} + \cdots\right)\psi_R$$
$$\equiv \mathcal{L}_0 + \mathcal{L}_{\mathrm{int}}. \tag{7}$$

The second line effectively defines the inverse of $\slashed{\partial} + ig\slashed{A}$ in terms of its expansion in the coupling constant g, or equivalently, in the external gauge field. (We treat the gauge field perturbatively, and thus the operator $\slashed{\partial} + ig\slashed{A}$ cannot have zero modes.) The vacuum polarization now consists of a bubble and a tadpole diagram. We start with the bubble diagram. Expanding $\exp(-\int d^d x \mathcal{L}_{\mathrm{int}})$ to second order and using Eq. (5) gives

$$\frac{1}{2}\left\langle\left(igm^2\bar{\psi}_R\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\psi_R\right)^2\right\rangle = \frac{g^2}{2}\mathrm{tr}\left(P_R\slashed{A}\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\right) = \Pi^L. \tag{8}$$

The tadpole contribution arises from expanding $\exp(-\int d^d x \mathcal{L}_{\mathrm{int}})$ to first order,

$$g^2 m^2\left\langle\bar{\psi}_R\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\psi_R\right\rangle = -g^2\mathrm{tr}\left(P_R\slashed{A}\frac{1}{\slashed{\partial}}\slashed{A}\frac{1}{\slashed{\partial}}\right) = -2\Pi^L. \tag{9}$$

The vacuum polarization of a RH propagator zero is thus

$$\Pi^L - 2\Pi^L = -\Pi^L = -\Pi^e + \Pi^o. \tag{10}$$

Comparing with Eq. (3) we see that the parity-even part flips its sign (the parity-odd part remains the same). A similar result is obtained for a LH propagator zero.

Equation (10) is our main result. To start, it demonstrates that a propagator zero acts as a coupled state at low energy in the gauge theory through the poles it generates in the vertices. Let us concentrate on the parity-even part Π^e, postponing the parity-odd part to our discussion of anomalies below. Differentiating it twice with respect to A_μ gives the familiar result

$$\Pi^e_{\mu\nu}(k) = -\frac{g^2}{2}\int\frac{d^d p}{(2\pi)^d}\frac{\mathrm{tr}(\slashed{p}\gamma_\mu(\slashed{p} + \slashed{k})\gamma_\nu)}{p^2(p+k)^2}. \tag{11}$$

In the Abelian case, this result is valid for a unit-charge field. In the non-Abelian case, the result is multiplied by $\mathrm{tr}(T_a T_b) = T\delta_{ab}$, where T is the group trace in the fermion representation.

The integral (11) is UV divergent and may be computed for example using dimensional regularization. In four dimensions, after renormalization, the result is

$$\Pi^e_{\mu\nu} = \frac{g^2}{24\pi^2}(k^2\delta_{\mu\nu} - k_\mu k_\nu)\log\left(\frac{k^2}{4\pi\mu^2}\right) + \cdots, \tag{12}$$

where μ is the renormalization scale. The dots stand for a finite term, which is subleading compared to the logarithm.

The logarithm in Eq. (12) determines the fermion contribution to the one-loop beta function. Because Π^e flips sign between Eqs. (3) and (10), it follows that the contribution of a propagator zero to the one-loop beta function has the same absolute value as a normal fermion, but an opposite sign. Similarly, the contribution of a propagator zero to the imaginary part of the vacuum polarization will be negative, in violation of the optical theorem. The propagator zero thus acts as a ghost state, which ruins the unitarity of the gauge theory in the SMG phase.

A similar result was found in the 1980s in Refs. [45,46], where a specific nonlocal lattice action [47] was studied in which the fermion doublers were replaced by poles in the action, or equivalently, zeros in the fermion propagator. We have employed an effective Lagrangian approach to generalize this result, finding that the effect occurs in complete generality.

Let us illustrate the consequences of these findings for the mirror fermion approach. For definiteness, we start with a four-dimensional lattice gauge theory of massless domain-wall fermions, in which the LH spectrum on one wall constitutes the fermion spectrum of the target chiral gauge theory. The RH spectrum on the other wall is the mirror spectrum. Before we turn on any interactions to gap the mirrors, the theory is vectorlike. Let the contribution of these Dirac fermions to the coefficient of the one-loop beta function be b_f. This contribution is split evenly between the LH and RH fields, with each contributing $b_f/2$. The one-loop contribution of the target chiral fermions is thus $b_f/2$.

