Axion Paradigm with Color-Mediated Neutrino Masses

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(Received 20 September 2023; revised 17 November 2023; accepted 2 January 2024; published 1 February 2024)

We propose a generalized Kim-Shifman-Vainshtein-Zakharov–type axion framework in which colored fermions and scalars act as two-loop Majorana neutrino-mass mediators. The global Peccei-Quinn symmetry under which exotic fermions are charged solves the strong CP problem. Within our general proposal, various setups can be distinguished by probing the axion-to-photon coupling at helioscopes and haloscopes. We also comment on axion dark-matter production in the early Universe.

DOI: 10.1103/PhysRevLett.132.051801

Introduction.—Despite its remarkable success, the standard model (SM) fails to account for neutrino masses [1,2] and cosmological dark matter (DM) [3,4], new physics being required to explain both phenomena. Most SM extensions either focus on only one of these problems or, when both are addressed, their solutions are not directly connected. However, there have been many attempts to relate DM to neutrino-mass generation. For example, in scotogenic scenarios neutrino masses arise radiatively from the exchange of DM states [5,6], a paradigm that has been further developed in many recent studies—see, e.g., Refs. [7–19]. An interesting alternative is to consider dark sectors that trigger lepton number violation and neutrino-mass generation within low-scale seesaw schemes [20–22].

Another long-standing drawback of the SM concerns the strong CP symmetry violation in QCD, encoded by the (arbitrary) phase parameter $\bar{\theta}$. The fact that severe experimental constraints on the neutron electric dipole moment [23,24] force $|\bar{\theta}| < 10^{-10}$ is known as the strong CP problem. One of the most elegant solutions to this problem is the Peccei-Quinn (PQ) mechanism [25,26] based on a classical QCD-anomalous $U(1)_{\mathrm{PQ}}$ symmetry. Spontaneous $U(1)_{\mathrm{PQ}}$ breaking leads to a pseudo-Goldstone boson—the axion a [27,28]. The ground state of a turns out to be such that it effectively sets $\bar{\theta} = 0$, providing a dynamical solution to the strong CP problem. Among the plethora of axion models, two main sets can be distinguished: the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) [29,30] and the Kim-Shifman-Vainshtein-Zakharov (KSVZ) [31,32] invisible-axion scenarios. In the former, SM quarks are charged under $U(1)_{\mathrm{PQ}}$, while in the latter the PQ-charged fields are exotic quarks (for a review see Ref. [33]).

Axions, which may be produced nonthermally in the early Universe via the so-called misalignment mechanism [34–36], can also be excellent alternatives [33] to weakly interacting massive particle DM [37]. It is then tempting to embed the axion paradigm in frameworks that simultaneously provide an explanation for small neutrino masses. This has been explored in the literature recently, for example by realizing the DFSZ or KSVZ axion within the type-I seesaw. Technically, natural setups that also address other SM shortcomings such as the baryon asymmetry of the Universe and inflation were proposed [38–42]. In this Letter, we suggest a new idea in which neutrino masses are generated at the quantum level via colored mediators that also provide a solution to the strong CP problem. This new class of KSVZ-type axion models connects three otherwise unrelated issues: small neutrino masses, the strong CP problem, and DM.

Framework.—The original KSVZ model [31,32] extends the SM with vectorlike fermions $\Psi_{L,R}$ in the fundamental representation of $SU(3)_c$, singlets under $SU(2)_L$, and with $Y = 0$. A complex scalar singlet σ breaks a $U(1)_{\mathrm{PQ}}$ symmetry spontaneously, providing mass to those exotic fermions. The phase of σ corresponds to the axion field a. The fact that left-handed and right-handed exotic fermions carry different PQ charges ensures the anomalous axion-gluon coupling, required to solve the strong CP problem.

In this Letter, we show that generic $\Psi_{L,R}$ fields in the $SU(3)_c$ complex representation (p, q) with $p > q = 0, 1, 2, \ldots$ can act as neutrino-mass mediators at the

TABLE I. Matter content and quantum numbers for our new KSVZ models with two-loop neutrino masses. Here, ω is the PQ charge, and $n = 1, 2, \ldots,\ p > q = 0, 1, 2, \ldots$.

