

Preparation of Matrix Product States with Log-Depth Quantum Circuits

Daniel Malz^{1,*}, Georgios Styliaris^{2,3,*}, Zhi-Yuan Wei^{2,3,*}, and J. Ignacio Cirac^{1,2,3}

¹*Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, 2100 Copenhagen, Denmark*

²*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, 85748 Garching, Germany*

³*Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, 80799 München, Germany*



(Received 1 August 2023; accepted 19 December 2023; published 24 January 2024)

We consider the preparation of matrix product states (MPS) on quantum devices via quantum circuits of local gates. We first prove that faithfully preparing translation-invariant normal MPS of N sites requires a circuit depth $T = \Omega(\log N)$. We then introduce an algorithm based on the renormalization-group transformation to prepare normal MPS with an error ϵ in depth $T = O[\log(N/\epsilon)]$, which is optimal. We also show that measurement and feedback leads to an exponential speedup of the algorithm to $T = O[\log \log(N/\epsilon)]$. Measurements also allow one to prepare arbitrary translation-invariant MPS, including long-range non-normal ones, in the same depth. Finally, the algorithm naturally extends to inhomogeneous MPS.

DOI: [10.1103/PhysRevLett.132.040404](https://doi.org/10.1103/PhysRevLett.132.040404)

One of the most important tasks in many-body physics and quantum information science is the preparation of useful or relevant states. This has spurred a large effort to find ways to prepare states, for example, adiabatically [1], dissipatively [2,3], or using quantum circuits. A natural class of states to consider are matrix product states (MPS), because they efficiently approximate ground states of gapped local Hamiltonians [4–6]. Moreover, many paradigmatic states can neatly be expressed as MPS, such as the cluster [7], Greenberger-Horne-Zeilinger (GHZ) [8], W [9], and Affleck-Kennedy-Lieb-Tasaki (AKLT) states [10,11].

Several ways are known to prepare MPS. Using unitary quantum circuits with strictly local gates, all MPS can be prepared using a sequential quantum circuit of depth $T \propto N$ [12]. This is provably optimal for long-range correlated states such as the GHZ state [13]. However, for so-called normal MPS [14], which have short-range correlations, shorter depths are possible. Indeed, when allowing for a small error ϵ , they can be obtained by acting on a product state with a constant-depth circuit of quasilocal gates—gates whose support grows (poly-)logarithmically with system size [15,16]. However, such quasilocal gates have to be compiled into gates with strictly local support, and in the worst case such a compilation leads to circuits with a depth scaling exponentially in the support, and thus as

$\text{poly}(N)$. However, since normal MPS all lie in the topologically trivial phase, one can construct adiabatic paths with a guaranteed gap [17], which means normal MPS can provably be prepared adiabatically in $T = O[\text{polylog}(N/\epsilon)]$ [18] (also see [19]).

Despite these results, it remains unclear if the scaling of the state-of-the-art algorithm [18] is optimal, or if there exist even faster algorithms to prepare normal MPS. Proving optimality requires finding a tight lower bound on the depth, or, equivalently, its complexity, which is believed to be difficult in general [13,20,21].

Here, we first resolve the question of asymptotically optimal preparation of normal translation-invariant (TI) MPS. We prove that any circuit faithfully preparing them requires a depth $T = \Omega(\log N)$, i.e., it has to scale at least logarithmically with N . We then introduce an algorithm that saturates this bound and prepares all normal TI-MPS in a circuit depth

$$T = O[\log(N/\epsilon)] \quad (1)$$

using strictly local gates. This is asymptotically faster than the previously fastest known algorithm (adiabatic preparation [18]) and also asymptotically optimal. Moreover, the algorithm naturally extends to inhomogeneous MPS that are suitably short-range correlated.

If one has additionally access to measurements and feedback, it is known that MPS can be prepared exactly in a depth $T = O(\log N)$ by expressing them in terms of the multiscale entanglement renormalization ansatz (MERA) [22]. Including measurements also yields a speedup for our algorithm, and allows us to extend it to non-normal MPS, such that all TI-MPS can provably be prepared in depth

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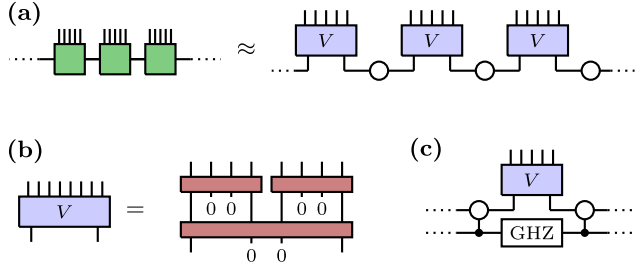


FIG. 1. Algorithm for MPS preparation. (a) After blocking, we approximate the state through its RG fixed point (nearest-neighbor entangled pairs for normal tensors [Eq. (12)]) combined with isometries V that encode the local structure of the state. (b) We use RG to construct an efficient circuit for V , which can be further expressed with a low-depth circuit of local gates. (c) Our algorithm extends to non-normal (i.e., long-range correlated) MPS with GHZ-like fixed points [Eq. (19)] depicted here. Using quantum circuits assisted by measurements, both the fixed-point states and the isometry V can be implemented efficiently.

$$T = O[\log \log(N/\epsilon)]. \quad (2)$$

This is exponentially faster than the best known measurement assisted protocol [22]. It also shows that our lower bound can be violated with access to measurements. As a byproduct, our work also proves that the finite-range MERA [23] can approximate normal TI-MPS in $O[\log \log(N/\epsilon)]$ layers.

Our algorithm fundamentally builds on the renormalization-group (RG) transformation. The RG procedure consists of blocking neighboring sites and discarding short-range correlations. After consecutive RG transformations, the state asymptotically converges to its fixed-point state [24], which has only nearest-neighbor correlations for normal TI-MPS [24,25]. This happens rapidly since it suffices to block only $O[\log(N/\epsilon)]$ sites to approximate well the fixed-point state [16]. Our algorithm first prepares this fixed point, and subsequently reintroduces the short-range correlations by applying an isometry of support $\log(N/\epsilon)$ [cf. Fig. 1(a)]. Our key contribution is that we can prove through an explicit construction (inspired by earlier works [12,22,24]) that this isometry can be implemented with a strictly local circuit of depth $T = O[\log(N/\epsilon)]$ [cf. Fig. 1(b)]. When assisted by measurements [cf. Fig. 1(c)], the depth of the isometry can be further reduced, while the GHZ-like fixed point of long-range correlated MPS can be prepared in constant depth [16,26]. Together, this leads to the circuit depth $T = O[\log \log(N/\epsilon)]$ to prepare almost arbitrary (including all TI) MPS.

