Sixfold Way of Traversable Wormholes in the Sachdev-Ye-Kitaev Model

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In the infrared limit, a nearly anti–de Sitter spacetime in two dimensions (AdS_2) perturbed by a weak double trace deformation and a two-site ($q > 2$)-body Sachdev-Ye-Kitaev (SYK) model with N Majoranas and a weak $2r$-body intersite coupling share the same near-conformal dynamics described by a traversable wormhole. We exploit this relation to propose a symmetry classification of traversable wormholes depending on N, q, and r, with $q > 2r$, and confirm it by a level statistics analysis using exact diagonalization techniques. Intriguingly, a time-reversed state never results in a new state, so only six universality classes occur—A, AI, BDI, CI, C, and D—and different symmetry sectors of the model may belong to distinct universality classes.

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A generic many-body quantum chaotic system that does not suffer from localization [1,2] eventually reaches an ergodic state governed by the symmetries of the system, rather than by the microscopic details of its Hamiltonian. Since this ergodic state only depends on global symmetries, it is possible to classify the dynamics by these symmetries. The study of level statistics is a powerful tool for establishing this classification because the level statistics of quantum chaotic systems [3,4] agree with the predictions of random matrix theory (RMT) [5–11]. Based on the theory of symmetric spaces, it was concluded that, after taking care of unitary symmetries, only ten universality classes exist, the so-called tenfold way of RMT [12]. The symmetry classification was later extended to non-Hermitian systems, where 38 universality classes exist [13–18].

The ten universality classes of Hermitian quantum chaotic systems have already been identified. Three classes related to the presence of time-reversal symmetry (TRS) (an antiunitary operator that commutes with the Hamiltonian) were reported in early studies in nuclear physics [5,6] and single-particle quantum chaotic systems [3,19], pertaining to systems with time-reversal invariance (class AI), broken time-reversal invariance (class A), and time-reversal invariance with broken rotational invariance and half-integer spin (class AII). Ensembles of antisymmetric [20] and anti-self-dual [21] Hermitian random matrices (classes D and C, respectively) were also discovered early on. Later, studies of the spectral properties of the QCD Dirac operator [22,23] revealed the existence of three more universality classes related to chiral symmetries represented by a unitary operator that anticommutes with the Hamiltonian (classes AIII, BDI, and CII). Shortly afterward, the classification was completed by adding chiral matrices with symmetric and antisymmetric off-diagonal blocks (classes CI and DIII, respectively) [12]. Physically, these classes are realized in superconducting systems with particle-hole symmetry (PHS) (an antiunitary operator that anticommutes with the Hamiltonian).

A related question is how many of the universality classes can be identified in more specific Hamiltonians describing a certain phenomenon. For instance, a full classificatory scheme was worked out for topological insulators in Ref. [24] and for systems at the Anderson transition [1] in Refs. [25–28]. More recently, the Sachdev-Ye-Kitaev (SYK) model [29–39] has been classified [40–49] in terms of RMT, thus providing a symmetry classification of quantum black holes in two-dimensional nearly anti-de Sitter (AdS_2) backgrounds. The relation between quantum gravity and the SYK model [37,39,50–52] has been extended to traversable [53], Euclidean [53,54], and Keldysh [55] wormholes. A study of level statistics [56] revealed that traversable wormholes [53] belong to the universality class of systems with TRS (class AI). A natural question to ask is whether this symmetry is a necessary condition for the existence of traversable wormholes.

The main goal of this Letter is to answer this question by providing an explicit symmetry classification of SYK configurations whose gravity dual is a nearly AdS_2 traversable wormhole that encompasses six universality classes. For that purpose, we introduce a two-site, left (L) and right (R), Hermitian SYK Hamiltonian $H = H_L + \alpha(-1)^{q/2}H_R + \lambda H_I$ [53,57–60] with left-right asymmetry parameter α and coupling constant λ. The two single-site q-body SYK Hamiltonians $H_{L,R}$ of N Majorana fermions and the $2r$-body Hamiltonian H_I coupling them are given by

$$H_{L,R} = \mathrm{i}^{q/2} \sum_{i_1<\cdots<i_q}^{N} J_{i_1\cdots i_q}\psi_{i_1}^{L,R}\cdots\psi_{i_q}^{L,R}, \tag{1}$$