Fields	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$U(1)_{\rm PQ}$	Multiplicity
Ψ_L	$[(p,q), 2n \pm 1, 0]$	ω	n_Ψ
Ψ_R	$[(p,q), 2n \pm 1, 0]$	0	n_Ψ
σ	$(\mathbf{1}, \mathbf{1}, 0)$	ω	1
η	$[(p,q), 2n, 1/2]$	0	n_η
χ	$[(p,q), 2n \pm 1, 0]$	0	n_χ

two-loop level. Two scalars η, χ with those same $SU(3)_c$ transformation properties are also required to put our mechanism at work. Both $\Psi_{L,R}$ and χ are hyperchargeless and transform as the same odd $SU(2)_L$ representation denoted by $2n \pm 1$. In contrast, η has $Y = 1/2$ and transforms as an even $SU(2)_L$ representation denoted by $2n$. As in the original KSVZ prescription, the complex scalar singlet σ with nonzero PQ charge ω is responsible for $U(1)_{\rm PQ}$ breaking, giving rise to the axion and to $\Psi_{L,R}$ masses (note that only Ψ_L carries PQ charge ω, Ψ_R is neutral). Table I lists all the new fields and their transformation properties under the SM and PQ symmetries.

The relevant new Yukawa terms are given by

$$-\mathcal{L}_{\rm Yuk.} \supset \mathbf{Y}_\Psi \overline{\Psi_L} \Psi_R \sigma + \frac{1}{2} \mathbf{Y}_{\chi_j} \Psi_R^T C \chi_j \Psi_R + \mathbf{Y}_i \bar{L} \eta_i^* \Psi_R + \text{H.c.}, \tag{1}$$

where L denotes the SM lepton doublet. For simplicity, from now on we omit color and $SU(2)_L$ indices. The multiplicities of Ψ, η and χ are n_Ψ, n_χ, and n_η, respectively. Thus, $\mathbf{Y}_\Psi$, $\mathbf{Y}_{\chi_j}$, and $\mathbf{Y}_i$ are $n_\Psi \times n_\Psi$, $n_\Psi \times n_\Psi$ and $3 \times n_\Psi$ complex Yukawa matrices, respectively, with $i = 1, \ldots, n_\eta$ and $j = 1, \ldots, n_\chi$.

The key scalar-potential terms responsible for neutrino mass generation are

$$V \supset \mu_{ijk} \chi_i \chi_j \chi_k + \kappa_{ij} \eta_i^\dagger \Phi \chi_j + \lambda_{ijk} \Phi^\dagger \eta_i \chi_j \chi_k + \text{H.c.}, \tag{2}$$

where $\Phi = (\phi^+, \phi^0)^T$ is the SM Higgs doublet. To preserve the $SU(3)_c$ symmetry the colored scalars η and χ must not acquire a vacuum expectation value, so that the only vacuum expectation values are $\langle \sigma \rangle = v_\sigma/\sqrt{2}$ breaking $U(1)_{\rm PQ}$, and $\langle \phi \rangle^0 = v/\sqrt{2} \simeq 174$ GeV triggering electroweak symmetry breaking.

Strong CP problem.—The PQ field $\sigma = (v_\sigma + \rho)\exp(ia/v_\sigma)/\sqrt{2}$ contains the axion a and the radial mode ρ. Once σ develops a nonzero v_σ, the PQ symmetry is spontaneously broken at a scale $f_{\rm PQ} = \langle \sigma \rangle = v_\sigma/\sqrt{2}$, leading to the axion decay constant

$$f_a = \frac{f_{\rm PQ}}{N} = \frac{v_\sigma}{\sqrt{2} N}, \tag{3}$$

where N is the color anomaly factor. The up-to-date QCD axion mass at next-to-leading order is [43]

$$m_a = 5.70(7) \left(\frac{10^{12}\ \text{GeV}}{f_a} \right) \mu\text{eV}. \tag{4}$$