Preliminaries.—For simplicity, we first consider (normalized) TI-MPS,

$$|\phi_N\rangle \propto \sum_{i_1, \dots, i_N=1}^d \text{Tr}(A^{i_1} \cdots A^{i_N}) |i_1 \cdots i_N\rangle, \quad (3)$$

and later extend to the inhomogeneous case. Above A^i are $D \times D$ matrices (D is the bond dimension) with $i = 1, \dots, d$ (physical dimension). We will extensively use graphical notation and identify $(A^i)_{jk} = j \text{---} \overset{i}{\square} \text{---} k$.

To each tensor A we associate its *transfer matrix*

$$E_A = \sum_{i=1}^d (A^i)^* \otimes A^i = \begin{array}{c} \square^{A^*} \\ | \\ \square^A \end{array}. \quad (4)$$

A tensor is called “normal” if (i) it is irreducible (A^i have no nontrivial common invariant subspace), and (ii) E_A has a unique largest eigenvalue $\lambda_1 = 1$ and no other of the same magnitude [4,14]. Its correlation length is defined via the subleading eigenvalue $\xi = -1/\ln(|\lambda_2|)$. After a gauge transformation [14], E_A of a normal tensor can be brought into the form

$$E_A = |\rho\rangle\langle\mathbb{1}| + R = \begin{array}{c} \rho \\ \circ \\ \mathbb{1} \end{array} \left[+ R, \quad (5)$$

where the leading right eigenvector $\rho > 0$ (Hermitian and positive definite) [14,27], $\langle\mathbb{1}|\rho\rangle = \text{Tr}(\rho) = 1$, and R has spectral radius less than 1.

Blocking q sites together yields a new tensor B

$$\square^B = \square^A \cdots \square^A \quad (6)$$

$\underbrace{\hspace{10em}}_q$

with physical dimension d^q , the same bond dimension D , and transfer matrix $E_B = E_A^q$. E_B approaches its fixed point in the limit $q \rightarrow \infty$ [24,25], which, for normal tensors, is $E_\infty = |\rho\rangle\langle\mathbb{1}|$.

Our goal is to devise an algorithm that approximates the target N -site MPS $|\phi_N\rangle$ by $|\tilde{\phi}_N\rangle$ with error $\epsilon = \epsilon(\phi_N, \tilde{\phi}_N)$, where

$$\epsilon(\phi, \psi) = 1 - |\langle\phi|\psi\rangle| \quad (7)$$

and $|\tilde{\phi}_N\rangle$ is prepared using a local quantum circuit. Our first result is that it is impossible to approximate well normal TI-MPS in depth $o(\log N)$. Subsequently, we provide an explicit algorithm with the asymptotically optimal depth $O[\log(N/\epsilon)]$.

Lower bound.—Given (i) $\{|\phi_N\rangle\}$ a sequence of normalized TI-MPS on N sites, generated by a normal tensor A , with finite correlation length $\xi > 0$ [cf. Eq. (3)], and (ii) $\{|\psi_N\rangle\}$ a sequence obtained from depth- T local quantum circuits applied to product states, we are interested in determining how fast T has to grow in order to approximate the MPS well, as measured by the error $\epsilon = \epsilon(\phi_N, \psi_N)$. We prove here that no quantum circuit with depth $T = o(\log N)$ can faithfully approximate this class.

Theorem 1.—If $T = o(\log N)$ there is some N_0 such that for all $N > N_0$ we have $\epsilon > 1/2$.

The proof can be found in the Appendix. To establish this result, we use the fact that $|\psi_N\rangle$ have a strictly finite light cone, whereas in a normal TI-MPS $|\phi_N\rangle$ correlation functions decay only exponentially. This leads to a mismatch in the expectation value of correlators outside the light cone, which gives a lower bound on the error between the two states. We additionally use the fact that sufficiently distant parts of the system are statistically independent, such that the error accumulates with increasing system size N , unless the circuit depth grows sufficiently quickly.

The algorithm.—We now present the key steps for our algorithm. We will (i) approximate $|\phi_N\rangle$ by $|\tilde{\phi}_N\rangle$, then (ii) show that $|\tilde{\phi}_N\rangle$ can be efficiently prepared, and (iii) prove that the approximation error decays sufficiently fast with N . We begin with the case of normal TI-MPS and return to the general case later.

Approximation through the fixed-point state.—To make the approximation, we follow the steps of the RG transformation [24]. After blocking q sites, we perform a polar decomposition on the blocked tensor B , interpreting it as a map from the D^2 -dimensional virtual space to the d^q -dimensional physical space [28]. This way we can write $B = VP$ where V is an isometry with $V^\dagger V = \mathbb{1}_{D^2}$ and $P > 0$ is positive definite. Thus,

$$E_B = \begin{array}{c} \text{---} B^* \text{---} \\ | \\ \text{---} B \text{---} \end{array} = \begin{array}{c} \text{---} P^* \text{---} \\ | \\ \text{---} P \text{---} \end{array} \xrightarrow{q \rightarrow \infty} \begin{array}{c} \text{---} P_\infty^* \text{---} \\ | \\ \text{---} P_\infty \text{---} \end{array} = \begin{array}{c} \text{---} \rho \text{---} \end{array}. \quad (8)$$

The approximation consists of replacing P by its fixed-point version P_∞ in the tensor B , while keeping the isometry V intact. Graphically,

$$\begin{array}{c} \text{---} B \text{---} \\ | \\ \text{---} P \text{---} \end{array} = \begin{array}{c} \text{---} V \text{---} \\ | \\ \text{---} P \text{---} \end{array} \approx \begin{array}{c} \text{---} V \text{---} \\ | \\ \text{---} \sqrt{\rho} \text{---} \end{array} = \begin{array}{c} \text{---} \tilde{B} \text{---} \end{array}. \quad (9)$$

Later we will assign meaning to the approximation sign in Eq. (9) by bounding the global error between the MPS $|\phi_N\rangle$ and $|\tilde{\phi}_N\rangle$ resulting from the two tensors, B and \tilde{B} . To obtain a vanishing error in the thermodynamic limit we will need $q \propto \log N$, which we assume for now and justify subsequently.