$$H_I = \mathrm{i}^r \frac{N^{1-r}}{r}\left(\sum_{i=1}^{N}\psi_i^L\psi_i^R\right)^r, \tag{2}$$

where the couplings $J_{i_1\cdots i_q}$ are Gaussian random variables with zero mean and variance $\sigma^2 = 2^{q-1}(q-1)!/(qN^{q-1})$. The Majorana fermions satisfy the commutation relation $\{\psi_i^A, \psi_j^B\} = \delta_{AB}\delta_{ij}$ ($i, j = 1, \ldots, N$ and $A, B = L, R$). The parameters N and q are taken to be even (see the Supplemental Material [61] for odd N).

Symmetry classification.—The class of H is determined by its behavior under antiunitary symmetries: (1) TRS: $THT^{-1} = +H$, $T\mathrm{i}T^{-1} = -\mathrm{i}$, $T^2 = \pm 1$; (2) PHS: $CHC^{-1} = -H$, $C\mathrm{i}C^{-1} = -\mathrm{i}$, $C^2 = \pm 1$. Any antiunitary symmetry A can be decomposed as $A = UK$, with U unitary and K complex conjugation, and we choose a basis such that

$$K\psi_i^L K^{-1} = \psi_i^L, \qquad K\psi_i^R K^{-1} = -\psi_i^R. \tag{3}$$

We define the unitary left and right parities,

$$S_L = (2\mathrm{i})^{N/2}\prod_{i=1}^{N}\psi_i^L, \qquad S_R = (2\mathrm{i})^{N/2}\prod_{i=1}^{N}\psi_i^R, \tag{4}$$

the total parity, $S = S_L S_R$, and the (exponential of the) spin operator,

$$Q = \exp\left\{-\frac{\pi}{4}\sum_{i=1}^{N}\psi_i^L\psi_i^R\right\} = 2^{-N/2}\prod_{i=1}^{N}\left(1 - 2\psi_i^L\psi_i^R\right). \tag{5}$$

They square to $S_L^2 = S_R^2 = S^2 = +1$ and $Q^2 = S$. Their action on the Majorana fermions is given by (recall that, throughout the main text, N is even)

$$\begin{aligned} S_{L,R}\psi_i^{L,R}S_{L,R}^{-1} &= -\psi_i^{L,R}, \qquad & S_{L,R}\psi_i^{R,L}S_{L,R}^{-1} &= \psi_i^{R,L},\\ Q\psi_i^R Q^{-1} &= -\psi_i^L, & Q\psi_i^L Q^{-1} &= \psi_i^R, \end{aligned} \tag{6}$$

i.e., the parity operators act inside each site, flipping the sign of the respective Majoranas, while, up to a sign, the spin operator exchanges the two species. Other operations are obtained by compositions, e.g., the total parity S reverses the signs of the Majoranas in both sites simultaneously. Using Eqs. (3) to (6), the transformation properties of $H_0 \equiv H_L + \alpha(-1)^{q/2}H_R$ and H_I under the unitary and antiunitary operators are

$$\begin{aligned} S_{L,R}H_0S_{L,R}^{-1} &= H_0, \qquad & S_{L,R}H_IS_{L,R}^{-1} &= (-1)^r H_I,\\ QH_0Q^{-1} &= (-1)^{q/2}H_0, & QH_IQ^{-1} &= H_I,\\ KH_0K^{-1} &= (-1)^{q/2}H_0, & KH_IK^{-1} &= H_I. \end{aligned} \tag{7}$$

Note the transformation of H_0 under Q holds only for $\alpha = 1$. If $\alpha \neq 1$, the left-right spin operator Q is not a symmetry of H.

The symmetry classification of H, which follows from Eq. (7), depends on N mod 4 [65] (in contrast to the one-site classification which depends on N mod 8 [40–42,46]), the parity of $q/2$ and r, and whether there is left-right symmetry ($\alpha = 1$) or not ($\alpha \neq 1$), see Tables I–IV. Below we provide a brief justification of this classification, see the Supplemental Material [61] for a detailed derivation.