This relation between m_a and f_a is a model-independent prediction of the QCD axion if the only explicit breaking of the PQ symmetry is by nonperturbative QCD effects. To be viable, the axion solution to the strong *CP* problem requires a nonvanishing anomaly factor N to ensure an axion-gluon coupling. For the models in Table I we get

$$N = 2 n_\Psi \omega (2n \pm 1) T(p, q), \tag{5}$$

with $T(p,q)$ the Dynkin index of the $SU(3)_c$ representation (p,q). As expected, N depends on the multiplicity of the colored fermions n_Ψ and on the Ψ_L PQ charge ω.

Neutrino-mass generation.—With the Yukawa and scalar interactions of Eqs. (1) and (2), two-loop Majorana neutrino masses arise from the diagrams in Fig. 1 [44].

Neutrino masses are mediated by colored particles, Ψ, η, and χ, transforming under the same $SU(3)_c$ representation. Moreover, since L and Φ are $SU(2)_L$ doublets, η must lie in an even $SU(2)_L$ representation, whereas Ψ and χ must be in an odd representation. The coupling between L and Ψ requires η to have $Y = 1/2$. In our scenario Ψ and χ carry no hypercharge, ensuring the Majorana nature of light neutrinos.

Among all generic scenarios in Table I, the simplest consistent realization of our idea is for $\Psi_{L,R}$ and η, χ to transform as triplets of $SU(3)_c$. Since the $SU(3)$ invariant

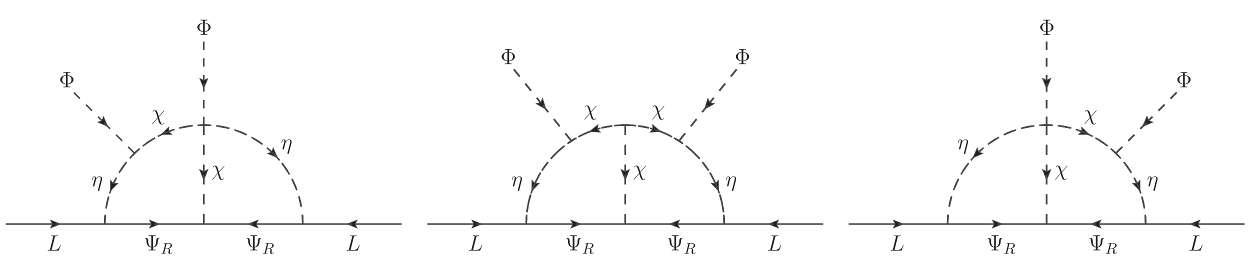

FIG. 1. Two-loop diagrams for neutrino-mass generation mediated by the colored particles of Table I.

coming from $(p,q) \otimes (p,q) \otimes (p,q)$ for $(p,q) \equiv \mathbf{3}$ is antisymmetric, the minimal required multiplicity is $n_\Psi = n_\chi = 2$, $n_\eta = 1$. For symmetric contractions, n_χ can also be as small as 1. Concerning $SU(2)_L$, η is a doublet while Ψ and χ are singlets. For $\omega = 1/2$, this setup predicts $N = 1$, just as in the original KSVZ model [31,32]. This minimal scenario simply extends the original KSVZ proposal with extra colored scalars η and χ, which mediate neutrino-mass generation. The resulting light-neutrino mass matrix is

$$(m_\nu)_{\alpha\beta} = \frac{N_c}{(16\pi^2)^2} \tilde{Y}^j_{a\alpha} (\tilde{Y}_\chi)^k_{ab} \tilde{Y}^l_{b\beta} \tilde{\mu}_{jkl} \mathcal{I}^{jkl}_{ab}, \tag{6}$$