Preparing the approximate state.—The approximate state $|\tilde{\phi}_N\rangle$ can be prepared by acting on the fixed-point state with a product of unitaries of support q (for simplicity $D = d$ in the illustration):

$$= \dots \begin{array}{c} \text{---} U \text{---} \\ | \\ \text{---} 0 \text{---} \end{array} \begin{array}{c} \text{---} U \text{---} \\ | \\ \text{---} 0 \text{---} \end{array} \dots \quad (10)$$

$L_i \quad C_i \quad R_i \quad L_{i+1} \quad C_{i+1} \quad R_{i+1}$

The unitary is constructed such that it implements the required isometry when acting on a product state $|0\rangle^{\otimes \ell}$ over the “central” region ($\ell = 2$ in the illustration):

$$\begin{array}{c} \text{---} U \text{---} \\ | \\ \text{---} 0 \text{---} \end{array} = \begin{array}{c} \text{---} V \text{---} \\ | \\ \text{---} \end{array}. \quad (11)$$

From dimension counting $D^2 d^\ell \geq d^q$, thus $\ell \sim q$. Note that for normal TI-MPS the fixed-point state $|\Omega\rangle = \bigotimes_{i=1}^{N/q} |\omega\rangle_{R_i L_{i+1}}$ is a tensor product of entangled pairs,

$$|\omega\rangle_{R_i L_{i+1}} = \begin{array}{c} \text{---} \text{---} \\ | \\ \text{---} \end{array} = (\mathbb{1} \otimes \sqrt{\rho}) \sum_{ii} |ii\rangle_{R_i L_{i+1}}, \quad (12)$$

each with support over the “right” and “left” Hilbert spaces of neighboring sites ($\dim R_i = \dim L_{i+1} = D$). It can thus be prepared from a product state with a constant-depth circuit.

So far, it is not obvious that the resulting circuit can be expressed efficiently in terms of strictly local gates, because the unitaries in Eq. (11) are only *quasilo*cal, i.e., having support $q \propto \log N$. While a naive bound on the circuit depth would be $\text{poly}(N)$, here we use the fact that U comes from an MPS to show that in reality it can be implemented in $T = O(q)$. We do this by providing two explicit and exact decompositions of U in terms of gates with constant support, the “sequential-RG” and the “tree-RG.”

The sequential-RG circuit.—We can express the unitary in Eq. (10) in terms of the original MPS by applying the inverse of P to its virtual legs [30],

$$\begin{array}{c} \text{---} U \text{---} \\ | \\ \text{---} 0 \text{---} \end{array} = \begin{array}{c} \text{---} A \text{---} A \text{---} A \text{---} A \text{---} \\ | \\ \text{---} P^{-1} \text{---} \end{array} = \begin{array}{c} \text{---} A' \text{---} A' \text{---} A' \text{---} C \text{---} \\ | \\ \text{---} \end{array}, \quad (13)$$

where in the last step we set $A^i = A^i \otimes \mathbb{1}_D$ and contracted P^{-1} with the rightmost A' to obtain C . As in sequential preparation of MPS [12] and in the left-canonical form [31], we can now iteratively apply singular value decompositions, starting from the tensor on the left and moving right, but stopping before the last tensor [32]. This defines a new set of tensors that describe the same isometry V but now each tensor is a local isometry (arrows indicate isometry direction, $q = 4$ in illustration):

$$V = V_q \dots V_1 = \begin{array}{c} \downarrow \\ \text{---} V_4 \text{---} V_3 \text{---} V_2 \text{---} \tilde{C} \text{---} \\ \uparrow \end{array}, \quad V_i : \mathbb{C}^{D'_i} \rightarrow \mathbb{C}^{d D'_{i+1}}, \quad (14)$$

with every V_i an isometry $V_i^\dagger V_i = \mathbb{1}_{D'_i}$ satisfying $D'_i \leq D^2$ ($D'_{q+1} = 1$). Importantly, $\tilde{C} = V_1$ is automatically also an isometry, as

$$V^\dagger V = \begin{array}{c} \boxed{V_4^*} \text{---} \boxed{V_3^*} \text{---} \boxed{V_2^*} \text{---} \boxed{\tilde{C}^*} \\ \boxed{V_4} \text{---} \boxed{V_3} \text{---} \boxed{V_2} \text{---} \boxed{C} \end{array} = \begin{array}{c} \boxed{\tilde{C}^*} \\ \boxed{C} \end{array} = \parallel. \quad (15)$$

Since this sequential circuit comprises q sites, its depth is $O(q)$. This scaling is unchanged if we additionally take into account that the inputs of the unitary in Eq. (13) are separated by $O(q)$ sites, which requires one to implement SWAP gates.

The tree-RG circuit.—Blocking two neighboring sites followed by a polar decomposition is the basis for the real-space RG transformation and halves the correlation length [24,35,36]. Instead of directly blocking q sites, we can repeatedly apply this transformation $k \sim \log_2 q$ times to the same effect [illustration below, and in Fig. 1(b)]. This generates a treelike circuit with k layers, in which each layer but the lowest consists of isometries from dimension D^2 to D^4 (below A_k is obtained from A by blocking k sites, and $q = 8$):

$$\begin{array}{c} \boxed{A_4} \text{---} \boxed{A_4} \\ \boxed{V^{(1)}} \text{---} \boxed{V^{(1)}} \\ \boxed{V^{(2)}} \text{---} \boxed{V^{(2)}} \\ \boxed{P^{(1)}} \text{---} \boxed{P^{(1)}} \end{array} = \begin{array}{c} \boxed{V^{(1)}} \text{---} \boxed{V^{(1)}} \\ \boxed{V^{(2)}} \text{---} \boxed{V^{(2)}} \\ \boxed{P^{(1)}} \text{---} \boxed{P^{(1)}} \end{array}. \quad (16)$$

In Eq. (16), the lowest layer is again the part that is replaced by the fixed-point state in our algorithm, i.e., a product of $|\omega\rangle$ [cf. Eq. (12)]. In this scheme, the lowest isometry $V^{(k)}$ acts across a distance q . Though not strictly local, this can be done in a depth $O(q)$ utilizing SWAP gates. Subsequent isometries act over distances $q/2$, $q/4$ and so forth, leading to an overall circuit depth $T = O(q)$.

Approximation error.—So far, we constructed efficient circuits for preparing $|\tilde{\phi}_N\rangle$, having assumed that we block q sites. The scaling of q is a consequence of the following Lemma, which is adapted from Ref. [16].

Lemma 1.—Given a sequence of TI-MPS generated from a normal tensor, and for all $\gamma < 1/2$,

$$\epsilon(\tilde{\phi}_N, \phi_N) = O\left(\frac{N}{q} e^{-\gamma q/\xi}\right). \quad (17)$$

The proof can be found in [37]. Using Lemma 1, it follows that $q = O[\log(N/\epsilon)]$. In particular, blocking $q = \lceil 2\xi(1+\eta) \ln N \rceil \propto \log N$ sites gives $\epsilon = O(N^{-\eta})$ for any $\eta > 0$.