Now let us assume that the RH fermions have been gapped in an SMG phase, with each RH propagator pole replaced by a propagator zero. What we find is that the total fermion contribution to the one-loop beta function will now vanish, because the contribution of the LH target chiral fermions, which is $b_f/2$, is canceled by that of the RH propagator zeros, which is $-b_f/2$.

In two dimensions, restricting ourselves to the Abelian case for simplicity, the vacuum polarization gives rise to a photon mass squared m_{ph}^2 which, in turn, is proportional to the sum of the U(1) charges squared [48] (for a textbook discussion, see Ref. [49]). The target chiral gauge theory, which consists of half of the massless fermion degrees of freedom of the initial domain-wall fermion lattice theory (and contains chiral fields of both chiralities), is expected to generate a mass squared $m_{\mathrm{ph}}^2/2$. Again, we find that the contribution of the propagator zeros has the same absolute value and an opposite sign, and is thus given by $-m_{\mathrm{ph}}^2/2$. The result is a vanishing photon mass in the SMG phase. These effects, in both four and two dimensions, reflect the role of the propagator zeros as coupled ghost states.

Propagator zero and anomalies.—We first briefly recall the calculation of the axial anomaly, starting with the two dimensional case. One way to perform the calculation is to introduce both a vector gauge field V_μ and an axial gauge field A_μ, and consider the Lagrangian

$$\mathcal{L} = \bar{\psi}\left(\partial + \frac{ig}{2}(\slashed{V} + \slashed{A}\gamma_5)\right)\psi. \tag{13}$$

If we set $V = A$, this reduces to

$$\mathcal{L} = \bar{\psi}(\partial + ig\slashed{A}P_R)\psi. \tag{14}$$

This Lagrangian describes a RH fermion coupled to the gauge field together with a free LH fermion. In dimensional regularization Eq. (13) is thus an adequate starting point for the calculation of the anomaly of a RH fermion. When calculating the anomaly using dimensional regularization, one must make sure to use a consistent definition of γ_5, or of its two-dimensional counterpart (see, for example, Refs. [49,50]). Below, we will be careful to only perform manipulations that are valid within dimensional regularization. This means, for example, that when we calculate a diagram we are allowed to anticommute γ_5 through $\slashed{A}$, because the external gauge field (and momenta) are two or four dimensional by assumption. However, we are not allowed to anticommute γ_5 through the fermion propagator, because of its evanescent part.

The two-dimensional axial anomaly arises from the bubble diagram with one V vertex and one A vertex. Starting from the Lagrangian (13), this diagram is given by

$$\Pi^{AV} = \frac{g^2}{4}\mathrm{tr}\left(\frac{1}{\partial}\slashed{V}\frac{1}{\partial}\slashed{A}\gamma_5\right). \tag{15}$$

In order to calculate the divergence of the axial current, one replaces $\slashed{A}$ by $\slashed{q}$, where q is the external momentum. The well-known result is $\propto \epsilon_{\mu\nu}\partial_\mu V_\nu$. Alternatively, using the two-dimensional relation $\gamma_\mu\gamma_5 \propto \epsilon_{\mu\nu}\gamma_\nu$ one can express the parity-odd part of the vacuum polarization in terms of the parity-even part, and then obtain the anomaly. For details see, e.g., Ref. [49].

Proceeding to the case of a propagator zero, our starting point is

$$\mathcal{L} = m^2\bar{\psi}\frac{1}{\partial + \frac{ig}{2}(\slashed{V} + \gamma_5\slashed{A})}\psi. \tag{16}$$

Note that this time γ_5 is to the left of $\slashed{A}$. If we set $V = A$, we now obtain

$$\mathcal{L} = m^2\bar{\psi}\frac{1}{\partial + igP_R\slashed{A}}\psi$$
$$= m^2\left(\bar{\psi}_R\frac{1}{\partial + ig\slashed{A}}\psi_R + \bar{\psi}_L\frac{1}{\partial}\psi_L\right). \tag{17}$$

The second equality holds in two and four dimensions, where γ_5 anticommutes with the Dirac operator, showing the adequacy of this starting point [compare Eq. (7)]. Again, for our perturbative calculations we define the

inverses of all differential operators that depend on external gauge fields through their expansions in powers of g.