where $j,k,l = 1,\ldots,6$ and $a,b = 1,2$. $N_c = 6$ is the color factor, $\tilde{Y}$ and $\tilde{Y}_\chi$ are Yukawa couplings, while $\tilde{\mu}$ denotes the cubic scalar couplings of Eq. (2), all written now in the mass basis. The loop function $\mathcal{I}^{jkl}_{ab}$ can be found in Refs. [45,53]. The above result can be estimated by

$$(m_\nu)_{\alpha\beta} \sim 0.1 \text{ eV} \left(\frac{\tilde{Y}^j_{a\alpha} (\tilde{Y}_\chi)^k_{ab} \tilde{Y}^l_{b\beta}}{10^{-3}} \right) \left(\frac{\tilde{\mu}_{jkl}}{10^8 \text{ GeV}} \right) \times \left(\frac{v}{246 \text{ GeV}} \right)^2 \left(\frac{10^8 \text{ GeV}}{m_\zeta} \right)^2, \tag{7}$$

where $m_\zeta = \sqrt{\lambda_{\text{eff}}} f_{\text{PQ}}$ is an effective colored scalar mass scale running in the loop with λ_{eff} being some quartic coupling parameter. A typical value for the PQ breaking scale is $f_{\text{PQ}} \sim 10^{12}$ GeV, so that axions account for the observed DM relic abundance. Hence, the scalars are expected to be heavy. The smallness of $\tilde{Y}_\chi$ and $\tilde{\mu}_{jkl}$ in Eq. (6) is symmetry-protected in t'Hooft's sense [58], as the Lagrangian acquires an additional $U(1)$ symmetry in their absence. Note also that, with only two copies of Ψ ($n_\Psi = 2$), one of the three light neutrinos is predicted to be massless due to the missing partner nature [59] of the underlying radiative seesaw mechanism. Charged lepton flavor violating processes would be mediated at one loop by the charged colored scalars and exotic fermions, but with very small rates [60].

In its minimal version, the above scenario implies that there is no cancellation in the $0\nu\beta\beta$ amplitude, even for normally ordered neutrino masses [12,17,61]. The resulting regions allowed by oscillation data correlate with the only free parameter available, i.e., the relative neutrino Majorana phase. One finds that, for inverted ordering, rates fall inside the expected sensitivities of the next round of experiments [62–65]. These would not only prove the Majorana nature by the black-box theorem [66], but could also ultimately determine the Majorana phase [67].

Probing the axion-to-photon coupling.—Indirect astrophysical and cosmological observations, as well as laboratory searches (for reviews see Refs. [33,68]), constrain the axion parameter space due to its couplings to photons, nucleons, and electrons. We now examine how to probe the various scenarios of Table I through their corresponding axion-to-photon coupling $g_{a\gamma\gamma}$.

TABLE II. E/N values for various $SU(3)_c \otimes SU(2)_L$ representation choices for Ψ [see Table I and Eq. (9)].

E/N		$SU(2)_L$ **3**	**5**	**7**	**9**	**11**
$SU(3)_c$	**3**	4	12	24	40	60
	6	8/5	24/5	48/5	16	24
	10	8/9	8/3	16/3	80/9	40/3
	15	1	3	6	10	15
	15′	4/7	12/7	24/7	40/7	60/7

In the KSVZ setup, the only chiral fermions charged under $U(1)_{\text{PQ}}$ are the new exotic fermions. Therefore, there are no model-dependent contributions to the axion coupling to nucleons and electrons. Using next-to-leading-order chiral Lagrangian techniques, one obtains [43]

$$g_{a\gamma\gamma} = \frac{\alpha_e}{2\pi f_a} \left[\frac{E}{N} - 1.92(4) \right], \tag{8}$$

where E and N are the model-dependent electromagnetic and color anomaly factors, respectively. For our class of models in Table I, we have

$$\frac{E}{N} = \frac{d(p,q)}{(2n \pm 1) T(p,q)} \sum_{j=0}^{2n\pm1-1} \left(\frac{2n \pm 1 - 1}{2} - j \right)^2, \tag{9}$$

with $d(q,p)$ being the dimension of $SU(3)_c$ representation. One sees that $E/N = 0$, as long as the hyperchargeless $\Psi_{L,R}$ are $SU(2)_L$ singlets. For higher weak multiplet representations $E/N \neq 0$, see Table II.