First, for the action of an antiunitary symmetry to be well-defined, all commuting unitary symmetries (conserved quantities) must be resolved. That is, in the common

TABLE I. Symmetry classification of the two-site SYK Hamiltonian for even $q/2$ and even r. Each line corresponds to a block of the Hamiltonian, labeled by the eigenvalues of the conserved quantities $S_{L,R}$ and Q. For each of the six blocks, we give its dimension and its symmetry class both for the left-right symmetric ($\alpha = 1$) and asymmetric ($\alpha \neq 1$) cases and for N mod 4 = 0, 2.

				N mod 4 = 0		N mod 4 = 2	
S_L	S_R	Q	Dimension	$\alpha = 1$	$\alpha \neq 1$	$\alpha = 1$	$\alpha \neq 1$
+1	+1	+1	$(2^N + 2^{N/2+1})/8$	AI	AI	A	A
		−1	$(2^N - 2^{N/2+1})/8$	AI		A	
−1	−1	+1	$(2^N + 2^{N/2+1})/8$	AI	AI	A	A
		−1	$(2^N - 2^{N/2+1})/8$	AI		A	
+1	−1	—	$2^N/4$	AI	AI	AI	A
−1	+1	—	$2^N/4$	AI	AI	AI	A

TABLE II. Same as Table I, but for even $q/2$ and odd r. There are four blocks labeled by the eigenvalues of S and Q. The results are independent of N.

S	Q	Dimension	$\alpha = 1$	$\alpha \neq 1$
$+1$	$+1$	$(2^N + 2^{N/2+1})/4$	AI	AI
	-1	$(2^N - 2^{N/2+1})/4$	AI	
-1	$+\mathrm{i}$	$2^N/4$	AI	AI
	$-\mathrm{i}$	$2^N/4$	AI	

TABLE IV. Same as Table I, but for odd $q/2$ and odd r. There are two blocks labeled by the eigenvalues of S.

		N mod $4 = 0$		N mod $4 = 2$	
S	Dimension	$\alpha = 1$	$\alpha \neq 1$	$\alpha = 1$	$\alpha \neq 1$
$+1$	$2^N/2$	BDI	D	CI	C
-1	$2^N/2$	BDI	D	CI	C

eigenbasis of H and its unitary symmetries, H assumes a block-diagonal structure, and the antiunitary symmetries T and C must act within a single block. The two-site SYK Hamiltonian can have two, four, or six blocks, as indicated in Tables I–IV. The total parity S is always conserved and, thus, H has at least two blocks identified by its eigenvalues $s = \pm 1$. For $\alpha = 1$, we have the following possibilities: (1) If $q/2$ and r are both odd, there is no other unitary symmetry and there are only two blocks (Table IV). (2) If r is odd but $q/2$ is even, Q is a symmetry of H and, because of $QS = SQ$, it splits the two blocks into two subblocks each and we get four blocks, labeled by the eigenvalues of S, $s = \pm 1$, and Q, $k = \pm 1, \pm \mathrm{i}$ (Table II). (3) When r is even, S_L and S_R are independently conserved, defining, at least, four blocks, $s_{L,R} = \pm 1$. If, moreover, $q/2$ is odd, these four blocks are the only blocks (Table III). (4) If, instead, $q/2$ is even (with r still even), and because $QS_{L,R} = SS_{L,R}Q$ [see Eq. (S8) of the Supplemental Material [61]), the two blocks with $s = +1$ get split by Q into two subblocks each, while the two blocks with $s = -1$ do not; in total, we get six blocks (Table I). If $\alpha \neq 1$, the blocks of $S_{L,R}$ and S are not split by Q (four blocks for r even, two for r odd).

Second, TRS is implemented by either $T = K$ (for even $q/2$ and any α) or $T = QK$ (for odd $q/2$ and $\alpha = 1$). In either case, we have $T^2 = +1$ and we conclude that H displays the same level statistics as random matrices from either the Gaussian orthogonal ensemble (GOE) [4,11]—if T acts within a single block of H—, or the Gaussian Unitary Ensemble (GUE) [4,11]—if T connects different blocks. This is determined by the commutator of T with the orthogonal projector onto the respective block, which must be checked on a case-by-case basis, see the Supplemental Material [61]. For odd $q/2$ and $\alpha \neq 1$, since Q is not a symmetry of H, there is no TRS and all blocks display GUE statistics.