Like Eq. (13), also Eq. (16) allows for the use of dimensional regularization. As in the case of the vacuum polarization, the two-dimensional anomaly of a propagator zero involves tadpole and bubble diagrams. The details are similar, and the final result is that the contribution of a propagator zero is Π^{AV}, with a contribution $-\Pi^{AV}$ from the bubble diagram [the minus sign comes from the location of γ_5 to the left of A in Eq. (16), instead of to the right in Eq. (13)] and a contribution $+2\Pi^{AV}$ from the tadpole diagram. In other words, a propagator pole and a propagator zero give rise to the same anomaly. This is consistent with our previous result, that the parity-odd part of the vacuum polarization is the same for a propagator pole and for a propagator zero.

The four-dimensional case is similar. Starting from the Lagrangian (13), the usual triangle diagram is given by

$$\Pi^{AVV} = -\frac{ig^3}{16}\operatorname{tr}\left(\frac{1}{\partial}\slashed{V}\frac{1}{\partial}\slashed{V}\frac{1}{\partial}\slashed{A}\gamma_5\right). \qquad (18)$$

For the contribution of a propagator zero, starting from the nonlocal Lagrangian (16) we have tadpole, bubble, and triangle diagrams. The triangle diagram for a propagator zero turns out to be the same as for a propagator pole (in this case the minus sign from the different placement of γ_5 is canceled by the overall relative sign of the three vertices contributing to the triangle diagram), while the bubble and tadpole diagrams cancel each other. The final result is again that a propagator pole and a propagator zero give rise to the same anomaly.

A formal but quick way to keep track of the relative signs is to note that the fermion partition function defined by the Lagrangian (13) is $\det[\partial + (ig/2)(\slashed{V} + \slashed{A}\gamma_5)]$, whereas the partition function defined by the nonlocal Lagrangian (16) is $\det^{-1}[\partial + (ig/2)(\slashed{V} + \gamma_5\slashed{A})]$. For the purpose of perturbative calculations, the latter partition function may alternatively be expressed as a path integral over a commuting complex field ϕ carrying a spinor index, with Lagrangian $\phi^\dagger[\partial + (ig/2)(\slashed{V} + \gamma_5\slashed{A})]\phi$. It is straightforward to see that the diagrammatic expansion of these determinants reproduces the relative signs we found above.

Let us revisit the mirror fermion approach to the construction of lattice chiral gauge theories in the light of our findings. As explained in the introduction, this approach was based on two premises. The first premise is that gapping the mirrors in an SMG phase will decouple them entirely from the low-energy gauge theory. The second is that the target chiral fermion spectrum must be anomaly free, in order to respect the gauge invariance of the lattice theory after the mirrors have been gapped.

What we have found is that, if the mirror propagator poles are traded with propagator zeros in the SMG phase, then they will remain coupled to the gauge theory at low energy. Gauge invariance is in fact always maintained, regardless of whether or not the fermion spectrum of the target chiral gauge theory is anomaly free. The reason is that each propagator zero generates the same anomaly as the corresponding propagator pole. Thus, the propagator zeros will always cancel the anomaly of the target chiral fermions, as did the original mirror propagator poles. The underlying reason for this result is gauge invariance, which was exact before the SMG dynamics was turned on, and remains so after. In two dimensions, an interesting relation was found between SMG and anomaly cancellation expressed through the "boundary fully gapping rules" of Ref. [34], but it turns out that this plays no role in arriving at our conclusions about anomaly cancellation between the target and mirror sectors of a model with symmetric mass generation.

At the same time, unitarity is always lost, because the propagator zeros contribute to the one-loop beta function in four dimensions, or to the photon mass squared in two-dimensional Abelian theories, as ghost states.

In principle, apart from the propagator zeros (and the target chiral fermions) there could exist bound states of the lattice fields that also contribute at large distances. Such states, if present, would remedy the theory only if they undo the effect of the propagator zeros both for the beta function and for the anomaly, without generating any other long distance effects. It is hard to see how this would come about.

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