We also numerically illustrate the exponential decay of Eq. (17) in [37] for preparing the 1D AKLT state [10,11] and an MPS family with tunable correlation length [44], which demonstrates that the circuit is also efficient in practice.

Inhomogeneous short-range correlated MPS.—Our results can be straightforwardly extended to MPS that have a finite correlation length, but are not TI. The setting

here is that we are given a sequence of MPS $\{|\phi_N\rangle\}$ with bond dimension at most D . We define such a sequence to have finite correlation length if, after blocking $q = O(\log N)$ times, the resulting states can be approximated up to quasilocal isometries by a state consisting of nearest-neighbor entangled pairs $|\Omega\rangle = \bigotimes_{i=1}^{N/q} |\omega^i\rangle_{R_i, L_{i+1}}$, with an error $\epsilon(\Omega, \phi_{\text{pos}}) \rightarrow 0$ as $N \rightarrow \infty$. Here,

$$|\phi_{\text{pos}}\rangle = \dots \text{---} \boxed{P_{i-1}} \text{---} \boxed{P_i} \text{---} \boxed{P_{i+1}} \text{---} \dots \quad (18)$$

arises after blocking q sites and keeping the positive part of the decomposition of $|\phi_N\rangle$. If the finite correlation assumption is satisfied, then the preparation scheme consists of preparing $|\Omega\rangle$ and implementing the isometry, decomposed with either of the two methods. The resulting total depth is again $O[\log(N/\epsilon)]$ with error $\epsilon(\Omega, \phi_{\text{pos}})$, as in the TI case.

In the Supplemental Material [37] we numerically show that this protocol can prepare inhomogeneous random MPS [45–48] efficiently. For that, we use a simple extension of the Evenbly-Vidal algorithm [49,50], which efficiently variationally finds $|\Omega\rangle$. This illustrates that our finite correlation length assumption, as defined earlier, is satisfied in a practical setting.

Preparations using measurements.—Measurements and subsequent conditional unitaries can make state preparation much faster [22,26,51–55]. Here, we elaborate how such measurements could be used in our algorithm.

Tree-RG circuit with measurements.—Local measurements and conditional local unitaries are the standard framework to perform quantum teleportation [56,57], which can be used to reduce the depth of the tree-RG circuit. Isometries appearing in Eq. (16) act on a constant number of sites, which, although spatially separated, can be teleported at neighboring registers with a constant overhead. This can be achieved by creating nearest-neighbor entangled pairs, then performing simultaneous measurements, and correcting (without postselection) based on the measurement outcomes [58] (this process is also detailed in Ref. [22]).

Therefore every isometry in Eq. (16) takes constant time using measurement. Crucially, however, the tree-RG circuit requires only $O[\log \log(N/\epsilon)]$ layers (in contrast to Ref. [22]). Since the fixed-point state can be prepared in constant time as before, this gives a preparation algorithm for short-range correlated MPS with depth $O[\log \log(N/\epsilon)]$.

Long-range MPS using measurements.—Another consequence of including measurements is that the creation of GHZ-like states $|\chi_M\rangle = \sum_{i=1}^b \alpha_i |i\rangle^{\otimes M}$ becomes possible in only constant depth [16,26,59]. These states are closely related to the fixed points of TI-MPS [33], which, up to an isometry, take the form [25]

$$|\Omega'\rangle = \sum_{j=1}^b \alpha_j^{(N)} \bigotimes_{i=1}^{N/q} |\omega_j\rangle_{R_i L_{i+1}}. \quad (19)$$

The normal case corresponds to $b = 1$ for which $|\Omega'\rangle = |\Omega\rangle$ while, in general, b is upper bounded by the number of blocks in the canonical form and $\alpha_j^{(N)}$ may depend on N [25].

Importantly, the different $|\omega_j\rangle$ are orthogonal [25], which suggests a preparation procedure for $|\Omega'\rangle$. First create $|\chi_{N/q}\rangle$, which can be done in constant depth with measurements (following, e.g., Ref [16]). Subsequently, apply in parallel the isometries $W: |j\rangle \mapsto |\omega_j\rangle_{R_i L_{i+1}}$ such that $|\Omega'\rangle = W^{\otimes N/q} |\chi_{N/q}\rangle$, which also takes constant depth.

In [37] we show how to explicitly obtain a state of the form Eq. (19) that approximates well the target $|\phi_N\rangle$ up to local isometries by blocking $q \propto \log(N/\epsilon)$ sites. As a result, following the same steps as in the tree-RG circuit with measurements we have a scheme that approximates all TI-MPS (short- or long-range correlated) with depth $T = O[\log \log(N/\epsilon)]$ [cf. Eq. (2)]. If instead measurements are only used for the preparation of $|\chi_{N/q}\rangle$, the depth is $O[\log(N/\epsilon)]$.

Our construction generalizes to inhomogeneous long-range correlated MPS exactly as in the short-range case.

Connection to MERA.—The circuit in the tree-RG scheme can be interpreted as a finite-range MERA with $O(\log \log N)$ layers, namely a shallow tensor tree acting on the fixed-point state. Specifically, the isometries $V^{(i)}$ [cf. Eq. (16)] are identified with the isometries in finite-range MERA, and all disentanglers are the identity, save for the first layer, which is identified with the single layer of unitaries that prepare the fixed-point state. Hence, within the approximation error ϵ ,

$$\text{normal TI-MPS} \subset \begin{array}{l} \text{finite-range MERA} \\ \text{with } O(\log \log N) \text{ layers.} \end{array} \quad (20)$$

Discussion and outlook.—Our results also imply that MPS in the same phase can be transformed into each other using a log-depth circuit, in contrast to the well-known quasilocal evolution corresponding to polylogarithmic depth circuit [17,60–62]. It would be interesting to explore whether our results could be exploited for applications other than state preparation. Specifically, a number of protocols [50,63–67] implicitly or explicitly depend on the ability to prepare (or disentangle) MPS using a sequential circuit. It may be possible to replace the sequential circuit with ours to reduce the circuit depth in these protocols. Another direction would be to extend our lower-bound proof and the preparation algorithm to prepare certain higher dimensional tensor network states [34].