Third, for all cases except for $q/2$ and r both odd, there is no PHS, then T is the only antiunitary symmetry, and all blocks belong either to class AI (if T acts within a single block) or class A (if it connects different blocks). When $q/2$ and r are both odd, there exists a PHS implemented by $C = S_L K$, which squares to $C^2 = (-1)^{N(N-1)/2}$, and commutes with the projector into a block with fixed S. In the left-right symmetric case ($\alpha = 1$), we thus have simultaneous TRS and PHS, and the blocks of H belong to class BDI (for N mod $4 = 0$) or CI (for N mod $4 = 2$). In the asymmetric case ($\alpha \neq 1$), we have only PHS and the blocks belong to class D (N mod $4 = 0$) or C (N mod $4 = 2$). Therefore, a slight asymmetry, $\alpha \approx 1$, substantially changes the universality class.

Comparing our results with the tenfold way [12], we can state the main results of this Letter. We have found a sixfold classification of the two-site SYK model Eq. (1): classes A, AI, BDI, CI, C, and D. Remarkably, for some parameters, different blocks of the same Hamiltonian belong to distinct symmetry classes, in contrast to the single-site SYK model [45]. Of the four remaining classes, class AIII—also not found in the standard single-site SYK model—could be realized by a Wishart extension of the model [49] based on the product of two SYKs with complex-conjugated couplings. On the other hand, no classes with symplectic symmetry (i.e., $T^2 = -1$, classes AII, CII, and DIII)—whose level statistics in the bulk are given by the Gaussian symplectic ensemble (GSE)—occur in the classification. The absence of these three classes, which indicates that a time-reversed state never results in a new state, arises as a fundamental restriction from the left-right symmetric intersite coupling of two SYKs. Universality classes with symplectic symmetry can still be realized if one considers a model with an asymmetric interaction, see the Supplemental Material [61]. The absence of $\mathrm{AII}^\dagger$ statistics (the equivalent of GSE statistics in non-Hermitian systems [16]) has been observed recently in Lindbladian quantum dissipative dynamics [65,66], which by construction has left-right symmetry. Therefore, generic coupled quantum systems with a left-right symmetric interaction do not have this symplectic symmetry.

Level statistics.—To confirm the proposed symmetry classification, we compare level correlations for different

TABLE III. Same as Table I, but for odd $q/2$ and even r. There are four blocks labeled by the eigenvalues of $S_{L,R}$.

			N mod $4 = 0$		N mod $4 = 2$	
S_L	S_R	Dimension	$\alpha = 1$	$\alpha \neq 1$	$\alpha = 1$	$\alpha \neq 1$
$+1$	$+1$	$2^N/4$	AI	A	A	A
-1	-1	$2^N/4$	AI	A	A	A
$+1$	-1	$2^N/4$	A	A	AI	A
-1	$+1$	$2^N/4$	A	A	AI	A

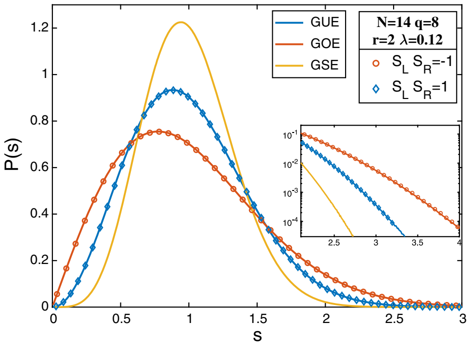

FIG. 1. Level spacing distributions $P(s)$ for $N = 14$, $r = 2$, $q = 8$, $\lambda = 0.12$, and $\alpha = 1$ that belong to the symmetry class A (GUE) or AI (GOE) depending on whether $S = S_L S_R = \pm 1$, see Table I. We find excellent agreement with RMT even in the tail of the spectrum (inset).