In Fig. 2, we display by oblique solid lines the axion-photon coupling $|g_{a\gamma\gamma}|$ in terms of m_a (bottom axis) and f_a (top axis). The black lines delimit the band of E/N values leading to the maximum and minimum $|g_{a\gamma\gamma}|$, corresponding to $E/N = 60$ for $\Psi \sim (\mathbf{3}, \mathbf{11}, 0)$ and $E/N = 12/7$ for $\Psi \sim (\mathbf{15'}, \mathbf{5}, 0)$, respectively [see Eq. (8) and Table II]. The $|g_{a\gamma\gamma}|$ corresponding to the popular KSVZ and DFSZ-I and II schemes are shown by the solid orange, light green, and dark green lines, respectively. The minimal KSVZ model featuring two-loop neutrino masses predicts $E/N = 0$ (solid orange line).

In the same plot we show the current bounds and future sensitivities from helioscopes and haloscopes. The CAST helioscope experiment [69] excludes the blue-shaded region, while haloscopes ADMX [70–72], RBF [73], CAPP [74], and HAYSTAC [75] exclude the magenta region. Projected sensitivities of IAXO [76], ADMX [77], and MADMAX [78] are indicated by the dashed blue, magenta, and purple contours, respectively. One sees that

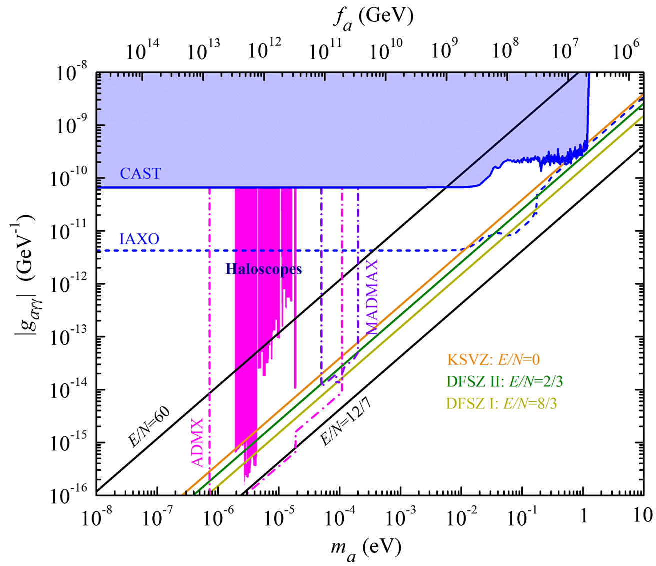

FIG. 2. $|g_{a\gamma\gamma}|$ versus m_a (bottom axis) and f_a (top axis) [see Eqs. (4) and (8)]. The black lines correspond to E/N values leading to maximum and minimum $|g_{a\gamma\gamma}|$ for the representations shown in Table II. The KSVZ and DFSZ I and II predictions are indicated by the orange, light green, and dark green lines, respectively. Shaded regions are presently excluded, while dash-dotted lines delimit projected sensitivities of several helioscope and haloscope experiments—see text for details.

the future (2025) IAXO experiment is expected to probe $g_{a\gamma\gamma}$ down to $(10^{-12}\text{–}10^{-11})$ GeV^{-1} reaching the popular QCD axion model predictions for $m_a \sim 0.1$ eV (blue-dashed contour). Out of all haloscope experiments, the most impressive is ADMX, which has already reached the KSVZ and DFSZ QCD axion lines for masses $m_a \sim 3$ μeV. Upcoming ADMX (dash-dotted magenta contour) should probe the full landscape of QCD axion models for masses 1 μeV $\lesssim m_a \lesssim$ 100 μeV. Moreover, MADMAX (2024) [78] is projected to cover the region 50 μeV $\lesssim m_a \lesssim$ 120 μeV (dash-dotted purple contour).