We thank Yujie Liu, Miguel Frías Pérez, and Rahul Trivedi for insightful discussions. D.M. acknowledges support from the Novo Nordisk Fonden under Grants

No. NNF22OC0071934 and No. NNF20OC0059939. G. S. is supported by the Alexander von Humboldt Foundation. The research is part of the Munich Quantum Valley, which is supported by the Bavarian State Government with funds from the Hightech Agenda Bayern Plus. We acknowledge funding from the German Federal Ministry of Education and Research (BMBF) through EQUAHUMO (Grant No. 13N16066) within the funding program quantum technologies—from basic research to market. The numerical calculations were performed using the ITensor Library [68].

Appendix: Proof of Theorem 1.—Before we present the proof, let us introduce the following lemma, which we will use to distinguish the states based on the mismatch between states with strictly finite correlation length and states with exponentially decaying correlations.

Lemma 2 (exponentially decaying correlations).—Let $\{|\phi_N\rangle\}$ be a sequence of TI-MPS generated by an injective tensor A with finite correlation length $\xi > 0$. Then, we can always find two local operators $\mathcal{O}_1, \mathcal{O}'_s$ acting on spins 1 and s with $\|\mathcal{O}\| = \|\mathcal{O}'\| = 1$ such that for any integer $s > 1$ and sufficiently large N ,

$$\langle \phi_N | \mathcal{O}_1 | \phi_N \rangle = \langle \phi_N | \mathcal{O}'_s | \phi_N \rangle = 0, \quad (A1a)$$

$$\langle \phi_N | \mathcal{O}_1 \mathcal{O}'_s | \phi_N \rangle \geq c e^{-(s-1)/\xi}, \quad (A1b)$$

where $c > 0$ is independent of N, s .

Proof.—Consider the connected correlation function

$$\Delta = \langle \phi_N | \mathcal{O}_1 \mathcal{O}'_s | \phi_N \rangle - \langle \phi_N | \mathcal{O}_1 | \phi_N \rangle \langle \phi_N | \mathcal{O}'_s | \phi_N \rangle, \quad (A2)$$

where \mathcal{O}_1 and \mathcal{O}'_s are two (potentially different) operators placed at sites 1 and s . We have

$$\Delta = \frac{1}{c_N^2} \left[\text{Tr} \left(E_1^{N-s-1} E_{\mathcal{O}} E_1^{s-1} E_{\mathcal{O}'} \right) - \text{Tr} \left(E_1^{N-1} E_{\mathcal{O}} \right) \text{Tr} \left(E_1^{N-1} E_{\mathcal{O}'} \right) \right], \quad (A3)$$

where the normalization $c_N = \sqrt{\text{Tr} \left(E_1^N \right)}$, and

$$E_{\mathcal{Q}} = \sum_{i,j=1}^d \langle i | \mathcal{Q} | j \rangle (A^i)^* \otimes A^j, \quad \mathcal{Q} \in \{1, \mathcal{O}, \mathcal{O}'\}, \quad (A4)$$

where d is the physical dimension of the MPS. Given the spectrum of E_1 we can always take N sufficiently large so that we can approximate with an arbitrarily small error, $E_1^{N-1} = E_1^{N-s-1} = |R_1\rangle \langle L_1| + O(e^{-N/\xi})$, where $\xi = -1/\ln(|\lambda_2|)$ and $1 = \lambda_1 > |\lambda_2| > \dots$ are the eigenvalues of E_1 . Note that in the main text we use the gauge in which $|R_1\rangle = |\rho\rangle$ and $|L_1\rangle = |1\rangle$. Here, $\langle L_1 | R_1 \rangle = 1$, so that $c_N \approx 1$ and

$$\Delta \approx \sum_{i=2}^{D^2} \lambda_i^{s-1} \langle L_1 | E_{\mathcal{O}} | R_i \rangle \langle L_i | E_{\mathcal{O}'} | R_1 \rangle, \quad (\text{A5})$$

where D is the bond dimension and we have written $E_{\mathbb{1}}^{s-1} = \sum_i \lambda_i^{s-1} |R_i\rangle \langle L_i|$. Since the tensor A is injective [14], we can always choose \mathcal{O} (and \mathcal{O}'), such that the corresponding transfer matrix $E_{\mathcal{O}} = |A\rangle \langle B|$ for arbitrary A, B (up to a normalization constant). In particular, we can use this to impose that

$$\langle L_i | E_{\mathcal{O}} | R_i \rangle = \langle L_i | E_{\mathcal{O}'} | R_i \rangle = 0, \quad \forall i \quad (\text{A6a})$$

$$\langle L_1 | E_{\mathcal{O}} | R_i \rangle = \langle L_i | E_{\mathcal{O}'} | R_1 \rangle = 0, \quad \forall i > 2, \quad (\text{A6b})$$

$$\langle L_1 | E_{\mathcal{O}} | R_2 \rangle \langle L_2 | E_{\mathcal{O}'} | R_1 \rangle = c' > 0. \quad (\text{A6c})$$

The first line ensures (A1a), while the second and third ensure (A1b) for sufficiently large N , with $c = c'/2$, where $1/2$ is an arbitrary constant chosen for concreteness. ■

Now, we can prove Theorem 1. Let $\{|\phi_N\rangle\}$ be a sequence of TI normalized MPS on N sites generated by a normal tensor A , and $\{|\psi_N\rangle\}$ a sequence of states obtained by applying a depth- T local quantum circuit to a product state and define the error $\epsilon = 1 - |\langle \phi_N | \psi_N \rangle|$.

Theorem 1 (restated).—If $T = o(\log N)$ there is some N_0 such that for all $N > N_0$ we have $\epsilon > 1/2$.

Proof.—Let us assume that $T = o[\log(N)]$ and $T > 2\xi$, since we can always add layers of identity operators to increase the depth of the circuit. We approximate $\{|\phi_N\rangle\}$ through $\{|\tilde{\phi}_N\rangle\}$ [cf. Eq. (10)] obtained by blocking $q_N = \lceil 2(1 + \eta)\xi \ln N \rceil$ with $\eta > 0$ and use Lemma 1 to bound the error as

$$\epsilon = 1 - |\langle \tilde{\phi}_N | \phi_N \rangle| < c_0 N^{-\eta} \quad (\text{A7})$$

for some constant c_0 independent of system size. We take N such that we have a large number of blocks, all of the same size, q_N , except for the last one, which may be larger. This is always possible, as $q_N = O(\log N)$. We also take N large enough to ensure $q_N > T$.