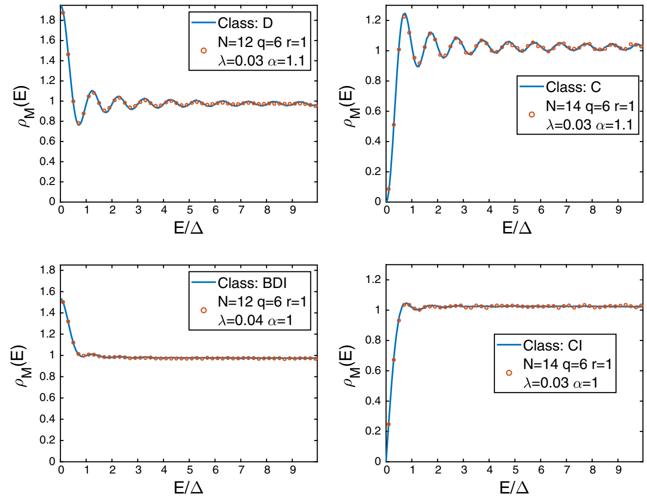

FIG. 2. Microscopic spectral density $\rho_M(E)$ in units of the mean level spacing Δ near $E_0 = 0$, for the values of the Hamiltonian Eq. (1) indicated in the legend. We find excellent agreement with the RMT result for the predicted universality class.

choices of parameters (N, q, r, and α), with the predictions of RMT for the corresponding universality classes. This procedure is justified because the SYK model is quantum chaotic and deviations from RMT only affect a few eigenvalues close to the ground state [56]. The spectrum of the Hamiltonian (1) is obtained by exact diagonalization techniques. At least 10^5 eigenvalues are used for a given set of parameters.

For the study of classes A and AI, we employ the distribution $P(s)$ of the level spacings $s_i = (E_i - E_{i-1})/\Delta$, where E_i is the set of ordered eigenvalues and Δ is the mean level spacing. We unfold the spectrum [4] using a low-order (at most sixth) polynomial fitting. We have found that blocks with $T^2 = +1$ (class AI) exhibit GOE level statistics, while blocks without TRS (class A) display GUE statistics. For $\lambda = 0$, both sites are uncorrelated, so the level statistics are given by Poisson statistics. Likewise, when $\lambda \to \infty$, the integrable H_I dominates and thus level statistics are not given by RMT either. Therefore, it is necessary to choose an intermediate value of λ, so that levels are sufficiently mixed by the interaction. As an illustrative example, Fig. 1 depicts the level spacing distribution in a case with even $q/2$ and r, N mod $4 = 2$, and $\alpha = 1$, where the classification predicts class A (in blocks with total parity $s = +1$) or AI ($s = -1$). These results confirm the agreement with the RMT prediction for the expected universality class, even in the tail of the distribution. An exhaustive confirmation of all the remaining cases, employing the spacing ratio distribution [67,68], is presented in the Supplemental Material [61].

The remaining four universality classes (BDI, CI, C, and D) are related to the existence of involutive symmetries that anticommute with the Hamiltonian. As a result, the spectrum is symmetric around $E_0 = 0$. Spectral correlations very close to E_0, probed by, e.g., the microscopic spectral density [23] expressed in units of the local mean level spacing, or the distribution of eigenvalues closest to E_0 [48,69–73], have distinct features that fully characterize the four universality classes. To illustrate this, in Fig. 2, we compare the microscopic spectral density around $E_0 = 0$ for odd $q/2$ and r, and different values of N mod 4 and α, corresponding to universality classes BDI ($\alpha = 1$, N mod $4 = 0$), CI ($\alpha = 1$, N mod $4 = 2$), D ($\alpha \neq 1$, N mod $4 = 0$), and C ($\alpha \neq 1$, N mod $4 = 2$), see Table IV. In all cases, we find excellent agreement with the RMT result. The complementary analysis in terms of the distribution of the eigenvalue closest to $E_0 = 0$, presented in the Supplemental Material [61], shows a similar agreement.

Traversable wormhole classification.—Having established the symmetry classification of the SYK Hamiltonian Eq. (1), we now study for which parameters (q, r, λ, and temperature T), this model is related to a traversable wormhole [53,74] in a near AdS_2 background [75,76]. First, we note that the traversable wormhole [53] requires a weak intersite coupling $\lambda \ll 1$, and a low temperature T, i.e., strong intrasite coupling. The small-λ condition is necessary to account rigorously [53,74] for the effect of a double trace deformation coupling the two boundaries in the gravitational path integral. In this limit, the holographic relation between the two-site SYK model and the gravity system is established by demonstrating that both models share the same low-energy effective action, which, in this case, is a generalized Schwarzian [53]. For $q = 4$ and $r = 1$, this program was carried out in Ref. [53]. A distinct feature of the wormhole phase for $r = 1$, confirmed by the numerical solution of the large-N Schwinger-Dyson (SD) equations [39], is the existence of a gapped ground state at low temperature. Since $\lambda \ll 1$, the gap $E_g \sim \lambda^\gamma$, $\gamma = 2/3 < 1$ for $q = 4$, is enhanced with respect to the perturbative result $E_g \sim \lambda$. Physically, this is interpreted as an enhanced tunneling rate induced by the strong intrasite interactions in the SYK model, and as a traversable wormhole on the