Axion dark matter and cosmology.—Axions are naturally light, weakly coupled with ordinary matter, cosmologically stable, and can be nonthermally produced in the early Universe. Indeed, they turn out to be an excellent DM candidate. In a preinflationary scenario, the PQ symmetry is broken before (or during) inflation and never restored during the reheating period of the Universe. Axion DM production occurs through the misalignment mechanism [34–36], with axion relic abundance given by [33]

$$\Omega_a h^2 \simeq \Omega_{\rm CDM} h^2 \frac{\theta_0^2}{2.15^2}\left(\frac{f_a}{2\times 10^{11}\ \text{GeV}}\right)^{\frac{7}{6}}, \qquad (10)$$

where the free parameter $|\theta_0| \in [0,\pi)$ is the initial misalignment angle and the observed cold DM (CDM) relic abundance obtained by Planck is $\Omega_{\rm CDM} h^2 = 0.1200 \pm 0.0012$ [4]. For $\theta_0 \sim \mathcal{O}(1)$ and $f_a \sim 5\times 10^{11}$ GeV, axions can account for the full DM abundance.

In a postinflationary scenario, where the PQ symmetry is broken after inflation, the observable Universe will be divided in patches with different values of the axion field (or θ phase). The initial misalignment angle is obtained through statistical average as $\langle\theta_0^2\rangle \simeq 2.15^2$ [33]. Thus, if the axion makes up the full CDM, $\Omega_a h^2 = \Omega_{\rm CDM} h^2$, so that f_a is predicted as in Eq. (10). Hence, if only the misalignment mechanism is at play, $f_a \lesssim 2\times 10^{11}$ GeV ensures that DM is not overproduced. However, the picture gets more complicated since topological defects (strings and domain walls) can also contribute to $\Omega_a h^2$ [79–83]. Note that the lightest state stemming from the colored fields Ψ, η, or χ can be cosmologically stable, and it can be thermally produced after inflation [84]. Searches in terrestrial, lunar, and meteoritic materials yield strong limits [85–89], practically ruling out such stable charged baryonic relics, unless some mechanism effectively suppresses their density or allows them to decay to ordinary matter [90,92–94]. (If electrically neutral, these relics might form viable bound-state DM [11,12].)

Turning to the preinflationary scenario, we assume that the exotic fermion and scalar masses lie above the reheating temperature of the Universe, i.e., $m_{\Psi,\eta,\chi} > T_{\rm RH}$. This is a reasonable assumption since their masses are proportional to $f_{\rm PQ} \gg v$, and $T_{\rm RH}$ is only bounded from below by big bang nucleosynthesis [95], $T_{\rm RH} \gtrsim 4.7$ MeV. This way, the abundance of stable baryonic or charged relics will be washed out during inflation, as well as topological defects. In preinflationary scenarios, the axion leaves an imprint in primordial fluctuations, reflected in the cosmic microwave background anisotropies and large-scale structure. The resulting isocurvature fluctuations are constrained by cosmic microwave background data [96], leading to an upper bound on the inflationary scale H_I [93]:

$$H_I \lesssim \frac{0.9\times 10^7}{\Omega_a h^2/\Omega_{\rm CDM} h^2}\left(\frac{\theta_0}{\pi}\frac{f_a}{10^{11}\ \text{GeV}}\right)\ \text{GeV}. \qquad (11)$$