We thus have

$$d(\phi_N, \psi_N) \geq d(\psi_N, \tilde{\phi}_N) - \sqrt{2c_0} N^{-\eta/2}, \quad (\text{A8})$$

where $d(\rho, \sigma) = \|\rho - \sigma\|_1/2$ is the trace distance [58], as well as an upper bound on trace distance from fidelity combined with Eq. (A7)

$$d(\phi_N, \tilde{\phi}_N) \leq \sqrt{1 - |\langle \phi_N | \tilde{\phi}_N \rangle|^2} \leq \sqrt{2c_0} N^{-\eta/2}. \quad (\text{A9})$$

In the following, we will find a lower bound to the first term in Eq. (A8) to make the difference larger than $1/2$. We will also drop the subscript N to simplify notation.

To obtain a bound on the distance of $|\psi_N\rangle$ and $|\tilde{\phi}_N\rangle$, let us consider instead a suitable subsystem. To that end, we divide the chain into $\lfloor N/(2q) \rfloor$ blocks of size $2q$ each, with the last block potentially smaller than $2q$. We then trace over all $2q$ spins at the sites contained in the intervals $[4mq + 1, 2(2m + 1)q]$, with $m = 0, 1, \dots$ in both states $|\tilde{\phi}\rangle$ and $|\psi\rangle$. In case the last block we constructed is smaller than $2q$, we trace it as well. If we perform such an operation on $|\tilde{\phi}\rangle \langle \tilde{\phi}|$, we obtain a product state

$$\rho = \rho_0^{\otimes k}, \quad (\text{A10})$$

which follows from the definition of $|\tilde{\phi}_N\rangle$ and the fact that it is invariant under translation by q sites. We have

$$k = \lfloor N/4q \rfloor. \quad (\text{A11})$$

Analogously, applying the same trace to $|\psi\rangle \langle \psi|$ we also obtain a product state, because $q > T$,

$$\sigma = \sigma_1 \otimes \dots \otimes \sigma_k. \quad (\text{A12})$$

Using the fact that the trace distance is contractive under tracing, and bounding it in terms of the Uhlmann fidelity, we have [58]

$$d(\tilde{\phi}, \psi) \geq d(\rho, \sigma) \geq 1 - F(\rho, \sigma), \quad (\text{A13})$$

with the Uhlmann fidelity between two density matrices ρ and σ defined as $F(\rho, \sigma) = \text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}}$.

Given that ρ and σ are product states, we have

$$F(\rho, \sigma) = \prod_{i=1}^k F(\rho_0, \sigma_i) \leq (1 - \delta)^{k/2}, \quad (\text{A14})$$

where $\delta = \min_i d(\rho_0, \sigma_i)^2$, and where we have used another bound between the fidelity and the trace distance [58].

Next we will lower bound δ using Lemma 2. However, we have to be a bit careful since this lemma applies to ϕ instead of $\tilde{\phi}$. Fortunately, we can use Eq. (A9) to replace one with the other. For the sake of concreteness, we will bound $d(\rho_0, \sigma_1)$ but the same analysis applies to every σ_i .

Let us take $s = 2T + 1$, and the operators $\mathcal{O}, \mathcal{O}'$ from Lemma 2 to define

$$a_{\mathcal{Q}} = \langle \phi | \mathcal{Q} | \phi \rangle, \quad (\text{A15})$$

$$\tilde{a}_{\mathcal{Q}} = \text{Tr}(\rho_0 \mathcal{Q}) = \langle \tilde{\phi} | \mathcal{Q} | \tilde{\phi} \rangle, \quad (\text{A16})$$

$$b_{\mathcal{Q}} = \text{Tr}(\sigma_1 \mathcal{Q}) = \langle \psi | \mathcal{Q} | \psi \rangle, \quad (\text{A17})$$

where $\mathcal{Q} \in \{\mathcal{O}_1, \mathcal{O}'_s, \mathcal{O}_1 \mathcal{O}'_s\}$. Given that $\|\mathcal{O}\| = \|\mathcal{O}'\| = 1$, we can bound

$$d(\rho_0, \sigma_1) \geq \max_{\mathcal{Q}=\mathcal{O}_1, \mathcal{O}'_s, \mathcal{O}_1, \mathcal{O}'_s} \left| |\tilde{a}_{\mathcal{Q}}| - |b_{\mathcal{Q}}| \right|. \quad (\text{A18})$$

According to Lemma 2, $a_{\mathcal{O}_1} = a_{\mathcal{O}'_s} = 0$, and

$$a_{\mathcal{O}_1, \mathcal{O}'_s} > \mu = c_1 e^{-2T/\xi}. \quad (\text{A19})$$

In order to use Eq. (A18), we need to connect $\tilde{a}_{\mathcal{Q}}$ to $a_{\mathcal{Q}}$. Using Eq. (A9), we choose sufficiently large N to obtain $d(\tilde{\phi}, \phi) \leq \mu/3$. This immediately implies that $|\tilde{a}_{\mathcal{O}_1}|, |\tilde{a}_{\mathcal{O}'_s}| < \mu/3$ and $\tilde{a}_{\mathcal{O}_1, \mathcal{O}'_s} > 2\mu/3$. Moreover, since ψ is created from a product state by a depth- T circuit, every connected correlation for operators at a distance larger than $2T$ vanishes. Since $s = 2T + 1$ we therefore have $b_{\mathcal{O}_1, \mathcal{O}'_s} = b_{\mathcal{O}_1} b_{\mathcal{O}'_s}$. We now show that for any choice of $b_{\mathcal{O}_1}, b_{\mathcal{O}'_s}$, the distance $d(\rho_0, \sigma_1)$ is bounded below by a constant. Thus, we minimize Eq. (A18) with respect to $x_1 = b_{\mathcal{O}_1}$ and $x_2 = b_{\mathcal{O}'_s}$, imposing $\mu < 1/2$:

$$\begin{aligned} d(\rho_0, \sigma_1) &\geq \min_{x_1, x_2} \left[\max \left(\left| x_1 - \frac{\mu}{3} \right|, \left| x_2 - \frac{\mu}{3} \right|, \left| \frac{2\mu}{3} - x_1 x_2 \right| \right) \right] \\ &= \min_x [\max(|x - \mu/3|, |2\mu/3 - x^2|)] > \mu/3. \end{aligned} \quad (\text{A20})$$

From this it immediately follows that $\delta > \mu^2/9$.

Putting Eqs. (A20) and (A14) into Eq. (A8), we arrive at

$$d(\phi, \psi) > 1 - (1 - \mu^2/9)^{k/2} - \sqrt{2c_0} N^{-\eta}. \quad (\text{A21})$$

The last term is negligible for large N . If then $k > 2/\delta$, one has that $(1 - \delta)^{k/2} < 1/e$ and thus $d(\phi, \psi) > \sqrt{3}/4$, which implies $\epsilon > 1/2$.