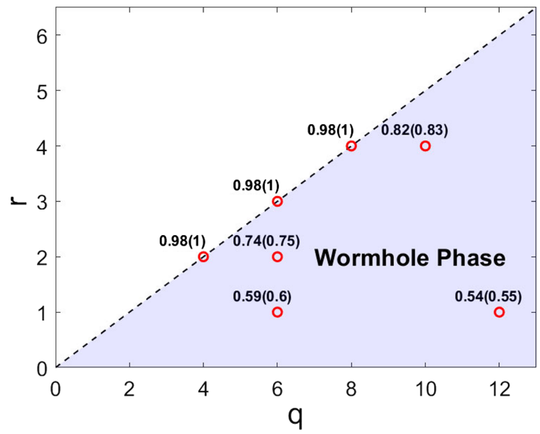

FIG. 3. Phase diagram of the Hamiltonian Eq. (1) as a function of q and r, obtained from the gap $E_g \propto \lambda^\gamma$. The region of parameters where we expect traversable wormhole physics (purple region) is characterized by $\gamma < 1$ and delimited by the dashed line $q = 2r$. The red circles give γ for the different values of q and r. The analytic result for E_g [in brackets, see Eq. (8)], is compared to the numerical value obtained from the exponential decay of Green's functions (see text and the Supplemental Material [61]).

gravity side. Generally, a gap $E_g \sim \lambda^\gamma$ with $\gamma < 1$ for $\lambda \ll 1$ is a defining feature of the traversable wormhole phase. Based on this definition, we constrain the previous symmetry classification to the values of q and r for which the SYK model Eq. (1) has an interaction-enhanced gap, i.e., $\gamma < 1$.

In order to proceed, we generalize the results of Ref. [53] by simply replacing $\Delta \equiv 1/q \to r/q$ in the generalized Schwarzian action of Ref. [53]. The resulting gap is given by

$$E_g \propto \lambda^{\frac{q}{2(q-r)}}. \tag{8}$$

We have confirmed this scaling with λ in the large-N limit by computing E_g numerically from the solution of the SD equations [39]. Results depicted in Fig. 3 for different q and r show an excellent agreement between the numerical result and analytic prediction $\gamma = q/[2(q-r)]$. For the technical procedure to solve the SD equations and extract the gap from the Green's function decay, we refer to both Refs. [53,56] and the Supplemental Material [61].

As a consequence of Eq. (8), only SYKs of the form (1) with $q > 2r$ (purple region in Fig. 3) can be dual to a traversable wormhole. For $q < 2r$ (white region in Fig. 3), $\gamma \geq 1$ and hence there is no tunneling enhancement, so there is no wormhole phase. The borderline case $q = 2r$ (dashed line in Fig. 3) would require further analysis to completely rule out the existence of a wormhole dual. While we have so far restricted ourselves to the case of identical SYKs, the observation of universality classes D and C requires $\alpha \neq 1$. This is not a problem because wormhole features are not qualitatively affected provided that α is sufficiently close to one [53,56]. Most importantly, the condition $q > 2r$ does not restrict the possible symmetry classes and all six occur for either $\alpha = 1$ or $\alpha \approx 1$. Finally, we note that another feature associated with a traversable wormhole, namely, the existence of a first-order phase transition in the free energy, also occurs in our SYK setting, see the Supplemental Material [61].

In conclusion, based on the relation between a two-site SYK model at low temperature and weak intersite coupling, we have identified AdS_2 traversable wormholes belonging to six universality classes: A, AI, BDI, CI, C, and D. Wormholes with symplectic symmetry (classes AII, CII, and DIII) are conspicuously missing. Moreover, enhanced tunneling that is a signature of wormhole physics only occurs for $q > 2r$, see Eq. (8). A natural extension of this work is the symmetry classification of coupled non-Hermitian SYKs, whose gravity dual are Euclidean and Keldysh wormholes [53–55,65,77].

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