In Fig. 3, we display θ_0 as a function of $f_{\rm PQ}$ [see Eq. (3)]. We highlight two cases, in red and blue, where the fermions, singlets under $SU(2)_L$, transform as $\Psi \sim 3$ and $\Psi \sim 15'$ under $SU(3)_c$, respectively. We take $\omega = 1/2$ and $n_\Psi = 2$. Along the solid lines we have $\Omega_a h^2 = 0.12$ [see Eq. (10)]. The region above these lines is excluded since it implies DM overabundance. Black hole superradiance sets $f_a \leq 6\times 10^{17}$ GeV [97,98] (shaded bands). Taking $\theta_0 \sim \mathcal{O}(1)$ leads to $f_a \gtrsim 5\times 10^{11}$ GeV, a region currently being probed by haloscope experiments—see Fig. 2. The dashed lines indicate different values of the inflationary scale H_I. Above these lines H_I is below the indicated value, in agreement with the isocurvature bound of Eq. (11). The allowed region for a given H_I lies above the dashed and below the solid contours. Taking $\theta_0 \sim \mathcal{O}(1)$ and $\Omega_a h^2 = 0.12$, we get a low scale for inflation $H_I \lesssim 10^7$ GeV (Planck currently probes $H_I \lesssim 10^{13}$ GeV [4]).

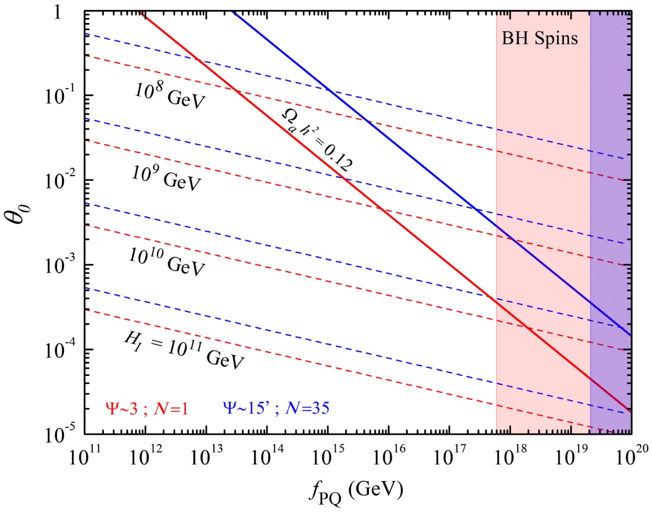

FIG. 3. Misalignment angle θ_0 as a function of $f_{\rm PQ}$. In red (blue) we show the scenario for $\Psi \sim 3$ ($\Psi \sim 15'$) under $SU(3)_c$, singlet under $SU(2)_L$, with $\omega = 1/2$ and $n_\Psi = 2$. Along the solid lines $\Omega_a h^2 = 0.12$, above (below) them we have DM over- or underabundance. Above the dashed lines the value of the inflationary scale H_I lies below the indicated value [see Eq. (11)]. Vertical bands are excluded by black hole superradiance.

Final remarks.—In this Letter, we proposed a connection between two seemingly unrelated facts: small neutrino masses and the strong CP problem. This was achieved within a novel class of KSVZ axion schemes, containing exotic colored fermions and scalars that act as neutrino-mass mediators at the two-loop level. The simplest realization of our proposal leads to promising $0\nu\beta\beta$ decay predictions.

Different representation assignments of the new fields under the SM and PQ symmetries yield distinct axion-to-photon couplings. This provides a way to differentiate the various realizations of our scheme at future helioscope and haloscope experiments such as IAXO, ADMX, and MADMAX.

Because of potentially dangerous colored relics, we have considered axion DM in the preinflationary scenario, where the PQ symmetry is broken before inflation. For an initial misalignment angle $\theta_0 \sim \mathcal{O}(1)$, axions can account for the full CDM budget, provided $f_a \sim 5 \times 10^{11}$ GeV, a region currently under scrutiny at haloscopes.

This research is supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects CFTP-FCT Unit UIDB/00777/2020 and UIDP/00777/2020, CERN/FIS-PAR/0019/2021, which are partially funded through POCTI (FEDER), COMPETE, QREN and EU. J. W. F. V. is supported by Spanish grants PID2020-113775GB-I00 (AEI/10.13039/501100011033) and Prometeo CIPROM/2021/054 (Generalitat Valenciana). The work of A. B. and H. B. C. is supported by the PhD FCT Grants No. UI/BD/154391/2023 and 2021.06340.BD, respectively. F. R. J. thanks the CERN Theoretical Physics Department for hospitality and financial support during the final stage of this work.

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