So we need to find N for which $k > 2/\delta$. Using Eq. (A11) and the definition of q , we have $k > N/(10\xi \ln N)$, where we chose $\eta < 1/4$. Using $\delta > \mu^2/9$ and the definition of μ [Eq. (A19)], we find $(18e^{4T/\xi})/c_1^2 > 2/\delta$. Putting this together, we need to find N such that

$$k > \frac{N}{5q} > \frac{N\gamma}{10\xi \log N} > \frac{18}{c_1^2} e^{4T/\xi} > \frac{2}{\delta}. \quad (\text{A22})$$

Since $T = o[\log(N)]$, we can always find an N_0 such that this is fulfilled for all $N > N_0$. ■

*These authors contributed equally to this work.

- [1] T. Albash and D. A. Lidar, Adiabatic quantum computation, *Rev. Mod. Phys.* **90**, 015002 (2018).
 [2] B. Kraus, H. P. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, Preparation of entangled states by quantum Markov processes, *Phys. Rev. A* **78**, 042307 (2008).

- [3] F. Verstraete, M. M. Wolf, and J. Ignacio Cirac, Quantum computation and quantum-state engineering driven by dissipation, *Nat. Phys.* **5**, 633 (2009).
 [4] M. Fannes, B. Nachtergaele, and R. F. Werner, Finitely correlated states on quantum spin chains, *Commun. Math. Phys.* **144**, 443 (1992).
 [5] F. Verstraete and J. I. Cirac, Matrix product states represent ground states faithfully, *Phys. Rev. B* **73**, 094423 (2006).
 [6] M. B. Hastings, An area law for one-dimensional quantum systems, *J. Stat. Mech.* (2007) P08024.
 [7] H. J. Briegel and R. Raussendorf, Persistent entanglement in arrays of interacting particles, *Phys. Rev. Lett.* **86**, 910 (2001).
 [8] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Going beyond Bell's theorem, in *Bell's Theorem, Quantum Theory and Conceptions of the Universe* (Springer, New York, 1989), pp. 69–72.
 [9] W. Dür, G. Vidal, and J. I. Cirac, Three qubits can be entangled in two inequivalent ways, *Phys. Rev. A* **62**, 062314 (2000).
 [10] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Rigorous results on valence-bond ground states in antiferromagnets, *Phys. Rev. Lett.* **59**, 799 (1987).
 [11] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Valence bond ground states in isotropic quantum antiferromagnets, *Commun. Math. Phys.* **115**, 477 (1988).
 [12] C. Schön, E. Solano, F. Verstraete, J. I. Cirac, and M. M. Wolf, Sequential generation of entangled multiqubit states, *Phys. Rev. Lett.* **95**, 110503 (2005).
 [13] S. Bravyi, M. B. Hastings, and F. Verstraete, Lieb-Robinson bounds and the generation of correlations and topological quantum order, *Phys. Rev. Lett.* **97**, 050401 (2006).
 [14] D. Perez-García, F. Verstraete, M. M. Wolf, and J. I. Cirac, Matrix product state representations, *Quantum Inf. Comput.* **7**, 401 (2007).
 [15] F. G. S. L. Brandão and M. J. Kastoryano, Finite correlation length implies efficient preparation of quantum thermal states, *Commun. Math. Phys.* **365**, 1 (2019).
 [16] L. Piroli, G. Styliaris, and J. I. Cirac, Quantum circuits assisted by local operations and classical communication: Transformations and phases of matter, *Phys. Rev. Lett.* **127**, 220503 (2021).
 [17] N. Schuch, D. Pérez-García, and I. Cirac, Classifying quantum phases using matrix product states and projected entangled pair states, *Phys. Rev. B* **84**, 165139 (2011).
 [18] Y. Ge, A. Molnár, and J. I. Cirac, Rapid adiabatic preparation of injective projected entangled pair states and Gibbs states, *Phys. Rev. Lett.* **116**, 080503 (2016).
 [19] S. Bachmann, W. D. Roeck, and M. Fraas, The adiabatic theorem and linear response theory for extended quantum systems, *Commun. Math. Phys.* **361**, 997 (2018).
 [20] F. G. S. L. Brandão, W. Chemissany, N. Hunter-Jones, R. Kueng, and J. Preskill, Models of quantum complexity growth, *PRX Quantum* **2**, 030316 (2021).
 [21] Y. Jia and M. M. Wolf, Hay from the haystack: Explicit examples of exponential quantum circuit complexity, *Commun. Math. Phys.* **402**, 141 (2023).

- [22] T.-C. Lu, L. A. Lessa, I. H. Kim, and T. H. Hsieh, Measurement as a shortcut to long-range entangled quantum matter, *PRX Quantum* **3**, 040337 (2022).
- [23] G. Evenbly and G. Vidal, Tensor network states and geometry, *J. Stat. Phys.* **145**, 891 (2011).
- [24] F. Verstraete, J. I. Cirac, J. I. Latorre, E. Rico, and M. M. Wolf, Renormalization-group transformations on quantum states, *Phys. Rev. Lett.* **94**, 140601 (2005).
- [25] J. I. Cirac, D. Perez-Garcia, N. Schuch, and F. Verstraete, Matrix product density operators: Renormalization fixed points and boundary theories, *Ann. Phys. (Amsterdam)* **378**, 100 (2017).
- [26] H. J. Briegel and R. Raussendorf, Persistent entanglement in arrays of interacting particles, *Phys. Rev. Lett.* **86**, 910 (2001).
- [27] D. E. Evans and R. Høegh-Krohn, Spectral properties of positive maps on C^* -algebras, *J. Lond. Math. Soc.* **17**, 345 (1977).
- [28] To do a polar decomposition we need q large enough that $d^q \geq D^2$. Later, we will have q scaling with the system size, so here we assume that B is injective [14]. This is always true for normal tensors after blocking finite (independent of N) sites [29].
- [29] M. Sanz, D. Perez-Garcia, M. M. Wolf, and J. I. Cirac, A quantum version of Wielandt's inequality, *IEEE Trans. Inf. Theory* **56**, 4668 (2010).
- [30] The subsequent derivation remains valid also for non-injective tensors B . In that case P^{-1} is understood as pseudoinverse.
- [31] U. Schollwöck, The density-matrix renormalization group in the age of matrix product states, *Ann. Phys. (Amsterdam)* **326**, 96 (2011).
- [32] The method can easily be generalized to absorb P^{-1} into any of the tensors. With C in the bulk, the sequential circuit is obtained by repeated singular value decomposition starting both left and right and stopping at C , as in the mixed-canonical form [31].
- [33] Here, we consider TI-MPS with constant D . Note that this does not include all states that are TI and have area law entanglement (e.g., W state [14,34]).
- [34] J. I. Cirac, D. Perez-Garcia, N. Schuch, and F. Verstraete, Matrix product states and projected entangled pair states: Concepts, symmetries, theorems, *Rev. Mod. Phys.* **93**, 045003 (2021).
- [35] M. Levin and C. P. Nave, Tensor renormalization group approach to two-dimensional classical lattice models, *Phys. Rev. Lett.* **99**, 120601 (2007).
- [36] G. Vidal, Entanglement renormalization, *Phys. Rev. Lett.* **99**, 220405 (2007).
- [37] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.132.040404>, which includes Refs. [38–43], for the proof of Lemma 1 and other numerical results.
- [38] G. K. Brennen and A. Miyake, Measurement-based quantum computer in the gapped ground state of a two-body Hamiltonian, *Phys. Rev. Lett.* **101**, 010502 (2008).
- [39] R. Iten, R. Colbeck, I. Kukuljan, J. Home, and M. Christandl, Quantum circuits for isometries, *Phys. Rev. A* **93**, 032318 (2016).
- [40] P. Rakyta and Z. Zimborás, Approaching the theoretical limit in quantum gate decomposition, *Quantum* **6**, 710 (2022).
- [41] C. Schön, K. Hammerer, M. M. Wolf, J. I. Cirac, and E. Solano, Sequential generation of matrix-product states in cavity QED, *Phys. Rev. A* **75**, 1 (2007).
- [42] X. Chen, Z.-C. Gu, and X.-G. Wen, Classification of gapped symmetric phases in one-dimensional spin systems, *Phys. Rev. B* **83**, 035107 (2011).
- [43] M. Hauru, M. Van Damme, and J. Haegeman, Riemannian optimization of isometric tensor networks, *SciPost Phys.* **10**, 040 (2021).
- [44] M. M. Wolf, G. Ortiz, F. Verstraete, and J. I. Cirac, Quantum phase transitions in matrix product systems, *Phys. Rev. Lett.* **97**, 110403 (2006).
- [45] S. Garnerone, T. R. de Oliveira, and P. Zanardi, Typicality in random matrix product states, *Phys. Rev. A* **81**, 032336 (2010).
- [46] J. Haferkamp, C. Bertoni, I. Roth, and J. Eisert, Emergent statistical mechanics from properties of disordered random matrix product states, *PRX Quantum* **2**, 040308 (2021).
- [47] C. Lancien and D. Pérez-Garcia, Correlation length in random MPS and PEPS, in *Annales Henri Poincaré* (Springer, New York, 2022), Vol. 23, pp. 141–222.
- [48] D. Haag, F. Baccari, and G. Styliaris, Typical correlation length of sequentially generated tensor network states, *PRX Quantum* **4**, 030330 (2023).
- [49] G. Evenbly and G. Vidal, Algorithms for entanglement renormalization, *Phys. Rev. B* **79**, 144108 (2009).
- [50] S.-H. Lin, R. Dilip, A. G. Green, A. Smith, and F. Pollmann, Real-and imaginary-time evolution with compressed quantum circuits, *PRX Quantum* **2**, 010342 (2021).
- [51] R. Raussendorf, S. Bravyi, and J. Harrington, Long-range quantum entanglement in noisy cluster states, *Phys. Rev. A* **71**, 062313 (2005).
- [52] M. Aguado, G. K. Brennen, F. Verstraete, and J. I. Cirac, Creation, manipulation, and detection of abelian and non-abelian anyons in optical lattices, *Phys. Rev. Lett.* **101**, 260501 (2008).
- [53] L. Piroli and J. I. Cirac, Quantum cellular automata, tensor networks, and area laws, *Phys. Rev. Lett.* **125**, 190402 (2020).
- [54] N. Tantivasadakarn, R. Thorngren, A. Vishwanath, and R. Verresen, Long-range entanglement from measuring symmetry-protected topological phases, [arXiv:2112.01519](https://arxiv.org/abs/2112.01519).
- [55] S. Bravyi, I. Kim, A. Kliesch, and R. Koenig, Adaptive constant-depth circuits for manipulating non-Abelian anyons, [arXiv:2205.01933](https://arxiv.org/abs/2205.01933).
- [56] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, *Phys. Rev. Lett.* **70**, 1895 (1993).
- [57] D. Gottesman and I. L. Chuang, Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations, *Nature (London)* **402**, 390 (1999).
- [58] M. A. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2002).

- [59] R. Verresen, N. Tantivasadakarn, and A. Vishwanath, Efficiently preparing Schrödinger’s cat, fractons and non-Abelian topological order in quantum devices, [arXiv:2112.03061](https://arxiv.org/abs/2112.03061).
- [60] T.J. Osborne, Efficient approximation of the dynamics of one-dimensional quantum spin systems, *Phys. Rev. Lett.* **97**, 157202 (2006).
- [61] J. Haah, M.B. Hastings, R. Kothari, and G.H. Low, Quantum algorithm for simulating real time evolution of lattice Hamiltonians, *SIAM J. Comput.* **52**, FOCS18 (2021).
- [62] A. Coser and D. Pérez-García, Classification of phases for mixed states via fast dissipative evolution, *Quantum* **3**, 174 (2019).
- [63] M. Cramer, M.B. Plenio, S.T. Flammia, R. Somma, D. Gross, S.D. Bartlett, O. Landon-Cardinal, D. Poulin, and Y.K. Liu, Efficient quantum state tomography, *Nat. Commun.* **1**, 147 (2010).
- [64] R. Haghshenas, J. Gray, A.C. Potter, and G.K.-L. Chan, Variational power of quantum circuit tensor networks, *Phys. Rev. X* **12**, 011047 (2022).
- [65] S.-J. Ran, Encoding of matrix product states into quantum circuits of one- and two-qubit gates, *Phys. Rev. A* **101**, 032310 (2020).
- [66] R. Dilip, Y.-J. Liu, A. Smith, and F. Pollmann, Data compression for quantum machine learning, *Phys. Rev. Res.* **4**, 043007 (2022).
- [67] M.S. Rudolph, J. Miller, J. Chen, A. Acharya, and A. Perdomo-Ortiz, Synergy between quantum circuits and tensor networks: Short-cutting the race to practical quantum advantage, [arXiv:2208.13673](https://arxiv.org/abs/2208.13673).
- [68] M. Fishman, S. White, and E. Stoudenmire, The itensor software library for tensor network calculations, *SciPost Phys. Codebases* 004 (2022).